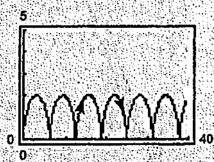
81. $\mathbf{r}(t) = (t - \sin t)\mathbf{i} + (1 - \cos t)\mathbf{j}$

(a)



The curve is a cycloid.

(b) $\mathbf{r}'(t) = (1 - \cos t)\mathbf{i} + \sin t\mathbf{j}$

$\mathbf{r}''(t) = \sin t\mathbf{i} + \cos t\mathbf{j}$

$\|\mathbf{r}'(t)\| = \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t} = \sqrt{2 - 2\cos t}$

Minimum of $\|\mathbf{r}'(t)\|$ is 0, $(t = 0)$

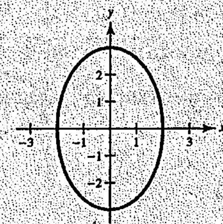
Maximum of $\|\mathbf{r}'(t)\|$ is 2, $(t = \pi)$

$\|\mathbf{r}''(t)\| = \sqrt{\sin^2 t + \cos^2 t} = 1$

Minimum and maximum of $\|\mathbf{r}''(t)\|$ is 1.

82. $\mathbf{r}(t) = 2\cos t\mathbf{i} + 3\sin t\mathbf{j}$

(a) Ellipse



(b) $\mathbf{r}'(t) = -2\sin t\mathbf{i} + 3\cos t\mathbf{j}$

$\mathbf{r}''(t) = -2\cos t\mathbf{i} - 3\sin t\mathbf{j}$

$\|\mathbf{r}'(t)\| = \sqrt{4\sin^2 t + 9\cos^2 t}$

Minimum of $\|\mathbf{r}'(t)\|$ is 2, $(t = \pi/2)$

Maximum of $\|\mathbf{r}'(t)\|$ is 3, $(t = 0)$

$\|\mathbf{r}''(t)\| = \sqrt{4\cos^2 t + 9\sin^2 t}$

Minimum of $\|\mathbf{r}''(t)\|$ is 2, $(t = 0)$

Maximum of $\|\mathbf{r}''(t)\|$ is 3, $(t = \pi/2)$

83. True

84. False. The definite integral is a vector, not a real number.

85. False. Let $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + \mathbf{k}$.

$\|\mathbf{r}(t)\| = \sqrt{2}$

$\frac{d}{dt}[\|\mathbf{r}(t)\|] = 0$

$\mathbf{r}'(t) = -\sin t\mathbf{i} + \cos t\mathbf{j}$

$\|\mathbf{r}'(t)\| = 1$

86. False

$D_t[\mathbf{r}(t) \cdot \mathbf{u}(t)] = \mathbf{r}(t) \cdot \mathbf{u}'(t) + \mathbf{r}'(t) \cdot \mathbf{u}(t)$

(See Theorem 11.2, part 4)

87. $\mathbf{r}(t) = e^t \sin t\mathbf{i} + e^t \cos t\mathbf{j}$

$\mathbf{r}'(t) = (e^t \cos t + e^t \sin t)\mathbf{i} + (e^t \cos t - e^t \sin t)\mathbf{j}$

$\mathbf{r}''(t) = (-e^t \sin t + e^t \cos t + e^t \sin t + e^t \cos t)\mathbf{i} + (e^t \cos t - e^t \sin t - e^t \sin t - e^t \cos t)\mathbf{j}$

$= 2e^t \cos t\mathbf{i} - 2e^t \sin t\mathbf{j}$

$\mathbf{r}(t) \cdot \mathbf{r}''(t) = 2e^{2t} \sin t \cos t - 2e^{2t} \sin t \cos t = 0$

Hence, $\mathbf{r}(t)$ is always perpendicular to $\mathbf{r}''(t)$.

Section 12.3 Velocity and Acceleration

1. $\mathbf{r}(t) = 3t\mathbf{i} + (t - 1)\mathbf{j}$

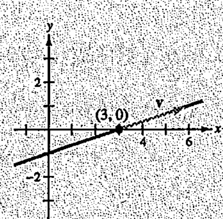
$\mathbf{v}(t) = \mathbf{r}'(t) = 3\mathbf{i} + \mathbf{j}$

$\mathbf{a}(t) = \mathbf{r}''(t) = \mathbf{0}$

$x = 3t,\ y = t - 1,\ y = \frac{x}{3} - 1$

At $(3, 0)$, $t = 1$.

$\mathbf{v}(1) = 3\mathbf{i} + \mathbf{j},\ \mathbf{a}(1) = \mathbf{0}$

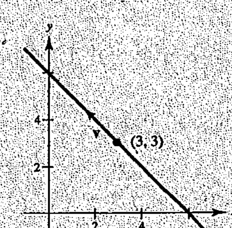


2. $\mathbf{r}(t) = (6 - t)\mathbf{i} + t\mathbf{j}$

$\mathbf{v}(t) = \mathbf{r}'(t) = -\mathbf{i} + \mathbf{j}$

$\mathbf{a}(t) = \mathbf{r}''(t) = \mathbf{0}$

$x = 6 - t,\ y = t,\ y = 6 - x$



3. $\mathbf{r}(t) = t^2\mathbf{i} + t\mathbf{j}$

$\mathbf{v}(t) = \mathbf{r}'(t) = 2t\mathbf{i} + \mathbf{j}$

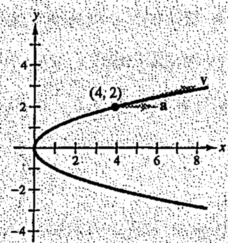
$\mathbf{a}(t) = \mathbf{r}''(t) = 2\mathbf{i}$

$x = t^2,\ y = t,\ x = y^2$

At $(4, 2)$, $t = 2$.

$\mathbf{v}(2) = 4\mathbf{i} + \mathbf{j}$

$\mathbf{a}(2) = 2\mathbf{i}$



4. $\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j}$

$\mathbf{v}(t) = \mathbf{r}'(t) = 2t\mathbf{i} + 3t^2\mathbf{j}$

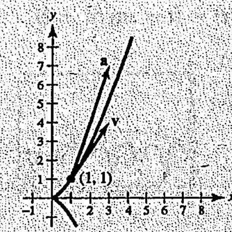
$\mathbf{a}(t) = \mathbf{r}''(t) = 2\mathbf{i} + 6t\mathbf{j}$

$x = t^2, y = t^3,\ x = y^{2/3}$

At $(1, 1)$, $t = 1$.

$\mathbf{v}(1) = 2\mathbf{i} + 3\mathbf{j}$

$\mathbf{a}(1) = 2\mathbf{i} + 6\mathbf{j}$



5. $\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j}$

$\mathbf{v}(t) = \mathbf{r}'(t) = -2\sin t\mathbf{i} + 2\cos t\mathbf{j}$

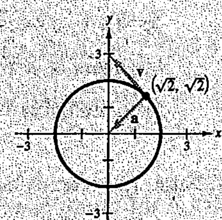
$\mathbf{a}(t) = \mathbf{r}''(t) = -2\cos t\mathbf{i} - 2\sin t\mathbf{j}$

$x = 2\cos t,\ y = 2\sin t,\ x^2 + y^2 = 4$

At $\left(\sqrt{2}, \sqrt{2}\right)$, $t = \dfrac{\pi}{4}$.

$\mathbf{v}\left(\dfrac{\pi}{4}\right) = -\sqrt{2}\mathbf{i} + \sqrt{2}\mathbf{j}$

$\mathbf{a}\left(\dfrac{\pi}{4}\right) = -\sqrt{2}\mathbf{i} - \sqrt{2}\mathbf{j}$



6. $\mathbf{r}(t) = 3\cos t\mathbf{i} + 2\sin t\mathbf{j}$

$\mathbf{v}(t) = -3\sin t\mathbf{i} + 2\cos t\mathbf{j}$

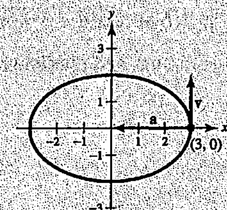
$\mathbf{a}(t) = -3\cos t\mathbf{i} - 2\sin t\mathbf{j}$

$x = 3\cos t, y = 2\sin t, \dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$ Ellipse

At $(3, 0)$, $t = 0$.

$\mathbf{v}(0) = 2\mathbf{j}$

$\mathbf{a}(0) = -3\mathbf{i}$



7. $\mathbf{r}(t) = \langle t - \sin t, 1 - \cos t\rangle$

$\mathbf{v}(t) = \mathbf{r}'(t) = \langle 1 - \cos t, \sin t\rangle$

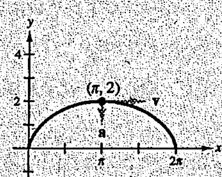
$\mathbf{a}(t) = \mathbf{r}''(t) = \langle \sin t, \cos t\rangle$

$x = t - \sin t,\ y = 1 - \cos t$ (cycloid)

At $(\pi, 2)$, $t = \pi$.

$\mathbf{v}(\pi) = \langle 2, 0\rangle = 2\mathbf{i}$

$\mathbf{a}(\pi) = \langle 0, -1\rangle = -\mathbf{j}$



8. $\mathbf{r}(t) = \langle e^{-t}, e^t\rangle$

$\mathbf{v}(t) = \mathbf{r}'(t) = \langle -e^{-t}, e^t\rangle$

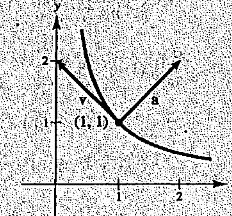
$\mathbf{a}(t) = \mathbf{r}''(t) = \langle e^{-t}, e^t\rangle$

$x = e^{-t} = \dfrac{1}{e^t}, y = e^t,\ y = \dfrac{1}{x}$

At $(1, 1)$, $t = 0$.

$\mathbf{v}(0) = \langle -1, 1\rangle = -\mathbf{i} + \mathbf{j}$

$\mathbf{a}(0) = \langle 1, 1\rangle = \mathbf{i} + \mathbf{j}$



9. $\mathbf{r}(t) = t\mathbf{i} + (2t - 5)\mathbf{j} + 3t\mathbf{k}$

$\mathbf{v}(t) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$

$s(t) = \|\mathbf{v}(t)\| = \sqrt{1 + 4 + 9} = \sqrt{14}$

$\mathbf{a}(t) = \mathbf{0}$

10. $\mathbf{r}(t) = 4t\mathbf{i} + 4t\mathbf{j} + 2t\mathbf{k}$

$\mathbf{v}(t) = 4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$

$s(t) = \|\mathbf{v}(t)\| = \sqrt{16 + 16 + 4} = 6$

$\mathbf{a}(t) = \mathbf{0}$

11. $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + \dfrac{t^2}{2}\mathbf{k}$

$\mathbf{v}(t) = \mathbf{i} + 2t\mathbf{j} + t\mathbf{k}$

$s(t) = \sqrt{1 + 4t^2 + t^2} = \sqrt{1 + 5t^2}$

$\mathbf{a}(t) = 2\mathbf{j} + \mathbf{k}$

12. $\mathbf{r}(t) = 3t\mathbf{i} + t\mathbf{j} + \dfrac{1}{4}t^2\mathbf{k}$

$\mathbf{v}(t) = 3\mathbf{i} + \mathbf{j} + \dfrac{1}{2}t\mathbf{k}$

$s(t) = \sqrt{9 + 1 + \dfrac{1}{4}t^2} = \sqrt{10 + \dfrac{1}{4}t^2}$

$\mathbf{a}(t) = \dfrac{1}{2}\mathbf{k}$

13. $\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + \sqrt{9 - t^2}\mathbf{k}$

$\mathbf{v}(t) = \mathbf{i} + \mathbf{j} - \dfrac{t}{\sqrt{9 - t^2}}\mathbf{k}$

$s(t) = \sqrt{1 + 1 + \dfrac{t^2}{9 - t^2}} = \sqrt{\dfrac{18 - t^2}{9 - t^2}}$

$\mathbf{a}(t) = -\dfrac{9}{(9 - t^2)^{3/2}}\mathbf{k}$

14. $\mathbf{r}(t) = t^2\mathbf{i} + t\mathbf{j} + 2t^{3/2}\mathbf{k}$

$\mathbf{v}(t) = 2t\mathbf{i} + \mathbf{j} + 3\sqrt{t}\mathbf{k}$

$s(t) = \sqrt{4t^2 + 1 + 9t} = \sqrt{4t^2 + 9t + 1}$

$\mathbf{a}(t) = 2\mathbf{i} + \dfrac{3}{2\sqrt{t}}\mathbf{k}$

15. $\mathbf{r}(t) = \langle 4t, 3\cos t, 3\sin t\rangle$

$\mathbf{v}(t) = \langle 4, -3\sin t, 3\cos t\rangle = 4\mathbf{i} - 3\sin t\mathbf{j} + 3\cos t\mathbf{k}$

$s(t) = \sqrt{16 + 9\sin^2 t + 9\cos^2 t} = 5$

$\mathbf{a}(t) = \langle 0, -3\cos t, -3\sin t\rangle = -3\cos t\mathbf{j} - 3\sin t\mathbf{k}$

16. $\mathbf{r}(t) = \langle e^t\cos t, e^t\sin t, e^t\rangle$

$\mathbf{v}(t) = (e^t\cos t - e^t\sin t)\mathbf{i} + (e^t\sin t + e^t\cos t)\mathbf{j} + e^t\mathbf{k}$

$s(t) = \sqrt{e^{2t}(\cos t - \sin t)^2 + e^{2t}(\cos t + \sin t)^2 + e^{2t}} = e^t\sqrt{3}$

$\mathbf{a}(t) = -2e^t\sin t\mathbf{i} + 2e^t\cos t\mathbf{j} + e^t\mathbf{k}$

17. (a) $\mathbf{r}(t) = \left\langle t, -t^2, \dfrac{t^3}{4}\right\rangle, t_0 = 1$

$\mathbf{r}'(t) = \left\langle 1, -2t, \dfrac{3t^2}{4}\right\rangle$

$\mathbf{r}'(1) = \left\langle 1, -2, \dfrac{3}{4}\right\rangle$

$x = 1 + t,\ y = -1 - 2t,\ z = \dfrac{1}{4} + \dfrac{3}{4}t$

(b) $\mathbf{r}(1 + 0.1) \approx \left\langle 1 + 0.1, -1 - 2(0.1), \dfrac{1}{4} + \dfrac{3}{4}(0.1)\right\rangle$

$= \langle 1.100, -1.200, 0.325\rangle$

18. (a) $\mathbf{r}(t) = \left\langle t, \sqrt{25 - t^2}, \sqrt{25 - t^2}\right\rangle, t_0 = 3$

$\mathbf{r}'(t) = \left\langle 1, \dfrac{-t}{\sqrt{25 - t^2}}, \dfrac{-t}{\sqrt{25 - t^2}}\right\rangle$

$\mathbf{r}'(3) = \left\langle 1, -\dfrac{3}{4}, -\dfrac{3}{4}\right\rangle$

$x = 3 + t,\ y = z = 4 - \dfrac{3}{4}t$

(b) $\mathbf{r}(3 + 0.1) \approx \left\langle 3 + 0.1, 4 - \dfrac{3}{4}(0.1), 4 - \dfrac{3}{4}(0.1)\right\rangle$

$= \langle 3.100, 3.925, 3.925\rangle$

19. $\mathbf{a}(t) = \mathbf{i} + \mathbf{j} + \mathbf{k}, \mathbf{v}(0) = \mathbf{0}, \mathbf{r}(0) = \mathbf{0}$

$\mathbf{v}(t) = \displaystyle\int(\mathbf{i} + \mathbf{j} + \mathbf{k})\,dt = t\mathbf{i} + t\mathbf{j} + t\mathbf{k} + \mathbf{C}$

$\mathbf{v}(0) = \mathbf{C} = \mathbf{0}, \mathbf{v}(t) = t\mathbf{i} + t\mathbf{j} + t\mathbf{k}, \mathbf{v}(t) = t(\mathbf{i} + \mathbf{j} + \mathbf{k})$

$\mathbf{r}(t) = \displaystyle\int(t\mathbf{i} + t\mathbf{j} + t\mathbf{k})\,dt = \dfrac{t^2}{2}(\mathbf{i} + \mathbf{j} + \mathbf{k}) + \mathbf{C}$

$\mathbf{r}(0) = \mathbf{C} = \mathbf{0}, \mathbf{r}(t) = \dfrac{t^2}{2}(\mathbf{i} + \mathbf{j} + \mathbf{k}),$

$\mathbf{r}(2) = 2(\mathbf{i} + \mathbf{j} + \mathbf{k}) = 2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

20. $\mathbf{a}(t) = 2\mathbf{i} + 3\mathbf{k}$

$\mathbf{v}(t) = \displaystyle\int(2\mathbf{i} + 3\mathbf{k})\,dt = 2t\mathbf{i} + 3t\mathbf{k} + \mathbf{C}$

$\mathbf{v}(0) = \mathbf{C} = 4\mathbf{j} \Longrightarrow \mathbf{v}(t) = 2t\mathbf{i} + 4\mathbf{j} + 3t\mathbf{k}$

$\mathbf{r}(t) = \displaystyle\int(2t\mathbf{i} + 4\mathbf{j} + 3t\mathbf{k})\,dt = t^2\mathbf{i} + 4t\mathbf{j} + \dfrac{3}{2}t^2\mathbf{k} + \mathbf{C}$

$\mathbf{r}(0) = \mathbf{C} = 0 \Longrightarrow \mathbf{r}(t) = t^2\mathbf{i} + 4t\mathbf{j} + \dfrac{3}{2}t^2\mathbf{k}$

$\mathbf{r}(2) = 4\mathbf{i} + 8\mathbf{j} + 6\mathbf{k}$

21. $\mathbf{a}(t) = t\mathbf{j} + t\mathbf{k}, \mathbf{v}(1) = 5\mathbf{j}, \mathbf{r}(1) = \mathbf{0}$

$\mathbf{v}(t) = \displaystyle\int(t\mathbf{j} + t\mathbf{k})\,dt = \dfrac{t^2}{2}\mathbf{j} + \dfrac{t^2}{2}\mathbf{k} + \mathbf{C}$

$\mathbf{v}(1) = \dfrac{1}{2}\mathbf{j} + \dfrac{1}{2}\mathbf{k} + \mathbf{C} = 5\mathbf{j} \Longrightarrow \mathbf{C} = \dfrac{9}{2}\mathbf{j} - \dfrac{1}{2}\mathbf{k}$

$\mathbf{v}(t) = \left(\dfrac{t^2}{2} + \dfrac{9}{2}\right)\mathbf{j} + \left(\dfrac{t^2}{2} - \dfrac{1}{2}\right)\mathbf{k}$

$\mathbf{r}(t) = \displaystyle\int\left[\left(\dfrac{t^2}{2} + \dfrac{9}{2}\right)\mathbf{j} + \left(\dfrac{t^2}{2} - \dfrac{1}{2}\right)\mathbf{k}\right]dt = \left(\dfrac{t^3}{6} + \dfrac{9}{2}t\right)\mathbf{j} + \left(\dfrac{t^3}{6} - \dfrac{1}{2}t\right)\mathbf{k} + \mathbf{C}$

$\mathbf{r}(1) = \dfrac{14}{3}\mathbf{j} - \dfrac{1}{3}\mathbf{k} + \mathbf{C} = \mathbf{0} \Longrightarrow \mathbf{C} = -\dfrac{14}{3}\mathbf{j} + \dfrac{1}{3}\mathbf{k}$

$\mathbf{r}(t) = \left(\dfrac{t^3}{6} + \dfrac{9}{2}t - \dfrac{14}{3}\right)\mathbf{j} + \left(\dfrac{t^3}{6} - \dfrac{1}{2}t + \dfrac{1}{3}\right)\mathbf{k}$

$\mathbf{r}(2) = \dfrac{17}{3}\mathbf{j} + \dfrac{2}{3}\mathbf{k}$

3. $\mathbf{r}(t)$ ... $= \mathbf{j} + \mathbf{k}, \mathbf{r}(0) = \mathbf{i}$

$\mathbf{v}(t)$ ... $\sin t\mathbf{i} + \cos t\mathbf{j} + \mathbf{C}$

a ... $= \mathbf{k}$

... $\mathbf{k}$

... $\cos t\mathbf{j} + \mathbf{k})\, dt = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k} + \mathbf{C}$

... $\mathbf{i} \implies \mathbf{C} = \mathbf{0}$

... $t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}$

... $(\cos 2)\mathbf{i} + (\sin 2)\mathbf{j} + 2\mathbf{k}$

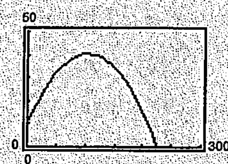
The velocity of an object involves both magnitude and direction of motion, whereas speed involves only magnitude.

24. (a) The speed is increasing.

(b) The speed is decreasing.

25. $\mathbf{r}(t) = (88 \cos 30^\circ)t\mathbf{i} + [10 + (88 \sin 30^\circ)t - 16t^2]\mathbf{j}$

$= 44\sqrt{3}t\mathbf{i} + (10 + 44t - 16t^2)\mathbf{j}$



26. $\mathbf{r}(t) = (900 \cos 45^\circ)t\mathbf{i} + [3 + (900 \sin 45^\circ)t - 16t^2]\mathbf{j}$

$= 450\sqrt{2}t\mathbf{i} + \left(3 + 450\sqrt{2}t - 16t^2\right)\mathbf{j}$

The maximum height occurs when $y'(t) = 450\sqrt{2} - 32t = 0$, which implies that $t = \left(225\sqrt{2}\right)/16$. The maximum height reached by the projectile is

$$y = 3 + 450\sqrt{2}\left(\frac{225\sqrt{2}}{16}\right) - 16\left(\frac{225\sqrt{2}}{16}\right)^2 = \frac{50,649}{8} = 6331.125 \text{ feet.}$$

The range is determined by setting $y(t) = 3 + 450\sqrt{2}t - 16t^2 = 0$ which implies that

$$t = \frac{-450\sqrt{2} - \sqrt{405,192}}{-32} \approx 39.779 \text{ seconds.}$$

Range: $x = 450\sqrt{2}\left(\frac{-450\sqrt{2} - \sqrt{405,192}}{-32}\right) \approx 25,315.500$ feet

27. $\mathbf{r}(t) = (v_0 \cos \theta)t\mathbf{i} + \left[h + (v_0 \sin \theta)t - \frac{1}{2}gt^2\right]\mathbf{j} = \frac{v_0}{\sqrt{2}}t\mathbf{i} + \left(3 + \frac{v_0}{\sqrt{2}}t - 16t^2\right)\mathbf{j}$

$\frac{v_0}{\sqrt{2}}t = 300$ when $3 + \frac{v_0}{\sqrt{2}}t - 16t^2 = 3.$

$$t = \frac{300\sqrt{2}}{v_0}, \frac{v_0}{\sqrt{2}}\left(\frac{300\sqrt{2}}{v_0}\right) - 16\left(\frac{300\sqrt{2}}{v_0}\right)^2 = 0, 300 - \frac{300^2(32)}{v_0^2} = 0$$

$v_0^2 = 300(32),\ v_0 = \sqrt{9600} = 40\sqrt{6},\ v_0 = 40\sqrt{6} \approx 97.98$ ft/sec

The maximum height is reached when the derivative of the vertical component is zero.

$$y(t) = 3 + \frac{tv_0}{\sqrt{2}} - 16t^2 = 3 + \frac{40\sqrt{6}}{\sqrt{2}}t - 16t^2 = 3 + 40\sqrt{3}t - 16t^2$$

$$y'(t) = 40\sqrt{3} - 32t = 0$$

$$t = \frac{40\sqrt{3}}{32} = \frac{5\sqrt{3}}{4}$$

Maximum height: $y\left(\frac{5\sqrt{3}}{4}\right) = 3 + 40\sqrt{3}\left(\frac{5\sqrt{3}}{4}\right) - 16\left(\frac{5\sqrt{3}}{4}\right)^2 = 78$ feet

28. 50 mph $= \frac{220}{3}$ ft/sec

$$\mathbf{r}(t) = \left(\frac{220}{3}\cos 15^\circ\right)t\mathbf{i} + \left[5 + \left(\frac{220}{3}\sin 15^\circ\right)t - 16t^2\right]\mathbf{j}$$

The ball is 90 feet from where it is thrown when

$$x = \frac{220}{3}\cos 15^\circ t = 90 \implies t = \frac{27}{22\cos 15^\circ} \approx 1.2706 \text{ seconds.}$$

The height of the ball at this time is

$$y = 5 + \left(\frac{220}{3}\sin 15^\circ\right)\left(\frac{27}{22\cos 15^\circ}\right) - 16\left(\frac{27}{22\cos 15^\circ}\right)^2 \approx 3.286 \text{ feet.}$$

29. $x(t) = t(v_0\cos\theta)$ or $t = \frac{x}{v_0\cos\theta}$

$y(t) = t(v_0\sin\theta) - 16t^2 + h$

$$y = \frac{x}{v_0\cos\theta}(v_0\sin\theta) - 16\left(\frac{x^2}{v_0^2\cos^2\theta}\right) + h = (\tan\theta)x - \left(\frac{16}{v_0^2}\sec^2\theta\right)x^2 + h$$

30. $y = x - 0.005x^2$

From Exercise 29 we know that $\tan\theta$ is the coefficient of x. Therefore, $\tan\theta = 1$, $\theta = (\pi/4)$ rad $= 45^\circ$. Also

$\frac{16}{v_0^2}\sec^2\theta =$ negative of coefficient of x^2

$\frac{16}{v_0^2}(2) = 0.005$ or $v_0 = 80$ ft/sec

$\mathbf{r}(t) = (40\sqrt{2}t)\mathbf{i} + (40\sqrt{2}t - 16t^2)\mathbf{j}$. Position function.

When $40\sqrt{2}t = 60$,

$$t = \frac{60}{40\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$\mathbf{v}(t) = 40\sqrt{2}\mathbf{i} + (40\sqrt{2} - 32t)\mathbf{j}$$

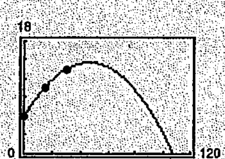
$$\mathbf{v}\left(\frac{3\sqrt{2}}{4}\right) = 40\sqrt{2}\mathbf{i} + (40\sqrt{2} - 24\sqrt{2})\mathbf{j} = 8\sqrt{2}(5\mathbf{i} + 2\mathbf{j}).$$ direction

$$\text{Speed} = \left\|\mathbf{v}\left(\frac{3\sqrt{2}}{4}\right)\right\| = 8\sqrt{2}\sqrt{25+4} = 8\sqrt{58} \text{ ft/sec}$$

31. $\mathbf{r}(t) = t\mathbf{i} + (-0.004t^2 + 0.3667t + 6)\mathbf{j}$

(a) $y = -0.004x^2 + 0.3667x + 6$

(b)



(c) $y' = -0.008x + 0.3667 = 0 \implies x = 45.8375$ and $y(45.8375) \approx 14.4$ feet.

(d) From Exercise 29,

$\tan\theta = 0.3667 \implies \theta \approx 20.14^\circ$

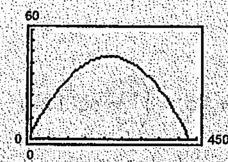
$$\frac{16\sec^2\theta}{v_0^2} = 0.004 \implies v_0^2 = \frac{16\sec^2\theta}{0.004} = \frac{4000}{\cos^2\theta}$$

$\implies v_0 \approx 67.4$ ft/sec.

32. $\mathbf{r}(t) = 140(\cos 22°)t\,\mathbf{i} + (2.5 + 140(\sin 22°)t - 16t^2)\mathbf{j}$

When $x = 375$, $t \approx 2.889$ and $y \approx 20.47$ feet.

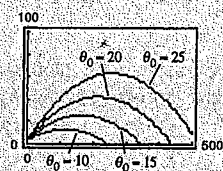
Thus, the ball clears the 10-foot fence.



33. $100 \text{ mph} = \left(100 \dfrac{\text{miles}}{\text{hr}}\right)\left(5280 \dfrac{\text{feet}}{\text{mile}}\right)/(3600 \text{ sec/hour}) = \dfrac{440}{3} \text{ ft/sec}$

(a) $\mathbf{r}(t) = \left(\dfrac{440}{3}\cos\theta_0\right)t\mathbf{i} + \left[3 + \left(\dfrac{440}{3}\sin\theta_0\right)t - 16t^2\right]\mathbf{j}$

(b)



Graphing these curves together with $y = 10$ shows that $\theta_0 = 20°$.

(c) We want

$$x(t) = \left(\frac{440}{3}\cos\theta\right)t \ge 400 \quad \text{and} \quad y(t) = 3 + \left(\frac{440}{3}\sin\theta\right)t - 16t^2 \ge 10.$$

From $x(t)$, the minimum angle occurs when $t = 30/(11\cos\theta)$. Substituting this for t in $y(t)$ yields:

$$3 + \left(\frac{440}{3}\sin\theta\right)\left(\frac{30}{11\cos\theta}\right) - 16\left(\frac{30}{11\cos\theta}\right)^2 = 10$$

$$400\tan\theta - \frac{14{,}400}{121}\sec^2\theta = 7$$

$$\frac{14{,}400}{121}(1 + \tan^2\theta) - 400\tan\theta + 7 = 0$$

$$14{,}400\tan^2\theta - 48{,}400\tan\theta + 15{,}247 = 0$$

$$\tan\theta = \frac{48{,}400 \pm \sqrt{48{,}400^2 - 4(14{,}400)(15{,}247)}}{2(14{,}400)}$$

$$\theta = \tan^{-1}\left(\frac{48{,}400 - \sqrt{1{,}464{,}332{,}800}}{28{,}800}\right) \approx 19.38°$$

34. $h = 7$ feet, $\theta = 35°$, 30 yards $= 90$ feet

$\mathbf{r}(t) = (v_0\cos 35°)t\mathbf{i} + [7 + (v_0\sin 35°)t - 16t^2]\mathbf{j}$

(a) $v_0\cos 35° t = 90$ when $7 + (v_0\sin 35°)t - 16t^2 = 4$

$$t = \frac{90}{v_0\cos 35°}$$

$$7 + (v_0\sin 35°)\left(\frac{90}{v_0\cos 35°}\right) - 16\left(\frac{90}{v_0\cos 35°}\right)^2 = 4$$

$$90\tan 35° + 3 = \frac{129{,}600}{v_0^2\cos^2 35°}$$

$$v_0^2 = \frac{129{,}600}{\cos^2 35°(90\tan 35° + 3)}$$

$$v_0 \approx 54.088 \text{ feet per second}$$

—CONTINUED—

34. —CONTINUED—

(b) The maximum height occurs when

$$y'(t) = v_0 \sin 35° - 32t = 0.$$

$$t = \frac{v_0 \sin 35}{32} \approx 0.969 \text{ second}$$

At this time, the height is $y(0.969) \approx 22.0$ feet.

(c) $x(t) = 90 \implies (v_0 \cos 35°)t = 90$

$$t = \frac{90}{54.088 \cos 35°} \approx 2.0 \text{ seconds}$$

35. $\mathbf{r}(t) = (v \cos \theta)t\mathbf{i} + [(v \sin \theta)t - 16t^2]\mathbf{j}$

(a) We want to find the minimum initial speed v as a function of the angle θ. Since the bale must be thrown to the position (16, 8), we have

$$16 = (v \cos \theta)t$$
$$8 = (v \sin \theta)t - 16t^2.$$

$t = 16/(v \cos \theta)$ from the first equation. Substituting into the second equation and solving for v, we obtain:

$$8 = (v \sin \theta)\left(\frac{16}{v \cos \theta}\right) - 16\left(\frac{16}{v \cos \theta}\right)^2$$

$$1 = 2\frac{\sin \theta}{\cos \theta} - 512\left(\frac{1}{v^2 \cos^2 \theta}\right)$$

$$512\frac{1}{v^2 \cos^2 \theta} = 2\frac{\sin \theta}{\cos \theta} - 1$$

$$\frac{1}{v^2} = \left(2\frac{\sin \theta}{\cos \theta} - 1\right)\frac{\cos^2 \theta}{512} = \frac{2 \sin \theta \cos \theta - \cos^2 \theta}{512}$$

$$v^2 = \frac{512}{2 \sin \theta \cos \theta - \cos^2 \theta}$$

We minimize $f(\theta) = \dfrac{512}{2 \sin \theta \cos \theta - \cos^2 \theta}$.

$$f'(\theta) = -512\frac{2 \cos^2 \theta - 2 \sin^2 \theta + 2 \sin \theta \cos \theta}{(2 \sin \theta \cos \theta - \cos^2 \theta)^2}$$

$$f'(\theta) = 0 \implies 2 \cos(2\theta) + \sin(2\theta) = 0$$

$$\tan(2\theta) = -2$$

$$\theta \approx 1.01722 \approx 58.28°$$

Substituting into the equation for v, $v \approx 28.78$ feet per second.

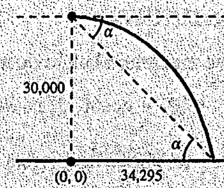
(b) If $\theta = 45°$,

$$16 = (v \cos \theta)t = v\frac{\sqrt{2}}{2}t$$

$$8 = (v \sin \theta)t - 16t^2 = v\frac{\sqrt{2}}{2}t - 16t^2$$

From part (a), $v^2 = \dfrac{512}{2(\sqrt{2}/2)(\sqrt{2}/2) - (\sqrt{2}/2)^2} = \dfrac{512}{1/2} = 1024 \implies v = 32$ ft/sec.

36. Place the origin directly below the plane. Then $\theta = 0$, $v_0 = 792$ and



$$\mathbf{r}(t) = (v_0 \cos \theta)t\mathbf{i} + (30{,}000 + (v_0 \sin \theta)t - 16t^2)\mathbf{j}$$
$$= 792t\mathbf{i} + (30{,}000 - 16t^2)\mathbf{j}$$
$$\mathbf{v}(t) = 792\mathbf{i} - 32t\mathbf{j}.$$

At time of impact, $30{,}000 - 16t^2 = 0 \implies t^2 = 1875 \implies t \approx 43.3$ seconds.

$$\mathbf{r}(43.3) = 34{,}294.6\mathbf{i}$$
$$\mathbf{v}(43.3) = 792\mathbf{i} - 1385.6\mathbf{j}$$
$$\|\mathbf{v}(43.3)\| = 1596 \text{ ft/sec} = 1088 \text{ mph}$$
$$\tan \alpha = \frac{30{,}000}{34{,}294.6} \approx 0.8748 \implies \alpha \approx 0.7187(41.18^\circ)$$

37. $\mathbf{r}(t) = (v_0 \cos \theta)t\mathbf{i} + [(v_0 \sin \theta)t - 16t^2]\mathbf{j}$

$(v_0 \sin \theta)t - 16t^2 = 0$ when $t = 0$ and $t = \dfrac{v_0 \sin \theta}{16}$.

The range is

$$x = (v_0 \cos \theta)t = (v_0 \cos \theta)\frac{v_0 \sin \theta}{16} = \frac{v_0^2}{32} \sin 2\theta.$$

Hence,

$$x = \frac{1200^2}{32} \sin(2\theta) = 3000 \implies \sin 2\theta = \frac{1}{15} \implies \theta \approx 1.91^\circ.$$

38. From Exercise 37, the range is

$$x = \frac{v_0^2}{32} \sin 2\theta.$$

Hence, $x = 150 = \dfrac{v_0^2}{32} \sin(24^\circ) \implies v_0^2 = \dfrac{4800}{\sin 24^\circ} \implies v_0 \approx 108.6$ ft/sec.

39. (a) $\theta = 10^\circ$, $v_0 = 66$ ft/sec

$$\mathbf{r}(t) = (66 \cos 10^\circ)t\mathbf{i} + [0 + (66 \sin 10^\circ)t - 16t^2]\mathbf{j}$$
$$\mathbf{r}(t) \approx (65t)\mathbf{i} + (11.46t - 16t^2)\mathbf{j}$$

Maximum height: 2.052 feet

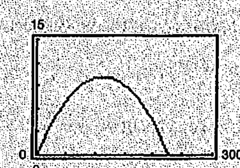
Range: 46.557 feet

(b) $\theta = 10^\circ$, $v_0 = 146$ ft/sec

$$\mathbf{r}(t) = (146 \cos 10^\circ)t\mathbf{i} + [0 + (146 \sin 10^\circ)t - 16t^2]\mathbf{j}$$
$$\mathbf{r}(t) \approx (143.78t)\mathbf{i} + (25.35t - 16t^2)\mathbf{j}$$

Maximum height: 10.043 feet

Range: 227.828 feet

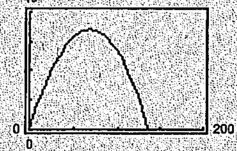


(c) $\theta = 45^\circ$, $v_0 = 66$ ft/sec

$$\mathbf{r}(t) = (66 \cos 45^\circ)t\mathbf{i} + [0 + (66 \sin 45^\circ)t - 16t^2]\mathbf{j}$$
$$\mathbf{r}(t) \approx (46.67t)\mathbf{i} + (46.67t - 16t^2)\mathbf{j}$$

Maximum height: 34.031 feet

Range: 136.125 feet

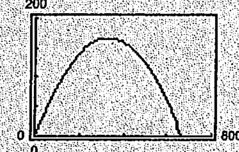


(d) $\theta = 45^\circ$, $v_0 = 146$ ft/sec

$$\mathbf{r}(t) = (146 \cos 45^\circ)t\mathbf{i} + [0 + (146 \sin 45^\circ)t - 16t^2]\mathbf{j}$$
$$\mathbf{r}(t) \approx (103.24t)\mathbf{i} + (103.24t - 16t^2)\mathbf{j}$$

Maximum height: 166.531 feet

Range: 666.125 feet



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39. —CONTINUED—

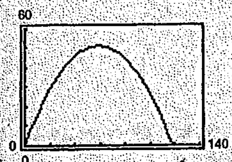
(e) $\theta = 60°,\ v_0 = 66$ ft/sec

$\mathbf{r}(t) = (66 \cos 60°)t\mathbf{i} + [0 + (66 \sin 60°)t - 16t^2]\mathbf{j}$

$\mathbf{r}(t) \approx (33t)\mathbf{i} + (57.16t - 16t^2)\mathbf{j}$

Maximum height: 51.047 feet

Range: 117.888 feet



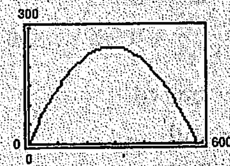
(f) $\theta = 60°,\ v_0 = 146$ ft/sec

$\mathbf{r}(t) = (146 \cos 60°)t\mathbf{i} + [0 + (146 \sin 60°)t - 16t^2]\mathbf{j}$

$\mathbf{r}(t) \approx (73t)\mathbf{i} + (126.44t - 16t^2)\mathbf{j}$

Maximum height: 249.797 feet

Range: 576.881 feet



40. (a) $\mathbf{r}(t) = t(v_0 \cos \theta)\mathbf{i} + (tv_0 \sin \theta - 16t^2)\mathbf{j}$

$t(v_0 \sin \theta - 16t) = 0$ when $t = \dfrac{v_0 \sin \theta}{16}$.

Range: $x = v_0 \cos \theta\left(\dfrac{v_0 \sin \theta}{32}\right) = \left(\dfrac{v_0^2}{32}\right) \sin 2\theta$

The range will be maximum when

$$\frac{dx}{dt} = \left(\frac{v_0^2}{32}\right) 2 \cos 2\theta = 0$$

or

$$2\theta = \frac{\pi}{2},\ \theta = \frac{\pi}{4} \text{ rad.}$$

(b) $y(t) = tv_0 \sin \theta - 16t^2$

$\dfrac{dy}{dt} = v_0 \sin \theta - 32t = 0$ when $t = \dfrac{v_0 \sin \theta}{32}$.

Maximum height:

$$y\left(\frac{v_0 \sin \theta}{32}\right) = \frac{v_0^2 \sin^2 \theta}{32} - 16\frac{v_0^2 \sin^2 \theta}{32^2} = \frac{v_0^2 \sin^2 \theta}{64}$$

Minimum height when $\sin \theta = 1$, or $\theta = \dfrac{\pi}{2}$.

41. $\mathbf{r}(t) = (v_0 \cos \theta)t\mathbf{i} + [h + (v_0 \sin \theta)t - 4.9t^2]\mathbf{j}$

$= (100 \cos 30°)t\mathbf{i} + [1.5 + (100 \sin 30°)t - 4.9t^2]\mathbf{j}$

The projectile hits the ground when $-4.9t^2 + 100\left(\frac{1}{2}\right)t + 1.5 = 0 \implies t \approx 10.234$ seconds.

The range is therefore $(100 \cos 30°)(10.234) \approx 886.3$ meters.

The maximum height occurs when $dy/dt = 0$.

$100 \sin 30 = 9.8t \implies t \approx 5.102$ sec

The maximum height is

$y = 1.5 + (100 \sin 30°)(5.102) - 4.9(5.102)^2 \approx 129.1$ meters.

42. $\mathbf{r}(t) = (v_0 \cos \theta)t\mathbf{i} + [h + (v_0 \sin \theta)t - 4.9t^2]\mathbf{j}$

$= (v_0 \cos 8°)t\mathbf{i} + [(v_0 \sin 8°)t - 4.9t^2]\mathbf{j}$

$x = 50$ when $(v_0 \cos 8°)t = 50 \implies t = \dfrac{50}{v_0 \cos 8°}$. For this value of t, $y = 0$:

$$(v_0 \sin 8°)\left(\frac{50}{v_0 \cos 8°}\right) - 4.9\left(\frac{50}{v_0 \cos 8°}\right)^2 = 0$$

$$50 \tan 8° = \frac{(4.9)(2500)}{v_0^2 \cos^2 8°} \implies v_0^2 = \frac{(4.9)50}{\tan 8° \cos^2 8°} \approx 1777.698$$

$$\implies v_0 \approx 42.2 \text{ m/sec}$$

43. $\mathbf{r}(t) = b(\omega t - \sin \omega t)\mathbf{i} + b(1 - \cos \omega t)\mathbf{j}$

$\mathbf{v}(t) = b(\omega - \omega \cos \omega t)\mathbf{i} + b\omega \sin \omega t\,\mathbf{j} = b\omega(1 - \cos \omega t)\mathbf{i} + b\omega \sin \omega t\mathbf{j}$

$\mathbf{a}(t) = (b\omega^2 \sin \omega t)\mathbf{i} + (b\omega^2 \cos \omega t)\mathbf{j} = b\omega^2[\sin(\omega t)\mathbf{i} + \cos(\omega t)\mathbf{j}]$

$\|\mathbf{v}(t)\| = \sqrt{2}\, b\omega\sqrt{1 - \cos(\omega t)}$

$\|\mathbf{a}(t)\| = b\omega^2$

(a) $\|\mathbf{v}(t)\| = 0$ when $\omega t = 0, 2\pi, 4\pi, \ldots$

(b) $\|\mathbf{v}(t)\|$ is maximum when $\omega t = \pi, 3\pi, \ldots,$ then $\|\mathbf{v}(t)\| = 2b\omega$.

44. $\mathbf{r}(t) = b(\omega t - \sin \omega t)\mathbf{i} + b(1 - \cos \omega t)\mathbf{j}$

$\mathbf{v}(t) = b\omega[(1 - \cos \omega t)\mathbf{i} + (\sin \omega t)\mathbf{j}]$

Speed $= \|\mathbf{v}(t)\| = \sqrt{2}b\omega\sqrt{1 - \cos \omega t}$ and has a maximum value of $2b\omega$ when $\omega t = \pi, 3\pi, \ldots$.

55 mph $= 80.67$ ft/sec $= 80.67$ rad/sec $= \omega$ since (since $b = 1$)

Therefore, the maximum speed of a point on the tire is twice the speed of the car:

$2(80.67)$ ft/sec $= 110$ mph

45. $\mathbf{v}(t) = -b\omega \sin(\omega t)\mathbf{i} + b\omega \cos(\omega t)\mathbf{j}$

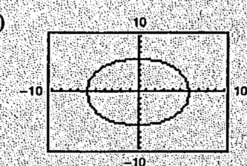
$\mathbf{r}(t) \cdot \mathbf{v}(t) = -b^2\omega \sin(\omega t)\cos(\omega t) + b^2\omega \sin(\omega t)\cos(\omega t) = 0$

Therefore, $\mathbf{r}(t)$ and $\mathbf{v}(t)$ are orthogonal.

46. (a) Speed $= \|\mathbf{v}\| = \sqrt{b^2\omega^2 \sin^2(\omega t) + b^2\omega^2 \cos^2(\omega t)}$

$= \sqrt{b^2\omega^2[\sin^2(\omega t) + \cos^2(\omega t)]} = b\omega$

(b)



The graphing utility draws the circle faster for greater values of ω.

47. $\mathbf{a}(t) = -b\omega^2 \cos(\omega t)\mathbf{i} - b\omega^2 \sin(\omega t)\mathbf{j} = -b\omega^2[\cos(\omega t)\mathbf{i} + \sin(\omega t)\mathbf{j}] = -\omega^2\mathbf{r}(t)$

$\mathbf{a}(t)$ is a negative multiple of a unit vector from $(0, 0)$ to $(\cos \omega t, \sin \omega t)$ and thus $\mathbf{a}(t)$ is directed toward the origin.

48. $\|\mathbf{a}(t)\| = b\omega^2\|\cos(\omega t)\mathbf{i} + \sin(\omega t)\mathbf{j}\| = b\omega^2$

49. $\|\mathbf{a}(t)\| = \omega^2 b,\ b = 2$

$1 = m(32)$

$\mathbf{F} = m(\omega^2 b) = \frac{1}{32}(2\omega^2) = 10$

$\omega = 4\sqrt{10}$ rad/sec

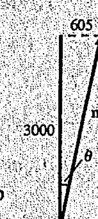
$\|\mathbf{v}(t)\| = b\omega = 8\sqrt{10}$ ft/sec

50. $\|\mathbf{v}(t)\| = 30$ mph $= 44$ ft/sec

$\omega = \frac{\|\mathbf{v}(t)\|}{b} = \frac{44}{300}$ rad/sec

$\|\mathbf{a}(t)\| = b\omega^2$

$\mathbf{F} = m(b\omega^2) = \frac{3000}{32}(300)\left(\frac{44}{300}\right)^2 = 605$ lb



Let $\mathbf{n}$ be normal to the road.

$\|\mathbf{n}\| \cos \theta = 3000$

$\|\mathbf{n}\| \sin \theta = 605$

Dividing the second equation by the first:

$\tan \theta = \frac{605}{3000}$

$\theta = \arctan\left(\frac{605}{3000}\right) \approx 11.4°$.

51. To find the range, set $y(t) = h + (v_0 \sin\theta)t - \frac{1}{2}gt^2 = 0$ then $0 = \left(\frac{1}{2}g\right)t^2 - (v_0 \sin\theta)t - h$.
By the Quadratic Formula, (discount the negative value)

$$t = \frac{v_0 \sin\theta + \sqrt{(-v_0 \sin\theta)^2 - 4[(1/2)g](-h)}}{2[(1/2)g]} = \frac{v_0 \sin\theta + \sqrt{v_0^2 \sin^2\theta + 2gh}}{g} \text{ seconds}$$

At this time,

$$x(t) = v_0 \cos\theta\left(\frac{v_0 \sin\theta + \sqrt{v_0^2\sin^2\theta + 2gh}}{g}\right) = \frac{v_0\cos\theta}{g}\left(v_0 \sin\theta + \sqrt{v_0^2\left(\sin^2\theta + \frac{2gh}{v_0^2}\right)}\right)$$

$$= \frac{v_0^2\cos\theta}{g}\left(\sin\theta + \sqrt{\sin^2\theta + \frac{2gh}{v_0^2}}\right) \text{ feet}$$

52. $h = 6$ feet, $v_0 = 45$ feet per second, $\theta = 42.5°$. From Exercise 47,

$$t = \frac{45 \sin 42.5° + \sqrt{(45)^2 \sin^2 42.5° + 2(32)(6)}}{32} \approx 2.08 \text{ seconds.}$$

At this time, $x(t) \approx 69.02$ feet.

53. $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$ Position vector

$\mathbf{v}(t) = x'(t)\mathbf{i} + y'(t)\mathbf{j} + z'(t)\mathbf{k}$ Velocity vector

$\mathbf{a}(t) = x''(t)\mathbf{i} + y''(t)\mathbf{j} + z''(t)\mathbf{k}$ Acceleration vector

$$\text{Speed} = \|\mathbf{v}(t)\| = \sqrt{(x'(t)^2 + y'(t)^2 + z'(t)^2} = C, \ C \text{ is a constant.}$$

$$\frac{d}{dt}[x'(t)^2 + y'(t)^2 + z'(t)^2] = 0$$

$$2x'(t)x''(t) + 2y'(t)y''(t) + 2z'(t)z''(t) = 0$$

$$2[x'(t)x''(t) + y'(t)y''(t) + z'(t)z''(t)] = 0$$

$$\mathbf{v}(t) \cdot \mathbf{a}(t) = 0$$

Orthogonal

54. $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$

$y(t) = m(x(t)) + b$, m and b are constants.

$\mathbf{r}(t) = x(t)\mathbf{i} + [m(x(t)) + b]\mathbf{j}$

$\mathbf{v}(t) = x'(t)\mathbf{i} + mx'(t)\mathbf{j}$

$s(t) = \sqrt{[x'(t)]^2 + [mx'(t)]^2} = C$, C is a constant.

Thus, $x'(t) = \dfrac{C}{\sqrt{1 + m^2}}$

$x''(t) = 0$

$\mathbf{a}(t) = x''(t)\mathbf{i} + mx''(t)\mathbf{j} = \mathbf{0}$.

55. $\mathbf{r}(t) = 6\cos t\,\mathbf{i} + 3\sin t\,\mathbf{j}$

(a) $\mathbf{v}(t) = \mathbf{r}'(t) = -6\sin t\,\mathbf{i} + 3\cos t\,\mathbf{j}$

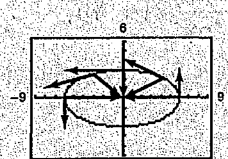
$$\|\mathbf{v}(t)\| = \sqrt{36\sin^2 t + 9\cos^2 t} = 3\sqrt{4\sin^2 t + \cos^2 t} = 3\sqrt{3\sin^2 t + 1}$$

$\mathbf{a}(t) = \mathbf{v}'(t) = -6\cos t\,\mathbf{i} - 3\sin t\,\mathbf{j}$

(b)

t	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π
Speed	3	$\frac{3}{2}\sqrt{10}$	6	$\frac{3}{2}\sqrt{13}$	3

(c)



(d) The speed is increasing when the angle between $\mathbf{v}$ and $\mathbf{a}$ is in the interval

$$\left[0, \frac{\pi}{2}\right).$$

The speed is decreasing when the angle is in the interval

$$\left(\frac{\pi}{2}, \pi\right].$$