89. True

90. False. The graph of $x = y = z = t^3$ represents a line.

91. False. Although $\mathbf{r}(4) = \langle 4, 16\rangle = \mathbf{u}(2)$, they do not collide. Their paths cross this point at different times.

92. True. $y^2 + z^2 = t^2\sin^2 t + t^2\cos^2 t = t^2 = x$

Section 12.2 Differentiation and Integration of Vector-Valued Functions

1. $\mathbf{r}(t) = t^2\mathbf{i} + t\mathbf{j},\ t_0 = 2$

$x(t) = t^2,\ y(t) = t$

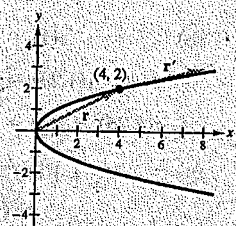
$x = y^2$

$\mathbf{r}(2) = 4\mathbf{i} + 2\mathbf{j}$

$\mathbf{r}'(t) = 2t\mathbf{i} + \mathbf{j}$

$\mathbf{r}'(2) = 4\mathbf{i} + \mathbf{j}$

$\mathbf{r}'(t_0)$ is tangent to the curve.



2. $\mathbf{r}(t) = t\mathbf{i} + t^3\mathbf{j},\ t_0 = 1$

$x(t) = t,\ y(t) = t^3$

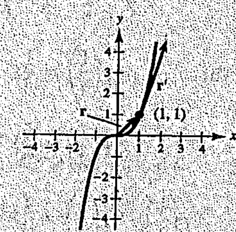
$y = x^3$

$\mathbf{r}(1) = \mathbf{i} + \mathbf{j}$

$\mathbf{r}'(t) = \mathbf{i} + 3t^2\mathbf{j}$

$\mathbf{r}'(1) = \mathbf{i} + 3\mathbf{j}$

$\mathbf{r}'(t_0)$ is tangent to the curve.



3. $\mathbf{r}(t) = t^2\mathbf{i} + \dfrac{1}{t}\mathbf{j},\ t_0 = 2$

$x(t) = t^2,\ y(t) = \dfrac{1}{t}$

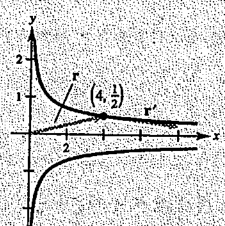
$x = \dfrac{1}{y^2}$

$\mathbf{r}(2) = 4\mathbf{i} + \dfrac{1}{2}\mathbf{j}$

$\mathbf{r}'(t) = 2t\mathbf{i} - \dfrac{1}{t^2}\mathbf{j}$

$\mathbf{r}'(2) = 4\mathbf{i} - \dfrac{1}{4}\mathbf{j}$

$\mathbf{r}'(t_0)$ is tangent to the curve.



4. (a) $\mathbf{r}(t) = (1 + t)\mathbf{i} + t^3\mathbf{j},\ t_0 = 1$

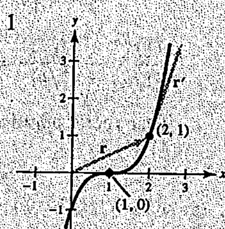
$x = 1 + t$
$y = t^3 = (x - 1)^3$

(b) $\mathbf{r}(1) = 2\mathbf{i} + \mathbf{j}$

$\mathbf{r}'(t) = \mathbf{i} + 3t^2\mathbf{j}$

$\mathbf{r}'(1) = \mathbf{i} + 3\mathbf{j}$

$\mathbf{r}'(1)$ is tangent to the curve.



5. $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j},\ t_0 = \dfrac{\pi}{2}$

$x(t) = \cos t,\ y(t) = \sin t$

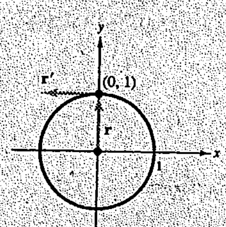
$x^2 + y^2 = 1$

$\mathbf{r}\left(\dfrac{\pi}{2}\right) = \mathbf{j}$

$\mathbf{r}'(t) = -\sin t\mathbf{i} + \cos t\mathbf{j}$

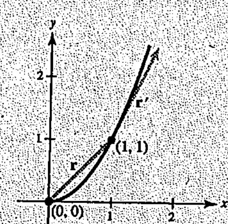
$\mathbf{r}'\left(\dfrac{\pi}{2}\right) = -\mathbf{i}$

$\mathbf{r}'(t_0)$ is tangent to the curve.



6. (a) $\mathbf{r}(t) = e^t\mathbf{i} + e^{2t}\mathbf{j},\ t_0 = 0$

$x = e^t, y = e^{2t} = x^2 \implies y = x^2, x > 0$



(b) $\mathbf{r}(0) = \mathbf{i} + \mathbf{j}$

$\mathbf{r}'(t) = e^t\mathbf{i} + 2e^{2t}\mathbf{j}$

$\mathbf{r}'(0) = \mathbf{i} + 2\mathbf{j}$

$\mathbf{r}'(0)$ is tangent to the curve.

55. $\mathbf{r}_1(t) = t\mathbf{i}, \quad 0 \le t \le 4 \quad (\mathbf{r}_1(0) = \mathbf{0}, \mathbf{r}_1(4) = 4\mathbf{i})$

$\mathbf{r}_2(t) = (4 - 4t)\mathbf{i} + 6t\mathbf{j}, \quad 0 \le t \le 1 \quad (\mathbf{r}_2(0) = 4\mathbf{i}, \mathbf{r}_2(1) = 6\mathbf{j})$

$\mathbf{r}_3(t) = (6 - t)\mathbf{j}, \quad 0 \le t \le 6 \quad (\mathbf{r}_3(0) = 6\mathbf{j}, \mathbf{r}_3(6) = \mathbf{0})$

(Other answers possible)

56. $\mathbf{r}_1(t) = t\mathbf{i}, \quad 0 \le t \le 10 \quad (\mathbf{r}_1(0) = \mathbf{0}, \mathbf{r}_1(10) = 10\mathbf{i})$

$\mathbf{r}_2(t) = 10(\cos t\mathbf{i} + \sin t\mathbf{j}), \quad 0 \le t \le \frac{\pi}{4} \quad \left(\mathbf{r}_2(0) = 10\mathbf{i}, \mathbf{r}_2\left(\frac{\pi}{4}\right) = 5\sqrt{2}\mathbf{i} + 5\sqrt{2}\mathbf{j}\right)$

$\mathbf{r}_3(t) = 5\sqrt{2}(1 - t)\mathbf{i} + 5\sqrt{2}(1 - t)\mathbf{j}, \quad 0 \le t \le 1 \quad \left(\mathbf{r}_3(0) = 5\sqrt{2}\mathbf{i} + 5\sqrt{2}\mathbf{j}, \mathbf{r}_3(1)\right) = \mathbf{0}$

(Other answers possible)

57. $\mathbf{r}_1(t) = t\mathbf{i} + t^2\mathbf{j}, \quad 0 \le t \le 2 \quad (y = x^2)$

$\mathbf{r}_2(t) = (2 - t)\mathbf{i} + 4\mathbf{j}, \quad 0 \le t \le 2$

$\mathbf{r}_3(t) = (4 - t)\mathbf{j}, \quad 0 \le t \le 4$

(Other answers possible)

58. $\mathbf{r}_1(t) = t\mathbf{i} + \sqrt{t}\mathbf{j}, \quad 0 \le t \le 1 \quad (y = \sqrt{x})$

$\mathbf{r}_2(t) = (1 - t)\mathbf{i} + (1 - t)\mathbf{j}, \quad 0 \le t \le 1 \quad (y = x)$

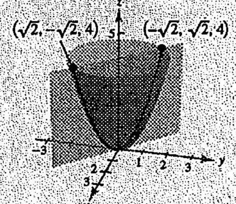
(Other answers possible)

59. $z = x^2 + y^2, \; x + y = 0$

Let $x = t$, then $y = -x = -t$ and $z = x^2 + y^2 = 2t^2$. Therefore,

$x = t, \; y = -t, \; z = 2t^2.$

$\mathbf{r}(t) = t\mathbf{i} - t\mathbf{j} + 2t^2\mathbf{k}$

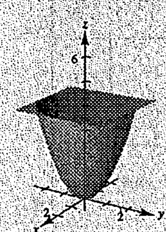


60. $z = x^2 + y^2, \; z = 4$

Therefore, $x^2 + y^2 = 4$ or

$x = 2\cos t, y = 2\sin t, z = 4.$

$\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j} + 4\mathbf{k}$



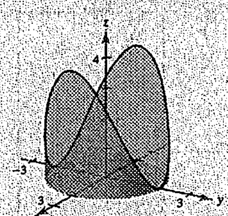
61. $x^2 + y^2 = 4, \; z = x^2$

$x = 2\sin t, \; y = 2\cos t$

$z = x^2 = 4\sin^2 t$

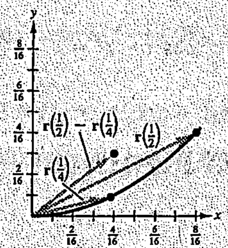
t	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
x	0	1	$\sqrt{2}$	2	$\sqrt{2}$	0
y	2	$\sqrt{3}$	$\sqrt{2}$	0	$-\sqrt{2}$	-2
z	0	1	2	4	2	0

$\mathbf{r}(t) = 2\sin t\mathbf{i} + 2\cos t\mathbf{j} + 4\sin^2 t\mathbf{k}$



7. $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j}$

(a)



(b) $$\mathbf{r}\left(\frac{1}{4}\right) = \frac{1}{4}\mathbf{i} + \frac{1}{16}\mathbf{j}$$

$$\mathbf{r}\left(\frac{1}{2}\right) = \frac{1}{2}\mathbf{i} + \frac{1}{4}\mathbf{j}$$

$$\mathbf{r}\left(\frac{1}{2}\right) - \mathbf{r}\left(\frac{1}{4}\right) = \frac{1}{4}\mathbf{i} + \frac{3}{16}\mathbf{j}$$

(c) $$\mathbf{r}'(t) = \mathbf{i} + 2t\mathbf{j}$$

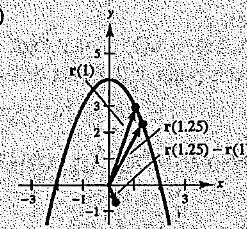
$$\mathbf{r}'\left(\frac{1}{4}\right) = \mathbf{i} + \frac{1}{2}\mathbf{j}$$

$$\frac{\mathbf{r}(1/2) - \mathbf{r}(1/4)}{(1/2) - (1/4)} = \frac{(1/4)\mathbf{i} + (3/16)\mathbf{j}}{1/4} = \mathbf{i} + \frac{3}{4}\mathbf{j}$$

This vector approximates $\mathbf{r}'\left(\frac{1}{4}\right)$.

8. $\mathbf{r}(t) = t\mathbf{i} + (4 - t^2)\mathbf{j}$

(a)



(b) $$\mathbf{r}(1) = \mathbf{i} + 3\mathbf{j}$$

$$\mathbf{r}(1.25) = 1.25\mathbf{i} + 2.4375\mathbf{j}$$

$$\mathbf{r}(1.25) - \mathbf{r}(1) = 0.25\mathbf{i} - 0.5625\mathbf{j}$$

(c) $$\mathbf{r}'(t) = \mathbf{i} - 2t\mathbf{j}$$

$$\mathbf{r}'(1) = \mathbf{i} - 2\mathbf{j}$$

$$\frac{\mathbf{r}(1.25) - \mathbf{r}(1)}{1.25 - 1} = \frac{0.25\mathbf{i} - 0.5625\mathbf{j}}{0.25} = \mathbf{i} - 2.25\mathbf{j}$$

This vector approximates $\mathbf{r}'(1)$.

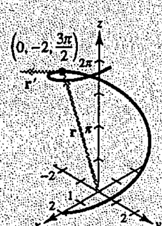
9. (a) and (b) $\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j} + t\mathbf{k}$, $t_0 = \dfrac{3\pi}{2}$

$x^2 + y^2 = 4$, $z = t$

$$\mathbf{r}'(t) = -2\sin t\mathbf{i} + 2\cos t\mathbf{j} + \mathbf{k}$$

$$\mathbf{r}\left(\frac{3\pi}{2}\right) = -2\mathbf{j} + \frac{3\pi}{2}\mathbf{k}$$

$$\mathbf{r}'\left(\frac{3\pi}{2}\right) = 2\mathbf{i} + \mathbf{k}$$



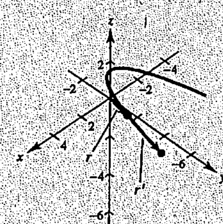
10. $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + \frac{3}{2}\mathbf{k}$, $t_0 = 2$

$y = x^2$, $z = \dfrac{3}{2}$

$$\mathbf{r}'(t) = \mathbf{i} + 2t\mathbf{j}$$

$$\mathbf{r}(2) = 2\mathbf{i} + 4\mathbf{j} + \frac{3}{2}\mathbf{k}$$

$$\mathbf{r}'(2) = \mathbf{i} + 4\mathbf{j}$$



11. $\mathbf{r}(t) = 6t\mathbf{i} - 7t^2\mathbf{j} + t^3\mathbf{k}$

$\mathbf{r}'(t) = 6\mathbf{i} - 14t\mathbf{j} + 3t^2\mathbf{k}$

12. $\mathbf{r}(t) = \dfrac{1}{t}\mathbf{i} + 16t\mathbf{j} + \dfrac{t^2}{2}\mathbf{k}$

$$\mathbf{r}'(t) = -\frac{1}{t^2}\mathbf{i} + 16\mathbf{j} + t\mathbf{k}$$

13. $\mathbf{r}(t) = a\cos^3 t\mathbf{i} + a\sin^3 t\mathbf{j} + \mathbf{k}$

$\mathbf{r}'(t) = -3a\cos^2 t\sin t\mathbf{i} + 3a\sin^2 t\cos t\mathbf{j}$

14. $\mathbf{r}(t) = 4\sqrt{t}\mathbf{i} + t^2\sqrt{t}\mathbf{j} + \ln t^2\mathbf{k}$

$$\mathbf{r}'(t) = \frac{2}{\sqrt{t}}\mathbf{i} + \left(2t\sqrt{t} + \frac{t^2}{2\sqrt{t}}\right)\mathbf{j} + \frac{2}{t}\mathbf{k}$$

$$= \frac{2}{\sqrt{t}}\mathbf{i} + \frac{5t^{3/2}}{2}\mathbf{j} + \frac{2}{t}\mathbf{k}$$

15. $\mathbf{r}(t) = e^{-t}\mathbf{i} + 4\mathbf{j}$

$\mathbf{r}'(t) = -e^{-t}\mathbf{i}$

16. $\mathbf{r}(t) = \langle \sin t - t\cos t, \cos t + t\sin t, t^2\rangle$

$\mathbf{r}'(t) = \langle t\sin t, t\cos t, 2t\rangle$

17. $\mathbf{r}(t) = \langle t\sin t, t\cos t, t\rangle$

$\mathbf{r}'(t) = \langle \sin t + t\cos t, \cos t - t\sin t, 1\rangle$

18. $\mathbf{r}(t) = \langle \arcsin t, \arccos t, 0\rangle$

$$\mathbf{r}'(t) = \left\langle \frac{1}{\sqrt{1-t^2}}, -\frac{1}{\sqrt{1-t^2}}, 0\right\rangle$$

19. $\mathbf{r}(t) = t^3\mathbf{i} + \frac{1}{2}t^2\mathbf{j}$

(a) $\mathbf{r}'(t) = 3t^2\mathbf{i} + t\mathbf{j}$

$\mathbf{r}''(t) = 6t\mathbf{i} + \mathbf{j}$

(b) $\mathbf{r}'(t) \cdot \mathbf{r}''(t) = 3t^2(6t) + t = 18t^3 + t$

20. $\mathbf{r}(t) = (t^2 + t)\mathbf{i} + (t^2 - t)\mathbf{j}$

(a) $\mathbf{r}'(t) = (2t + 1)\mathbf{i} + (2t - 1)\mathbf{j}$

$\mathbf{r}''(t) = 2\mathbf{i} + 2\mathbf{j}$

(b) $\mathbf{r}'(t) \cdot \mathbf{r}''(t) = (2t + 1)(2) + (2t - 1)(2) = 8t$

21. $\mathbf{r}(t) = 4\cos t\mathbf{i} + 4\sin t\mathbf{j}$

(a) $\mathbf{r}'(t) = -4\sin t\mathbf{i} + 4\cos t\mathbf{j}$

$\mathbf{r}''(t) = -4\cos t\mathbf{i} - 4\sin t\mathbf{j}$

(b) $\mathbf{r}'(t) \cdot \mathbf{r}''(t) = (-4\sin t)(-4\cos t) + 4\cos t(-4\sin t)$

$= 0$

22. $\mathbf{r}(t) = 8\cos t\mathbf{i} + 3\sin t\mathbf{j}$

(a) $\mathbf{r}'(t) = -8\sin t\mathbf{i} + 3\cos t\mathbf{j}$

$\mathbf{r}''(t) = -8\cos t\mathbf{i} - 3\sin t\mathbf{j}$

(b) $\mathbf{r}'(t) \cdot \mathbf{r}''(t) = (-8\sin t)(-8\cos t) + 3\cos t(-3\sin t)$

$= 55\sin t\cos t$

23. $\mathbf{r}(t) = \frac{1}{2}t^2\mathbf{i} - t\mathbf{j} + \frac{1}{6}t^3\mathbf{k}$

(a) $\mathbf{r}'(t) = t\mathbf{i} - \mathbf{j} + \frac{1}{2}t^2\mathbf{k}$

$\mathbf{r}''(t) = \mathbf{i} + t\mathbf{k}$

(b) $\mathbf{r}'(t) \cdot \mathbf{r}''(t) = t(1) - 1(0) + \frac{1}{2}t^2(t) = t + \frac{t^3}{2}$

24. $\mathbf{r}(t) = t\mathbf{i} + (2t + 3)\mathbf{j} + (3t - 5)\mathbf{k}$

(a) $\mathbf{r}'(t) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$

$\mathbf{r}''(t) = 0$

(b) $\mathbf{r}'(t) \cdot \mathbf{r}''(t) = 0$

25. $\mathbf{r}(t) = \langle \cos t + t\sin t, \sin t - t\cos t, t\rangle$

(a) $\mathbf{r}'(t) = \langle -\sin t + \sin t + t\cos t, \cos t - \cos t + t\sin t, 1\rangle$

$= \langle t\cos t, t\sin t, 1\rangle$

$\mathbf{r}''(t) = \langle \cos t - t\sin t, \sin t + t\cos t, 0\rangle$

(b) $\mathbf{r}'(t) \cdot \mathbf{r}''(t) = (t\cos t)(\cos t - t\sin t) + (t\sin t)(\sin t + t\cos t) = t$

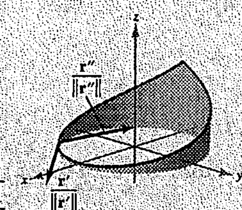
26. $\mathbf{r}(t) = \langle e^{-t}, t^2, \tan(t)\rangle$

(a) $\mathbf{r}'(t) = \langle -e^{-t}, 2t, \sec^2 t\rangle$

$\mathbf{r}''(t) = \langle e^{-t}, 2, 2\sec^2 t\tan t\rangle$

(b) $\mathbf{r}'(t) \cdot \mathbf{r}''(t) = -e^{-2t} + 4t + 2\sec^4 t\tan t$

27. $$\mathbf{r}(t) = \cos(\pi t)\mathbf{i} + \sin(\pi t)\mathbf{j} + t^2\mathbf{k},\ t_0 = -\frac{1}{4}$$



$$\mathbf{r}'(t) = -\pi\sin(\pi t)\mathbf{i} + \pi\cos(\pi t)\mathbf{j} + 2t\mathbf{k}$$

$$\mathbf{r}'\left(-\frac{1}{4}\right) = \frac{\sqrt{2}\pi}{2}\mathbf{i} + \frac{\sqrt{2}\pi}{2}\mathbf{j} - \frac{1}{2}\mathbf{k}$$

$$\left\|\mathbf{r}'\left(\frac{1}{4}\right)\right\| = \sqrt{\left(\frac{\sqrt{2}\pi}{2}\right)^2 + \left(\frac{\sqrt{2}\pi}{2}\right)^2 + \left(-\frac{1}{2}\right)^2} = \sqrt{\pi^2 + \frac{1}{4}} = \frac{\sqrt{4\pi^2+1}}{2}$$

$$\frac{\mathbf{r}'(-1/4)}{\|\mathbf{r}'(-1/4)\|} = \frac{1}{\sqrt{4\pi^2+1}}\left(\sqrt{2}\pi\mathbf{i} + \sqrt{2}\pi\mathbf{j} - \mathbf{k}\right)$$

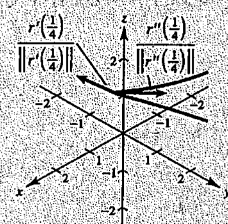
$$\mathbf{r}''(t) = -\pi^2\cos(\pi t)\mathbf{i} - \pi^2\sin(\pi t)\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{r}''\left(-\frac{1}{4}\right) = -\frac{\sqrt{2}\pi^2}{2}\mathbf{i} + \frac{\sqrt{2}\pi^2}{2}\mathbf{j} + 2\mathbf{k}$$

$$\left\|\mathbf{r}''\left(-\frac{1}{4}\right)\right\| = \sqrt{\left(-\frac{\sqrt{2}\pi^2}{2}\right)^2 + \left(\frac{\sqrt{2}\pi^2}{2}\right)^2 + (2)^2} = \sqrt{\pi^4+4}$$

$$\frac{\mathbf{r}''(-1/4)}{\|\mathbf{r}'(-1/4)\|} = \frac{1}{2\sqrt{\pi^4+4}}\left(-\sqrt{2}\pi^2\mathbf{i} + \sqrt{2}\pi^2\mathbf{j} + 4\mathbf{k}\right)$$

28. $$\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + e^{0.75t}\mathbf{k},\ t_0 = \frac{1}{4}$$



$$\mathbf{r}'(t) = \mathbf{i} + 2t\mathbf{j} + 0.75e^{0.75t}\mathbf{k}$$

$$\mathbf{r}'\left(\frac{1}{4}\right) = \mathbf{i} + \frac{1}{2}\mathbf{j} + 0.75e^{0.1875}\mathbf{k} = \mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{3}{4}e^{3/16}\mathbf{k}$$

$$\left\|\mathbf{r}'\left(\frac{1}{4}\right)\right\| = \sqrt{1^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{3}{4}e^{3/16}\right)^2} = \sqrt{\frac{5}{4} + \frac{9}{16}e^{3/8}} = \frac{\sqrt{20+9e^{3/8}}}{4}$$

$$\frac{\mathbf{r}'(1/4)}{\|\mathbf{r}'(1/4)\|} = \frac{1}{\sqrt{20+9e^{3/8}}}(4\mathbf{i} + 2\mathbf{j} + 3e^{3/16}\mathbf{k})$$

$$\mathbf{r}''(t) = 2\mathbf{i} + \frac{9}{16}e^{0.75t}\mathbf{k}$$

$$\mathbf{r}''\left(\frac{1}{4}\right) = 2\mathbf{i} + \frac{9}{16}e^{3/16k}$$

$$\left\|\mathbf{r}''\left(\frac{1}{4}\right)\right\| = \sqrt{2^2 + \left(\frac{9}{16}e^{3/16}\right)^2} = \sqrt{4 + \frac{81}{256}e^{3/8}} = \frac{\sqrt{1024+81e^{3/8}}}{16}$$

$$\frac{\mathbf{r}''(1/4)}{\|\mathbf{r}'(1/4)\|} = \frac{1}{\sqrt{1024+81e^{3/8}}}(32\mathbf{j} + 9e^{3/16}\mathbf{k})$$

29. $\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j}$

$\mathbf{r}'(t) = 2t\mathbf{i} + 3t^2\mathbf{j}$

$\mathbf{r}'(0) = \mathbf{0}$

Smooth on $(-\infty, 0)$, $(0, \infty)$

30. $\mathbf{r}(t) = \frac{1}{t-1}\mathbf{i} + 3t\mathbf{j}$

$\mathbf{r}'(t) = -\frac{1}{(t-1)^2}\mathbf{i} + 3\mathbf{j}$

Not continuous when $t = 1$

Smooth on $(-\infty, 1)$, $(1, \infty)$

31. $\mathbf{r}(\theta) = 2\cos^3\theta\mathbf{i} + 3\sin^3\theta\mathbf{j}$

$\mathbf{r}'(\theta) = -6\cos^2\theta\sin\theta\mathbf{i} + 9\sin^2\theta\cos\theta\mathbf{j}$

$\mathbf{r}'\left(\frac{n\pi}{2}\right) = \mathbf{0}$

Smooth on $\left(\frac{n\pi}{2}, \frac{(n+1)\pi}{2}\right)$, n any integer.

32. $\mathbf{r}(\theta) = (\theta + \sin\theta)\mathbf{i} + (1 - \cos\theta)\mathbf{j}$

$\mathbf{r}'(\theta) = (1 + \cos\theta)\mathbf{i} + \sin\theta\mathbf{j}$

$\mathbf{r}'((2n-1)\pi) = \mathbf{0}$, n any integer

Smooth on $((2n-1)\pi, (2n+1)\pi)$

33. $\mathbf{r}(\theta) = (\theta - 2\sin\theta)\mathbf{i} + (1 - 2\cos\theta)\mathbf{j}$

$\mathbf{r}'(\theta) = (1 - 2\cos\theta)\mathbf{i} + (2\sin\theta)\mathbf{j}$

$\mathbf{r}'(\theta) \neq \mathbf{0}$ for any value of θ

Smooth on $(-\infty, \infty)$

34. $\mathbf{r}(t) = \frac{2t}{8+t^3}\mathbf{i} + \frac{2t^2}{8+t^3}\mathbf{j}$

$\mathbf{r}'(t) = \frac{16-4t^3}{(t^3+8)^2}\mathbf{i} + \frac{32t-2t^4}{(t^3+8)^2}\mathbf{j}$

$\mathbf{r}'(t) \neq \mathbf{0}$ for any value of t.

$\mathbf{r}$ is not continuous when $t = -2$.

Smooth on $(-\infty, -2), (-2, \infty)$

35. $\mathbf{r}(t) = (t-1)\mathbf{i} + \frac{1}{t}\mathbf{j} - t^2\mathbf{k}$

$\mathbf{r}'(t) = \mathbf{i} - \frac{1}{t^2}\mathbf{j} - 2t\mathbf{k} \neq \mathbf{0}$

$\mathbf{r}$ is smooth for all $t \neq 0$: $(-\infty, 0), (0, \infty)$

36. $\mathbf{r}(t) = e^t\mathbf{i} - e^{-t}\mathbf{j} + 3t\mathbf{k}$

$\mathbf{r}'(t) = e^t\mathbf{i} + e^{-t}\mathbf{j} + 3\mathbf{k} \neq \mathbf{0}$

$\mathbf{r}$ is smooth for all t: $(-\infty, \infty)$

37. $\mathbf{r}(t) = t\mathbf{i} - 3t\mathbf{j} + \tan t\mathbf{k}$

$\mathbf{r}'(t) = \mathbf{i} - 3\mathbf{j} + \sec^2 t\mathbf{k} \neq \mathbf{0}$

$\mathbf{r}$ is smooth for all $t \neq \frac{\pi}{2} + n\pi = \frac{2n+1}{2}\pi$.

Smooth on intervals of form $\left(-\frac{\pi}{2} + n\pi, \frac{\pi}{2} + n\pi\right)$

38. $\mathbf{r}(t) = \sqrt{t}\mathbf{i} + (t^2-1)\mathbf{j} + \frac{1}{4}t\mathbf{k}$

$\mathbf{r}'(t) = \frac{1}{2\sqrt{t}}\mathbf{i} + 2t\mathbf{j} + \frac{1}{4}\mathbf{k} \neq \mathbf{0}$

$\mathbf{r}$ is smooth for all $t > 0$: $(0, \infty)$

39. $\mathbf{r}(t) = t\mathbf{i} + 3t\mathbf{j} + t^2\mathbf{k}$, $\mathbf{u}(t) = 4t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$

(a) $\mathbf{r}'(t) = \mathbf{i} + 3\mathbf{j} + 2t\mathbf{k}$

(b) $\mathbf{r}''(t) = 2\mathbf{k}$

(c) $\mathbf{r}(t) \cdot \mathbf{u}(t) = 4t^2 + 3t^3 + t^5$

$D_t[\mathbf{r}(t) \cdot \mathbf{u}(t)] = 8t + 9t^2 + 5t^4$

(d) $3\mathbf{r}(t) - \mathbf{u}(t) = -t\mathbf{i} + (9t - t^2)\mathbf{j} + (3t^2 - t^3)\mathbf{k}$

$D_t[3\mathbf{r}(t) - \mathbf{u}(t)] = -\mathbf{i} + (9 - 2t)\mathbf{j} + (6t - 3t^2)\mathbf{k}$

(e) $\mathbf{r}(t) \times \mathbf{u}(t) = 2t^4\mathbf{i} - (t^4 - 4t^3)\mathbf{j} + (t^3 - 12t^2)\mathbf{k}$

$D_t[\mathbf{r}(t) \times \mathbf{u}(t)] = 8t^3\mathbf{i} + (12t^2 - 4t^3)\mathbf{j} + (3t^2 - 24t)\mathbf{k}$

(f) $\|\mathbf{r}(t)\| = \sqrt{10t^2 + t^4} = t\sqrt{10 + t^2}$

$D_t[\|\mathbf{r}(t)\|] = \frac{10 + 2t^2}{\sqrt{10 + t^2}}$

40. $\mathbf{r}(t) = t\mathbf{i} + 2\sin t\mathbf{j} + 2\cos t\mathbf{k}$

$\mathbf{u}(t) = \frac{1}{t}\mathbf{i} + 2\sin t\mathbf{j} + 2\cos t\mathbf{k}$

(a) $\mathbf{r}'(t) = \mathbf{i} + 2\cos t\mathbf{j} - 2\sin t\mathbf{k}$

(b) $\mathbf{r}''(t) = -2\sin t\mathbf{j} - 2\cos t\mathbf{k}$

(c) $\mathbf{r}(t) \cdot \mathbf{u}(t) = 1 + 4\sin^2 t + 4\cos^2 t = 5$

$D_t[\mathbf{r}(t) \cdot \mathbf{u}(t)] = 0, t \neq 0$

(d) $3\mathbf{r}(t) - \mathbf{u}(t) = \left(3t - \frac{1}{t}\right)\mathbf{i} + 4\sin t\mathbf{j} + 4\cos t\mathbf{k}$

$D_t[3\mathbf{r}(t) - \mathbf{u}(t)] = \left(3 - \frac{1}{t^2}\right)\mathbf{i} + 4\cos t\mathbf{j} - 4\sin t\mathbf{k}$

—CONTINUED—

40. —CONTINUED—

(e) $\mathbf{r}(t) \times \mathbf{u}(t) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ t & 2\sin t & 2\cos t \\ 1/t & 2\sin t & 2\cos t \end{vmatrix}$

$= 2\cos t\left(\frac{1}{t} - t\right)\mathbf{j} + 2\sin t\left(t - \frac{1}{t}\right)\mathbf{k}$

$D_t[\mathbf{r}(t) - \mathbf{u}(t)] = \left[-2\sin t\left(\frac{1}{t} - t\right) + 2\cos t\left(-\frac{1}{t^2} - 1\right)\right]\mathbf{j}$

$+ \left[2\cos t\left(t - \frac{1}{t}\right) + 2\sin t\left(1 + \frac{1}{t^2}\right)\right]\mathbf{k}$

(f) $\|\mathbf{r}(t)\| = \sqrt{t^2 + 4}$

$D_t(\|\mathbf{r}(t)\|) = \frac{1}{2}(t^2 + 4)^{-1/2}(2t) = \frac{t}{\sqrt{t^2 + 4}}$

41. $\mathbf{r}(t) = t\mathbf{i} + 2t^2\mathbf{j} + t^3\mathbf{k}$, $\mathbf{u}(t) = t^4\mathbf{k}$

(a) $\mathbf{r}(t) \cdot \mathbf{u}(t) = t^7$

$D_t[\mathbf{r}(t) \cdot \mathbf{u}(t)] = 7t^6$

Alternate Solution:

$D_t[\mathbf{r}(t) \cdot \mathbf{u}(t)] = \mathbf{r}(t) \cdot \mathbf{u}'(t) + \mathbf{r}'(t) \cdot \mathbf{u}(t)$

$= (t\mathbf{i} + 2t^2\mathbf{j} + t^3\mathbf{k}) \cdot (4t^3\mathbf{k}) + (\mathbf{i} + 4t\mathbf{j} + 3t^2\mathbf{k}) \cdot (t^4\mathbf{k})$

$= 4t^6 + 3t^6 = 7t^6$

(b) $\mathbf{r}(t) \times \mathbf{u}(t) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ t & 2t^2 & t^3 \\ 0 & 0 & t^4 \end{vmatrix} = 2t^6\mathbf{i} - t^5\mathbf{j}$

$D_t[\mathbf{r}(t) \times \mathbf{u}(t)] = 12t^5\mathbf{i} - 5t^4\mathbf{j}$

Alternate Solution:

$D_t[\mathbf{r}(t) \times \mathbf{u}(t)] = \mathbf{r}(t) \times \mathbf{u}'(t) + \mathbf{r}'(t) \times \mathbf{u}(t)$

$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ t & 2t^2 & t^3 \\ 0 & 0 & 4t^3 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 4t & 3t^2 \\ 0 & 0 & t^4 \end{vmatrix} = 12t^5\mathbf{i} - 5t^4\mathbf{j}$

42. $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}$, $\mathbf{u}(t) = \mathbf{j} + t\mathbf{k}$

(a) $\mathbf{r}(t) \cdot \mathbf{u}(t) = \sin t + t^2$

$D_t[\mathbf{r}(t) \cdot \mathbf{u}(t)] = \cos t + 2t$

Alternate Solution:

$D_t[\mathbf{r}(t) \cdot \mathbf{u}(t)] = \mathbf{r}(t) \cdot \mathbf{u}'(t) + \mathbf{r}'(t) \cdot \mathbf{u}(t)$

$= (\cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}) \cdot \mathbf{k} + (-\sin t\mathbf{i} + \cos t\mathbf{j} + \mathbf{k}) \cdot (\mathbf{j} + t\mathbf{k})$

$= t + \cos t + t = 2t + \cos t$

(b) $\mathbf{r}(t) \times \mathbf{u}(t) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos t & \sin t & t \\ 0 & 1 & t \end{vmatrix} = (t\sin t - t)\mathbf{i} - (t\cos t)\mathbf{j} + \cos t\mathbf{k}$

$D_t[\mathbf{r}(t) \times \mathbf{u}(t)] = (t\cos t + \sin t - 1)\mathbf{i} - (\cos t - t\sin t)\mathbf{j} - \sin t\mathbf{k}$

Alternate Solution:

$D_t[\mathbf{r}(t) \times \mathbf{u}(t)] = \mathbf{r}(t) \times \mathbf{u}'(t) + \mathbf{r}'(t) \times \mathbf{u}(t)$

$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos t & \sin t & t \\ 0 & 0 & 1 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -\sin t & \cos t & 1 \\ 0 & 1 & t \end{vmatrix} = (\sin t + t\cos t - 1)\mathbf{i} + (t\sin t - \cos t)\mathbf{j} - \sin t\mathbf{k}$

43.

$$\mathbf{r}(t) = 3\sin t\mathbf{i} + 4\cos t\mathbf{j}$$

$$\mathbf{r}'(t) = 3\cos t\mathbf{i} - 4\sin t\mathbf{j}$$

$$\mathbf{r}(t)\cdot\mathbf{r}'(t) = 9\sin t\cos t - 16\cos t\sin t = -7\sin t\cos t$$

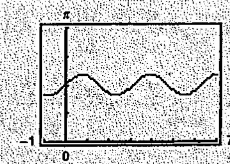
$$\cos\theta = \frac{\mathbf{r}(t)\cdot\mathbf{r}'(t)}{\|\mathbf{r}(t)\|\,\|\mathbf{r}'(t)\|} = \frac{-7\sin t\cos t}{\sqrt{9\sin^2 t + 16\cos^2 t}\sqrt{9\cos^2 t + 16\sin^2 t}}$$

$$\theta = \arccos\left[\frac{-7\sin t\cos t}{\sqrt{(9\sin^2 t + 16\cos^2 t)(9\cos^2 t + 16\sin^2 t)}}\right]$$

$\theta = 1.855$ maximum at $t = 3.927 = \left(\frac{5\pi}{4}\right)$ and $t = 0.785 = \left(\frac{\pi}{4}\right)$.

$\theta = 1.287$ minimum at $t = 2.356 = \left(\frac{3\pi}{4}\right)$ and $t = 5.498 = \left(\frac{7\pi}{4}\right)$.

$\theta = \frac{\pi}{2} = (1.571)$ for $t = \frac{n\pi}{2}$, $n = 0, 1, 2, 3, \ldots$



44.

$$\mathbf{r}(t) = t^2\mathbf{i} + t\mathbf{j}$$

$$\mathbf{r}'(t) = 2t\mathbf{i} + \mathbf{j}$$

$$\mathbf{r}(t)\cdot\mathbf{r}'(t) = 2t^3 + t$$

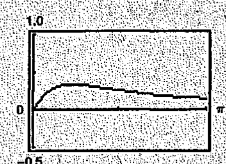
$$\|\mathbf{r}(t)\| = \sqrt{t^4 + t^2},\ \|\mathbf{r}'(t)\| = \sqrt{4t^2 + 1}$$

$$\cos\theta = \frac{2t^3 + t}{\sqrt{t^4 + t^2}\sqrt{4t^2 + 1}}$$

$$\theta = \arccos\frac{2t^3 + t}{\sqrt{t^4 + t^2}\sqrt{4t^2 + 1}}$$

$\theta = 0.340$ ($\approx 19.47°$)maximum at $t = 0.707 = \left(\frac{\sqrt{2}}{2}\right)$.

$\theta \neq \frac{\pi}{2}$ for any t.



45. $$\mathbf{r}'(t) = \lim_{\Delta t\to 0}\frac{\mathbf{r}(t+\Delta t) - \mathbf{r}(t)}{\Delta t} = \lim_{\Delta t\to 0}\frac{[3(t+\Delta t) + 2]\mathbf{i} + [1 - (t+\Delta t)^2]\mathbf{j} - (3t+2)\mathbf{i} - (1-t^2)\mathbf{j}}{\Delta t}$$

$$= \lim_{\Delta t\to 0}\frac{(3\Delta t)\mathbf{i} - (2t(\Delta t) + (\Delta t)^2)\mathbf{j}}{\Delta t} = \lim_{\Delta t\to 0} 3\mathbf{i} - (2t + \Delta t)\mathbf{j} = 3\mathbf{i} - 2t\mathbf{j}$$

46. $$\mathbf{r}'(t) = \lim_{\Delta t\to 0}\frac{\mathbf{r}(t+\Delta t) - \mathbf{r}(t)}{\Delta t}$$

$$= \lim_{\Delta t\to 0}\frac{\left[\sqrt{t+\Delta t}\,\mathbf{i} + \frac{3}{t+\Delta t}\mathbf{j} - 2(t+\Delta t)\mathbf{k}\right] - \left[\sqrt{t}\,\mathbf{i} + \frac{3}{t}\mathbf{j} - 2t\mathbf{k}\right]}{\Delta t}$$

$$= \lim_{\Delta t\to 0}\left[\frac{\sqrt{t+\Delta t} - \sqrt{t}}{\Delta t}\mathbf{i} + \frac{\frac{3}{t+\Delta t} - \frac{3}{t}}{\Delta t}\mathbf{j} - 2\mathbf{k}\right]$$

$$= \lim_{\Delta t\to 0}\left[\frac{\Delta t}{\Delta t\left(\sqrt{t+\Delta t} + \sqrt{t}\right)}\mathbf{i} + \frac{-3\Delta t}{(t+\Delta t)t(\Delta t)}\mathbf{j} - 2\mathbf{k}\right]$$

$$= \lim_{\Delta t\to 0}\left[\frac{1}{\sqrt{t+\Delta t} + \sqrt{t}}\mathbf{i} - \frac{3}{(t+\Delta t)t}\mathbf{j} - 2\mathbf{k}\right]$$

$$= \frac{1}{2\sqrt{t}}\mathbf{i} - \frac{3}{t^2}\mathbf{j} - 2\mathbf{k}$$

47. $$\mathbf{r}'(t) = \lim_{\Delta t\to 0} \frac{\mathbf{r}(t+\Delta t)-\mathbf{r}(t)}{\Delta t}$$
$$= \lim_{\Delta t\to 0} \frac{\langle (t+\Delta t)^2, 0, 2(t+\Delta t)\rangle - \langle t^2, 0, 2t\rangle}{\Delta t}$$
$$= \lim_{\Delta t\to 0} \frac{\langle 2t\Delta t + (\Delta t)^2, 0, 2\Delta t\rangle}{\Delta t}$$
$$= \lim_{\Delta t\to 0} \langle 2t+\Delta t, 0, 2\rangle$$
$$= \langle 2t, 0, 2\rangle$$

48. $$\mathbf{r}'(t) = \lim_{\Delta t\to 0} \frac{\mathbf{r}(t+\Delta t)-\mathbf{r}(t)}{\Delta t}$$
$$= \lim_{\Delta t\to 0} \frac{\langle 0, \sin(t+\Delta t), 4(t+\Delta t)\rangle - \langle 0, \sin t, 4t\rangle}{\Delta t}$$
$$= \lim_{\Delta t\to 0} \frac{\langle 0, \sin t\cdot\cos(\Delta t) + \sin(\Delta t)\cos t - \sin t, 4\Delta t\rangle}{\Delta t}$$
$$= \lim_{\Delta t\to 0} \left\langle 0, \frac{\sin t(\cos(\Delta t)-1)}{\Delta t} + \cos t\left(\frac{\sin(\Delta t)}{\Delta t}\right), 4\right\rangle$$
$$= \langle 0, 0+\cos t, 4\rangle$$
$$= \langle 0, \cos t, 4\rangle$$

49. $$\int (2t\mathbf{i}+\mathbf{j}+\mathbf{k})\,dt = t^2\mathbf{i} + t\mathbf{j} + t\mathbf{k} + \mathbf{C}$$

50. $$\int \left(4t^3\mathbf{i} + 6t\mathbf{j} - 4\sqrt{t}\mathbf{k}\right)dt = t^4\mathbf{i} + 3t^2\mathbf{j} - \frac{8}{3}t^{3/2}\mathbf{k} + \mathbf{C}$$

51. $$\int \left(\frac{1}{t}\mathbf{i} + \mathbf{j} - t^{3/2}\mathbf{k}\right)dt = \ln t\mathbf{i} + t\mathbf{j} - \frac{2}{5}t^{5/2}\mathbf{k} + \mathbf{C}$$

52. $$\int \left[\ln t\mathbf{i} + \frac{1}{t}\mathbf{j} + \mathbf{k}\right]dt = (t\ln t - t)\mathbf{i} + \ln t\mathbf{j} + t\mathbf{k} + \mathbf{C}$$

(Integration by parts)

53. $$\int \left[(2t-1)\mathbf{i} + 4t^3\mathbf{j} + 3\sqrt{t}\mathbf{k}\right]dt = (t^2-t)\mathbf{i} + t^4\mathbf{j} + 2t^{3/2}\mathbf{k} + \mathbf{C}$$

54. $$\int [e^t\mathbf{i} + \sin t\mathbf{j} + \cos t\mathbf{k}]\,dt = e^t\mathbf{i} - \cos t\mathbf{j} + \sin t\mathbf{k} + \mathbf{C}$$

55. $$\int \left[\sec^2 t\mathbf{i} + \frac{1}{1+t^2}\mathbf{j}\right]dt = \tan t\mathbf{i} + \arctan t\mathbf{j} + \mathbf{C}$$

56. $$\int [e^{-t}\sin t\mathbf{i} + e^{-t}\cos t\mathbf{j}]\,dt = \frac{e^{-t}}{2}(-\sin t - \cos t)\mathbf{i} + \frac{e^{-t}}{2}(-\cos t + \sin t)\mathbf{j} + \mathbf{C}$$

57. $$\int_0^1 (8t\mathbf{i} + t\mathbf{j} - \mathbf{k})\,dt = \Big[4t^2\mathbf{i}\Big]_0^1 + \left[\frac{t^2}{2}\mathbf{j}\right]_0^1 - \Big[t\mathbf{k}\Big]_0^1 = 4\mathbf{i} + \frac{1}{2}\mathbf{j} - \mathbf{k}$$

58. $$\int_{-1}^1 (t\mathbf{i} + t^3\mathbf{j} + \sqrt[3]{t}\mathbf{k})\,dt = \left[\frac{t^2}{2}\mathbf{i}\right]_{-1}^1 + \left[\frac{t^4}{4}\mathbf{j}\right]_{-1}^1 + \left[\frac{3}{4}t^{4/3}\mathbf{k}\right]_{-1}^1 = \mathbf{0}$$

59. $$\int_0^{\pi/2} [(a\cos t)\mathbf{i} + (a\sin t)\mathbf{j} + \mathbf{k}]\,dt = \Big[a\sin t\mathbf{i}\Big]_0^{\pi/2} - \Big[a\cos t\mathbf{j}\Big]_0^{\pi/2} + \Big[t\mathbf{k}\Big]_0^{\pi/2} = a\mathbf{i} + a\mathbf{j} + \frac{\pi}{2}\mathbf{k}$$

60. $$\int_0^{\pi/4} [(\sec t\tan t)\mathbf{i} + (\tan t)\mathbf{j} + (2\sin t\cos t)\mathbf{k}]\,dt = \Big[\sec t\mathbf{i} + \ln|\sec t|\mathbf{j} + \sin^2 t\mathbf{k}\Big]_0^{\pi/4}$$
$$= \left(\sqrt{2}-1\right)\mathbf{i} + \ln\sqrt{2}\,\mathbf{j} + \frac{1}{2}\mathbf{k}$$

61. $$\int_0^2 (t\mathbf{i} + e^t\mathbf{j} - te^t\mathbf{k})\,dt = \left[\frac{t^2}{2}\mathbf{i}\right]_0^2 + \Big[e^t\mathbf{j}\Big]_0^2 - \Big[(t-1)e^t\mathbf{k}\Big]_0^2$$
$$= 2\mathbf{i} + (e^2-1)\mathbf{j} - (e^2+1)\mathbf{k}$$

62. $$\|t\mathbf{i} + t^2\mathbf{j}\| = \sqrt{t^2+t^4} = t\sqrt{1+t^2} \text{ for } t \ge 0$$
$$\int_0^3 \|t\mathbf{i} + t^2\mathbf{j}\|\,dt = \int_0^3 t\sqrt{1+t^2}\,dt$$
$$= \left[\frac{1}{3}(1+t^2)^{3/2}\right]_0^3$$
$$= \frac{1}{3}(10^{3/2}-1)$$

63. $\mathbf{r}(t) = \int (4e^{2t}\mathbf{i} + 3e^t\mathbf{j})\,dt = 2e^{2t}\mathbf{i} + 3e^t\mathbf{j} + \mathbf{C}$

$\mathbf{r}(0) = 2\mathbf{i} + 3\mathbf{j} + \mathbf{C} = 2\mathbf{i} \Rightarrow \mathbf{C} = -3\mathbf{j}$

$\mathbf{r}(t) = 2e^{2t}\mathbf{i} + 3(e^t - 1)\mathbf{j}$

64. $\mathbf{r}(t) = \int (3t^2\mathbf{j} + 6\sqrt{t}\mathbf{k})\,dt = t^3\mathbf{j} + 4t^{3/2}\mathbf{k} + \mathbf{C}$

$\mathbf{r}(0) = C = \mathbf{i} + 2\mathbf{j}$

$\mathbf{r}(t) = \mathbf{i} + (2 + t^3)\mathbf{j} + 4t^{3/2}\mathbf{k}$

65. $\mathbf{r}'(t) = \int -32\mathbf{j}\,dt = -32t\mathbf{j} + \mathbf{C}_1$

$\mathbf{r}'(0) = \mathbf{C}_1 = 600\sqrt{3}\mathbf{i} + 600\mathbf{j}$

$\mathbf{r}'(t) = 600\sqrt{3}\mathbf{i} + (600 - 32t)\mathbf{j}$

$\mathbf{r}(t) = \int \left[600\sqrt{3}\mathbf{i} + (600 - 32t)\mathbf{j}\right] dt$

$= 600\sqrt{3}t\mathbf{i} + (600t - 16t^2)\mathbf{j} + \mathbf{C}$

$\mathbf{r}(0) = \mathbf{C} = \mathbf{0}$

$\mathbf{r}(t) = 600\sqrt{3}t\mathbf{i} + (600t - 16t^2)\mathbf{j}$

66. $\mathbf{r}''(t) = -4\cos t\mathbf{j} - 3\sin t\mathbf{k}$

$\mathbf{r}'(t) = -4\sin t\mathbf{j} + 3\cos t\mathbf{k} + \mathbf{C}_1$

$\mathbf{r}'(0) = 3\mathbf{k} = 3\mathbf{k} + \mathbf{C}_1 \Rightarrow \mathbf{C}_1 = \mathbf{0}$

$\mathbf{r}(t) = 4\cos t\mathbf{j} + 3\sin t\mathbf{k} + \mathbf{C}_2$

$\mathbf{r}(0) = 4j + \mathbf{C}_2 = 4\mathbf{j} \Rightarrow \mathbf{C}_2 = 0$

$\mathbf{r}(t) = 4\cos t\mathbf{j} + 3\sin t\mathbf{k}$

67. $\mathbf{r}(t) = \int (te^{-t^2}\mathbf{i} - e^{-t}\mathbf{j} + \mathbf{k})\,dt = -\frac{1}{2}e^{-t^2}\mathbf{i} + e^{-t}\mathbf{j} + t\mathbf{k} + \mathbf{C}$

$\mathbf{r}(0) = -\frac{1}{2}\mathbf{i} + \mathbf{j} + \mathbf{C} = \frac{1}{2}\mathbf{i} - \mathbf{j} + \mathbf{k} \Rightarrow \mathbf{C} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$

$\mathbf{r}(t) = \left(1 - \frac{1}{2}e^{-t^2}\right)\mathbf{i} + (e^{-t} - 2)\mathbf{j} + (t + 1)\mathbf{k} = \left(\frac{2 - e^{-t^2}}{2}\right)\mathbf{i} + (e^{-t} - 2)\mathbf{j} + (t + 1)\mathbf{k}$

68. $\mathbf{r}(t) = \int \left[\frac{1}{1 + t^2}\mathbf{i} + \frac{1}{t^2}\mathbf{j} + \frac{1}{t}\mathbf{k}\right] dt = \arctan t\mathbf{i} - \frac{1}{t}\mathbf{j} + \ln t\mathbf{k} + \mathbf{C}$

$\mathbf{r}(1) = \frac{\pi}{4}\mathbf{i} - \mathbf{j} + \mathbf{C} = 2\mathbf{i} \Rightarrow \mathbf{C} = \left(2 - \frac{\pi}{4}\right)\mathbf{i} + \mathbf{j}$

$\mathbf{r}(t) = \left[2 - \frac{\pi}{4} + \arctan t\right]\mathbf{i} + \left(1 - \frac{1}{t}\right)\mathbf{j} + \ln t\mathbf{k}$

69. See "Definition of the Derivative of a Vector-Valued Function" and Figure 12.8 on page 840.

70. To find the integral of a vector-valued function, you integrate each component function separately. The constant of integration **C** is a constant vector.

71. At $t = t_0$, the graph of $\mathbf{u}(t)$ is increasing in the x, y, and z directions simultaneously.

72. The graph of $\mathbf{u}(t)$ does not change position relative to the xy-plane.

73. Let $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$. Then $c\mathbf{r}(t) = cx(t)\mathbf{i} + cy(t)\mathbf{j} + cz(t)\mathbf{k}$ and

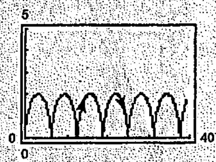
$$D_t[c\mathbf{r}(t)] = cx'(t)\mathbf{i} + cy'(t)\mathbf{j} + cz'(t)\mathbf{k}$$
$$= c[x'(t)\mathbf{i} + y'(t)\mathbf{j} + z'(t)\mathbf{k}] = c\mathbf{r}'(t).$$

74. Let $\mathbf{r}(t) = x_1(t)\mathbf{i} + y_1(t)\mathbf{j} + z_1(t)\mathbf{k}$ and $\mathbf{u}(t) = x_2(t)\mathbf{i} + y_2(t)\mathbf{j} + z_2(t)\mathbf{k}$.

$$\mathbf{r}(t) \pm \mathbf{u}(t) = [x_1(t) \pm x_2(t)]\mathbf{i} + [y_1(t) \pm y_2(t)]\mathbf{j} + [z_1(t) \pm z_2(t)]\mathbf{k}$$
$$D_t[\mathbf{r}(t) \pm \mathbf{u}(t)] = [x_1'(t) \pm x_2'(t)]\mathbf{i} + [y_1'(t) \pm y_2'(t)]\mathbf{j} + [z_1'(t) \pm z_2'(t)]\mathbf{k}$$
$$= [x_1'(t)\mathbf{i} + y_1'(t)\mathbf{j} + z_1'(t)\mathbf{k}] \pm [x_2'(t)\mathbf{i} + y_2'(t)\mathbf{j} + z_2'(t)\mathbf{k}]$$
$$= \mathbf{r}'(t) \pm \mathbf{u}'(t)$$

81. $\mathbf{r}(t) = (t - \sin t)\mathbf{i} + (1 - \cos t)\mathbf{j}$

(a)



The curve is a cycloid.

(b) $\mathbf{r}'(t) = (1 - \cos t)\mathbf{i} + \sin t\mathbf{j}$

$\mathbf{r}''(t) = \sin t\mathbf{i} + \cos t\mathbf{j}$

$\|\mathbf{r}'(t)\| = \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t} = \sqrt{2 - 2\cos t}$

Minimum of $\|\mathbf{r}'(t)\|$ is 0, $(t = 0)$

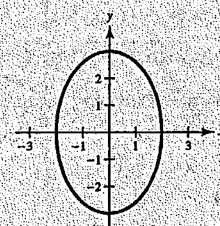
Maximum of $\|\mathbf{r}'(t)\|$ is 2, $(t = \pi)$

$\|\mathbf{r}''(t)\| = \sqrt{\sin^2 t + \cos^2 t} = 1$

Minimum and maximum of $\|\mathbf{r}''(t)\|$ is 1.

82. $\mathbf{r}(t) = 2\cos t\mathbf{i} + 3\sin t\mathbf{j}$

(a) Ellipse



(b) $\mathbf{r}'(t) = -2\sin t\mathbf{i} + 3\cos t\mathbf{j}$

$\mathbf{r}''(t) = -2\cos t\mathbf{i} - 3\sin t\mathbf{j}$

$\|\mathbf{r}'(t)\| = \sqrt{4\sin^2 t + 9\cos^2 t}$

Minimum of $\|\mathbf{r}'(t)\|$ is 2, $(t = \pi/2)$

Maximum of $\|\mathbf{r}'(t)\|$ is 3, $(t = 0)$

$\|\mathbf{r}''(t)\| = \sqrt{4\cos^2 t + 9\sin^2 t}$

Minimum of $\|\mathbf{r}''(t)\|$ is 2, $(t = 0)$

Maximum of $\|\mathbf{r}''(t)\|$ is 3, $(t = \pi/2)$

83. True

84. False. The definite integral is a vector, not a real number.

85. False. Let $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + \mathbf{k}$.

$\|\mathbf{r}(t)\| = \sqrt{2}$

$\frac{d}{dt}[\|\mathbf{r}(t)\|] = 0$

$\mathbf{r}'(t) = -\sin t\mathbf{i} + \cos t\mathbf{j}$

$\|\mathbf{r}'(t)\| = 1$

86. False

$D_t[\mathbf{r}(t) \cdot \mathbf{u}(t)] = \mathbf{r}(t) \cdot \mathbf{u}'(t) + \mathbf{r}'(t) \cdot \mathbf{u}(t)$

(See Theorem 11.2, part 4)

87. $\mathbf{r}(t) = e^t \sin t\mathbf{i} + e^t \cos t\mathbf{j}$

$\mathbf{r}'(t) = (e^t \cos t + e^t \sin t)\mathbf{i} + (e^t \cos t - e^t \sin t)\mathbf{j}$

$\mathbf{r}''(t) = (-e^t \sin t + e^t \cos t + e^t \sin t + e^t \cos t)\mathbf{i} + (e^t \cos t - e^t \sin t - e^t \sin t - e^t \cos t)\mathbf{j}$

$= 2e^t \cos t\mathbf{i} - 2e^t \sin t\mathbf{j}$

$\mathbf{r}(t) \cdot \mathbf{r}''(t) = 2e^{2t} \sin t \cos t - 2e^{2t} \sin t \cos t = 0$

Hence, $\mathbf{r}(t)$ is always perpendicular to $\mathbf{r}''(t)$.

Section 12.3 Velocity and Acceleration

1. $\mathbf{r}(t) = 3t\mathbf{i} + (t - 1)\mathbf{j}$

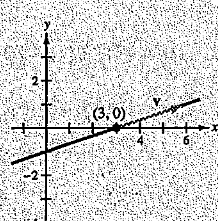
$\mathbf{v}(t) = \mathbf{r}'(t) = 3\mathbf{i} + \mathbf{j}$

$\mathbf{a}(t) = \mathbf{r}''(t) = \mathbf{0}$

$x = 3t,\ y = t - 1,\ y = \frac{x}{3} - 1$

At $(3, 0)$, $t = 1$.

$\mathbf{v}(1) = 3\mathbf{i} + \mathbf{j}, \mathbf{a}(1) = \mathbf{0}$

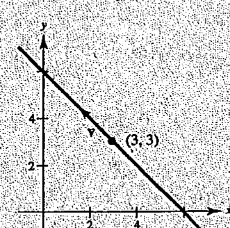


2. $\mathbf{r}(t) = (6 - t)\mathbf{i} + t\mathbf{j}$

$\mathbf{v}(t) = \mathbf{r}'(t) = -\mathbf{i} + \mathbf{j}$

$\mathbf{a}(t) = \mathbf{r}''(t) = \mathbf{0}$

$x = 6 - t,\ y = t,\ y = 6 - x$



3. $\mathbf{r}(t) = t^2\mathbf{i} + t\mathbf{j}$

$\mathbf{v}(t) = \mathbf{r}'(t) = 2t\mathbf{i} + \mathbf{j}$

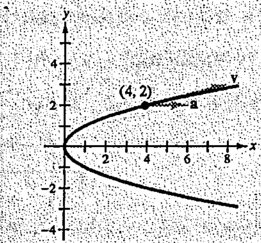
$\mathbf{a}(t) = \mathbf{r}''(t) = 2\mathbf{i}$

$x = t^2,\ y = t,\ x = y^2$

At $(4, 2)$, $t = 2$.

$\mathbf{v}(2) = 4\mathbf{i} + \mathbf{j}$

$\mathbf{a}(2) = 2\mathbf{i}$



4. $\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j}$

$\mathbf{v}(t) = \mathbf{r}'(t) = 2t\mathbf{i} + 3t^2\mathbf{j}$

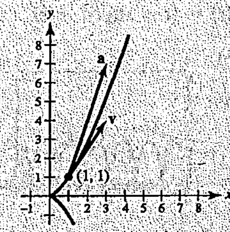
$\mathbf{a}(t) = \mathbf{r}''(t) = 2\mathbf{i} + 6t\mathbf{j}$

$x = t^2, y = t^3,\ x = y^{2/3}$

At $(1, 1)$, $t = 1$.

$\mathbf{v}(1) = 2\mathbf{i} + 3\mathbf{j}$

$\mathbf{a}(1) = 2\mathbf{i} + 6\mathbf{j}$



5. $\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j}$

$\mathbf{v}(t) = \mathbf{r}'(t) = -2\sin t\mathbf{i} + 2\cos t\mathbf{j}$

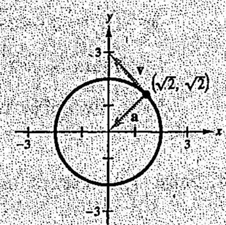
$\mathbf{a}(t) = \mathbf{r}''(t) = -2\cos t\mathbf{i} - 2\sin t\mathbf{j}$

$x = 2\cos t,\ y = 2\sin t,\ x^2 + y^2 = 4$

At $\left(\sqrt{2}, \sqrt{2}\right)$, $t = \dfrac{\pi}{4}$.

$\mathbf{v}\left(\dfrac{\pi}{4}\right) = -\sqrt{2}\mathbf{i} + \sqrt{2}\mathbf{j}$

$\mathbf{a}\left(\dfrac{\pi}{4}\right) = -\sqrt{2}\mathbf{i} - \sqrt{2}\mathbf{j}$



6. $\mathbf{r}(t) = 3\cos t\mathbf{i} + 2\sin t\mathbf{j}$

$\mathbf{v}(t) = -3\sin t\mathbf{i} + 2\cos t\mathbf{j}$

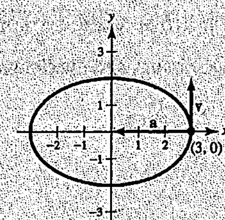
$\mathbf{a}(t) = -3\cos t\mathbf{i} - 2\sin t\mathbf{j}$

$x = 3\cos t, y = 2\sin t, \dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$ Ellipse

At $(3, 0)$, $t = 0$.

$\mathbf{v}(0) = 2\mathbf{j}$

$\mathbf{a}(0) = -3\mathbf{i}$



7. $\mathbf{r}(t) = \langle t - \sin t, 1 - \cos t\rangle$

$\mathbf{v}(t) = \mathbf{r}'(t) = \langle 1 - \cos t, \sin t\rangle$

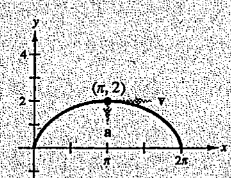
$\mathbf{a}(t) = \mathbf{r}''(t) = \langle \sin t, \cos t\rangle$

$x = t - \sin t,\ y = 1 - \cos t$ (cycloid)

At $(\pi, 2)$, $t = \pi$.

$\mathbf{v}(\pi) = \langle 2, 0\rangle = 2\mathbf{i}$

$\mathbf{a}(\pi) = \langle 0, -1\rangle = -\mathbf{j}$



8. $\mathbf{r}(t) = \langle e^{-t}, e^t\rangle$

$\mathbf{v}(t) = \mathbf{r}'(t) = \langle -e^{-t}, e^t\rangle$

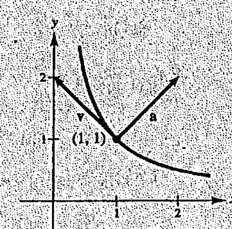
$\mathbf{a}(t) = \mathbf{r}''(t) = \langle e^{-t}, e^t\rangle$

$x = e^{-t} = \dfrac{1}{e^t},\ y = e^t,\ y = \dfrac{1}{x}$

At $(1, 1)$, $t = 0$.

$\mathbf{v}(0) = \langle -1, 1\rangle = -\mathbf{i} + \mathbf{j}$

$\mathbf{a}(0) = \langle 1, 1\rangle = \mathbf{i} + \mathbf{j}$



9. $\mathbf{r}(t) = t\mathbf{i} + (2t - 5)\mathbf{j} + 3t\mathbf{k}$

$\mathbf{v}(t) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$

$s(t) = \|\mathbf{v}(t)\| = \sqrt{1 + 4 + 9} = \sqrt{14}$

$\mathbf{a}(t) = \mathbf{0}$

10. $\mathbf{r}(t) = 4t\mathbf{i} + 4t\mathbf{j} + 2t\mathbf{k}$

$\mathbf{v}(t) = 4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$

$s(t) = \|\mathbf{v}(t)\| = \sqrt{16 + 16 + 4} = 6$

$\mathbf{a}(t) = \mathbf{0}$

11. $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + \dfrac{t^2}{2}\mathbf{k}$

$\mathbf{v}(t) = \mathbf{i} + 2t\mathbf{j} + t\mathbf{k}$

$s(t) = \sqrt{1 + 4t^2 + t^2} = \sqrt{1 + 5t^2}$

$\mathbf{a}(t) = 2\mathbf{j} + \mathbf{k}$

12. $\mathbf{r}(t) = 3t\mathbf{i} + t\mathbf{j} + \dfrac{1}{4}t^2\mathbf{k}$

$\mathbf{v}(t) = 3\mathbf{i} + \mathbf{j} + \dfrac{1}{2}t\mathbf{k}$

$s(t) = \sqrt{9 + 1 + \dfrac{1}{4}t^2} = \sqrt{10 + \dfrac{1}{4}t^2}$

$\mathbf{a}(t) = \dfrac{1}{2}\mathbf{k}$

13. $\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + \sqrt{9 - t^2}\mathbf{k}$

$\mathbf{v}(t) = \mathbf{i} + \mathbf{j} - \dfrac{t}{\sqrt{9 - t^2}}\mathbf{k}$

$s(t) = \sqrt{1 + 1 + \dfrac{t^2}{9 - t^2}} = \sqrt{\dfrac{18 - t^2}{9 - t^2}}$

$\mathbf{a}(t) = -\dfrac{9}{(9 - t^2)^{3/2}}\mathbf{k}$