12.3 Velocity and Acceleration (vector-valued function)

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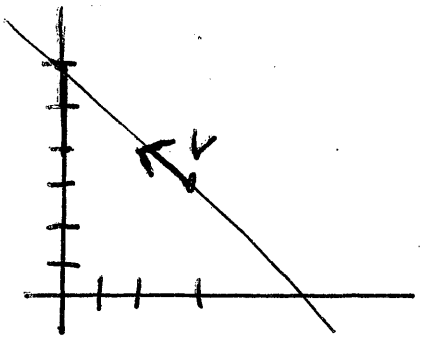
2) * sketch graph and sketch velocity, acceleration vectors at given point.

$r(t) = (6-t)i + tj \qquad (3,3)$

$v(t) = r'(t) = -i + j$

$a(t) = r''(t) = 0$

$x = 6 - t \quad y = t \quad y = 6 - x$



4) $r(t) = t^2 i + t^3 j \qquad (1,1)$

$v(t) = r'(t) = 2ti + 3t^2 j$

$a(t) = r''(t) = 2i + 6tj$

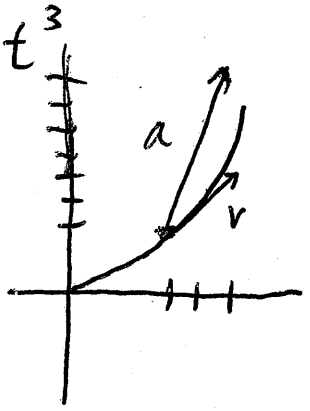
$v(1) = 2i + 3j$

$a(1) = 2i + 6j$

$x = t^2 \quad y = t^3$

$y = x^{3/2}$

at $(1,1)$, $t = 1$



8) $r(t) = \langle e^{-t}, e^t \rangle$

$v(t) = r'(t) = \langle -e^{-t}, e^t \rangle$

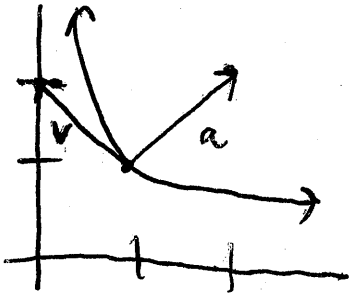
$a(t) = r''(t) = \langle e^{-t}, e^t \rangle$

$x = e^{-t} = \frac{1}{e^t} \quad y = e^t \quad y = \frac{1}{x}$

at $(1,1)$ $t = 0$

$v(0) = \langle -1, 1 \rangle$

$a(0) = \langle 1, 1 \rangle$



10) Find velocity, speed, acceleration given the position vector

$r(t) = 4t\,i + 4t\,j + 2t\,k$

$v(t) = 4i + 4j + 2k$

$\text{speed} = \|v(t)\| = \sqrt{4^2+4^2+2^2} = 6$

$a(t) = 0$

14) $r(t) = t^2 i + t\,j + 2t^{3/2}k$

$v(t) = 2t\,i + j + 3\sqrt{t}\,k$

$\text{speed} = \sqrt{4t^2+1+9t}$

$a(t) = 2i + \frac{3}{2\sqrt{t}}k$

16) $r(t) = \langle e^t\cos t, e^t\sin t, e^t \rangle$

$v(t) = (e^t\cos t - e^t\sin t)i + (e^t\sin t + e^t\cos t)j + e^t k$

$\text{speed} = \sqrt{e^{2t}(\cos t - \sin t)^2 + e^{2t}(\sin t + \cos t)^2 + e^{2t}} = \boxed{e^t\sqrt{3}}$

$a(t) = -2e^t\sin t\,i + 2e^t\cos t\,j + e^t k$

20) Use the acceleration function to find velocity, position vectors

$a(t) = 2i + 3k \quad v(0) = 4j \quad r(0) = 0$ Find $r(2)$

$v(t) = \int 2i + 3k\,dt = 2t\,i + 3t\,k + C \quad v(0) = C = 4j \quad v(t) = 2t\,i + 4j + 3t\,k$

$r(t) = \int 2t\,i + 4j + 3t\,k\,dt = t^2 i + 4t\,j + \frac{3}{2}t^2 k + C \quad r(0) = 0 = C$

$r(t) = t^2 i + 4t\,j + \frac{3}{2}t^2 k \quad \boxed{r(2) = 4i + 8j + 6k}$

22) $a(t) = -\cos t\,i - \sin t\,j \quad v(0) = j + k \quad r(0) = i$

$v(t) = \int(-\cos t\,i - \sin t\,j)\,dt = -\sin t\,i + \cos t\,j + C$

$v(0) = j + C = j + k = C = k$

$v(t) = -\sin t\,i + \cos t\,j + k$

$r(t) = \int(-\sin t\,i + \cos t\,j + k)\,dt = \cos t\,i + \sin t\,j + tk + C$

$r(0) = i + C = i \quad C = 0$

$r(t) = \cos t\,i + \sin t\,j + tk$

$r(2) = (\cos 2)i + (\sin 2)j + 2k$

25) projectile motion $r(t) = (v_0 \cos\theta)t\,i + \left[h + (v_0 \sin\theta)t - \frac{1}{2}gt^2\right]j$

height = 10 ft

$v_0 = 88 \quad \theta = 30°$

$r(t) = (88\cos 30)t\,i + \left[10 + 88\sin 30)t - 16t^2\right]j$

$= 44\sqrt{3}\,t\,i + (10 + 44t - 16t^2]j$

27) $v_0 \cos\theta\, t = 300 \quad \theta = 45°$

$v_0 \cos 45° t = 300$

$v_0\left(\frac{\sqrt{2}}{2}\right)t = 300$

$t = \frac{300\sqrt{2}}{v_0}$

$3 + \frac{v_0}{\sqrt{2}}t - 16t^2 = 3$

$\frac{v_0}{\sqrt{2}}\left(\frac{300\sqrt{2}}{v_0}\right) - 16\left(\frac{300\sqrt{2}}{v_0}\right)^2 = 0$

$v_0^2 = 300(32) \quad v_0 = 40\sqrt{6} \approx 97.98$ ft/s

$y(t) = 3 + \frac{40\sqrt{6}}{\sqrt{2}}t - 16t^2$

$y'(t) = 40\sqrt{3} - 32t = 0$

$t = \frac{40\sqrt{3}}{32} = \frac{5\sqrt{3}}{4}$ sec.

Max height: $y\left(\frac{5\sqrt{3}}{4}\right) = 3 + 40\sqrt{3}\left(\frac{5\sqrt{3}}{4}\right) - 16\left(\frac{5\sqrt{3}}{4}\right)^2 =$ **78 feet**

29) $y = -\frac{16\sec^2\theta}{v_0^2}x^2 + (\tan\theta)x + h$ ← show

$x(t) = (v_0\cos\theta)t \qquad t = \frac{x}{v_0\cos\theta}$

$y = (v_0\sin\theta)t - 16t^2 + h$

$y = v_0\sin\theta\left(\frac{x}{v_0\cos\theta}\right) - 16\left(\frac{x}{\cos\theta}\right)^2 + h = (\tan\theta)x - \left(\frac{16}{v_0^2}\sec^2\theta\right)x^2 + h$

31) a) $y = -0.004x^2 + 3.667x + 6$

c) $y' = -0.008x + 0.3667 \qquad x = 45.8375$ ft

$y(45.8375) \approx \boxed{144 \text{ ft.}}$

d) Find initial velocity, angle thrown

$y = -0.004x^2 + 3.667x + 6$

$y = \left(\frac{-16\sec^2\theta}{v_0^2}\right)x^2 + (\tan\theta)x + h$

$\tan\theta = 3.667$

$\boxed{\theta \approx 20.14^\circ}$

$\frac{-16\sec^2\theta}{v_0^2} = -0.004$

$\frac{16}{v_0^2\cos^2(20.14)} = 0.004$

$v_0^2 = \frac{4000}{\cos^2(20.14)} = 4537.99$

$v_0 = \sqrt{4537.99} \approx \boxed{67.4 \text{ ft/sec}}$