12.2 Differentiation, Integration (Vector-valued Functions)

p.846 11-62 ETQ

* $r(t) = f(t)i + g(t)j \quad r'(t) = f'(t)i + g'(t)j$

14) $r(t) = 4\sqrt{t}\,i + t^3\sqrt{t}\,j + \ln t^2 k$

$$r'(t) = \frac{2}{\sqrt{t}}i + \left(2t\sqrt{t} + \frac{t^2}{2\sqrt{t}}\right)j + \frac{2}{t}k$$

$$= \boxed{\frac{2}{\sqrt{t}}i + \frac{5t^{3/2}}{2}j + \frac{2}{t}k}$$

20) Find a) $r''(t)$ b) $r'(t) \cdot r''(t)$

$r(t) = (t^2+t)i + (t^2-t)j$

$r'(t) = (2t+1)i + (2t-1)j$

$r''(t) = 2i + 2j$

b) $r'(t) \cdot r''(t) =$

$(2t+1)(2) + (2t-1)(2) = \boxed{8t}$

26) $r(t) = \langle e^{-t}, t^2, \tan(t) \rangle$

$r'(t) = \langle -e^{-t}, 2t, \sec^2 t \rangle$

$r''(t) = \langle e^{-t}, 2, 2\sec^2 t \tan t \rangle$

b) $\boxed{r'(t) \cdot r''(t) = -e^{-2t} + 4t + 2\sec^4 t \tan t}$

32) Find open intervals where curve is smooth.

$r(\theta) = (\theta + \sin\theta)i + (1 - \cos\theta)j$

$r'(\theta) = (1 + \cos\theta)i + \sin\theta j$

$r'[(2n-1)\pi] = 0$, n being any integer

smooth on $((2n-1)\pi, (2n+1)\pi)$

42) Find (a) $D_t[r(t) \cdot u(t)]$ and (b) $D_t[r(t) \times u(t)]$

$r(t) = \cos t\, i + \sin t\, j + tk$, $u(t) = j + tk$

a) $r(t) \cdot u(t) = r(t) \cdot u'(t) + r'(t) \cdot u(t)$

$= (\cos t\, i + \sin t\, j + tk)(1k) + (-\sin t\, i + \cos t\, j + 1k)(j + tk)$

$= t + \cos t + t = \boxed{2t + \cos t}$

48) $r(t) = \langle 0, \sin t, 4t \rangle$ * Find $r'(t)$ using definition of derivative

$$r'(t) = \lim_{\Delta t \to 0} \frac{r(t+\Delta t) - r(t)}{\Delta t} = \frac{\langle 0, \sin(t+\Delta t), 4(t+\Delta t) \rangle - \langle 0, \sin t, 4t \rangle}{\Delta t}$$

* $\sin(x+y) = \sin x \cos y + \cos x \sin y$

$$= \frac{\langle 0, \sin t \cos \Delta t + \cos t \sin \Delta t - \sin t, 4\Delta t \rangle}{\Delta t}$$

$\lim_{\Delta t \to 0}$

$\langle 0, \cos t, 4 \rangle$

54) $\int [e^t i + \sin t\, j + \cos t\, k]\, dt = \boxed{e^t i - \cos t\, j + \sin t\, k + c}$

60) $\int_0^{\pi/4} \sec t \tan t\, i + \tan t\, j + 2\sin t \cos t\, k\, dt$

$= \sec t\, i + \ln|\sec t|\, j + \sin^2 t\, k \Big]_0^{\pi/4} = (\sqrt{2} - 1) i + \ln\sqrt{2}\, j + \frac{1}{2} k$