Accel Pre-Calculus   Name: _____

2.05 Graphing Sine and Cosine- Phase Shift Notes

For $y = a\sin[b(\theta - c)] + d$ or $y = a\cos[b(\theta - c)] + d$

Phase Shift- horizontal shift; 'c' tells you the phase shift; direction is opposite of what you think; <u>you must factor out 'b' to see the phase shift!</u>

Examples: Identify the transformations that occur on the parent function and graph the function.

1. $y = \sin\left(\theta - \frac{3\pi}{4}\right)$ shift right by $\frac{3\pi}{4}$ units

2. $y = \cos(2\theta + \pi)$

$y = \cos\left[2\left(\theta + \frac{\pi}{2}\right)\right]$

* shift left by $\frac{\pi}{2}$ units
* shrink by factor of 2 (period)

2.05 Practice: Graphing Sine & Cosine with Amplitude, Period, and Phase Shift

Write an equation of the sine function given the following information.

period $= \frac{2\pi}{b}$

$\frac{14\pi}{1} = \frac{2\pi}{b}$

$14\pi b = 2\pi$

1) amp. = 3.5 and a period of 14π

$b = \frac{2}{14} = \frac{1}{7}$

$\boxed{y = 3.5\sin\left(\frac{1}{7}\theta\right)}$

2) amp. $= \frac{1}{2}$, period $= \frac{1}{3}$, and phase shift right $\frac{1}{2}$

$\frac{1}{3} = \frac{2\pi}{b}$

$b = 6\pi$

$y = \frac{1}{2}\sin\left[6\pi\left(\theta - \frac{1}{2}\right)\right]$

Write an equation of the cosine function given the following information.

3) amp = 1.25, period=6π, & phase shift right 3π down 3

$\frac{6\pi}{1} = \frac{2\pi}{b}$

$6\pi b = 2\pi$

$b = \frac{1}{3}$

$y = 1.25\cos\left[\frac{1}{3}(\theta - 3\pi)\right] - 3$

4) amp. $= \frac{2}{3}$, period $= \frac{3}{4}$, and phase shift left 2

$\frac{3}{4} = \frac{2\pi}{b}$

$3b = 8\pi$

$b = \frac{8\pi}{3}$

$y = \frac{2}{3}\cos\left[\frac{8\pi}{3}(\theta + 2)\right]$

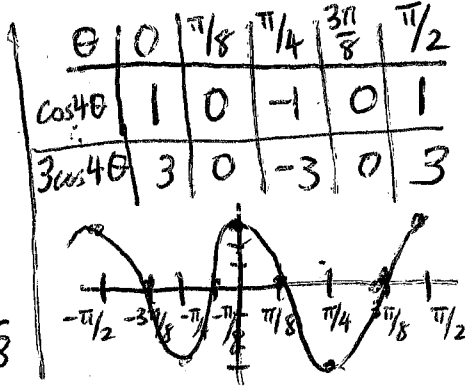**For each function, state the amplitude and period. Then label the axes appropriately and sketch the graph.**

$a = 3$

5) $y = 3\cos(4\theta)$ $b = 4$

Amp: __3__ Per: _____ PS: __—__

period $= \frac{2\pi}{b}$

period $= \frac{2\pi}{4} = \frac{\pi}{2}$

$I = \frac{1}{4} \cdot P$

$I = \frac{1}{4} \cdot \frac{\pi}{2} = \frac{\pi}{8}$

θ	0	$\pi/8$	$\pi/4$	$\frac{3\pi}{8}$	$\pi/2$
$\cos4\theta$	1	0	-1	0	1
$3\cos4\theta$	3	0	-3	0	3

$\frac{2\pi}{\pi/2}$

$a = -4$ period $= \frac{2\pi}{\pi/2}$

6) $y = -4\sin\frac{\pi\theta}{2}$ $b = \frac{\pi}{2}$ $p = 2\pi \cdot \frac{2}{\pi} = 4$

Amp: __4__ Per: __4__ PS: __—__

$I = \frac{4}{4} = 1$

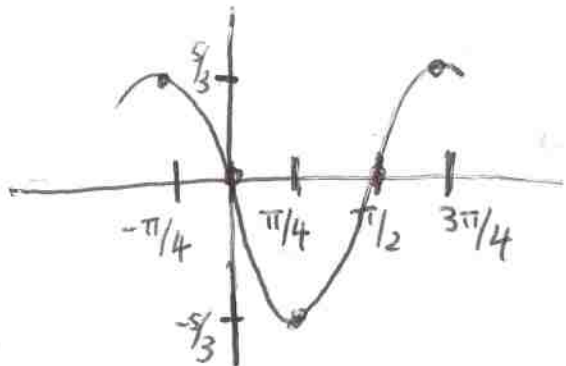| θ	0	1	2	3	4
$\sin\left(\frac{\pi\theta}{2}\right)$	0	1	0	-1	0
$-4\sin\left(\frac{\theta\pi}{2}\right)$	0	-4	0	4	0

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7) $y = \frac{5}{3}\cos\left(2\theta + \frac{\pi}{2}\right)$

$y = \frac{5}{3}\cos\left[2\left(\theta + \frac{\pi}{4}\right)\right]$

$a = \frac{5}{3}$ $b = 2$ PS: left $\frac{\pi}{4}$

period $= \frac{2\pi}{b} \to \frac{2\pi}{2} = \pi$ $I = \frac{\pi}{4}$

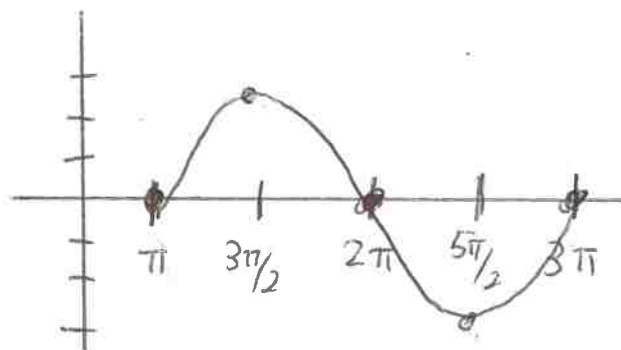| θ	$0 - \frac{\pi}{4}$	$\frac{\pi}{4} - \frac{\pi}{4}$	$\frac{\pi}{2} - \frac{\pi}{4}$	$\frac{3\pi}{4} - \frac{\pi}{4}$	$\pi - \frac{\pi}{4}$
	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$
$\cos(2\theta)$	1	0	-1	0	1
$\frac{5}{3}\cos(2\theta)$	$\frac{5}{3}$	0	$\frac{-5}{3}$	0	$\frac{5}{3}$

8) $y = 2.5\sin(\theta - \pi)$

$a = 2.5$ $b = 1$ $c = \pi$ P.S. right π units

θ	$0 + \pi$	$\frac{\pi}{2} + \pi$	$\pi + \pi$	$\frac{3\pi}{2} + \pi$	$2\pi + \pi$
	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π
$\sin(\theta - \pi)$	0	1	0	-1	0
$2.5\sin(\theta - \pi)$	0	2.5	0	-2.5	0

pg. 11 (a)

9)

$$y = 6\cos\left(\frac{\theta}{4} - \frac{\pi}{8}\right)$$

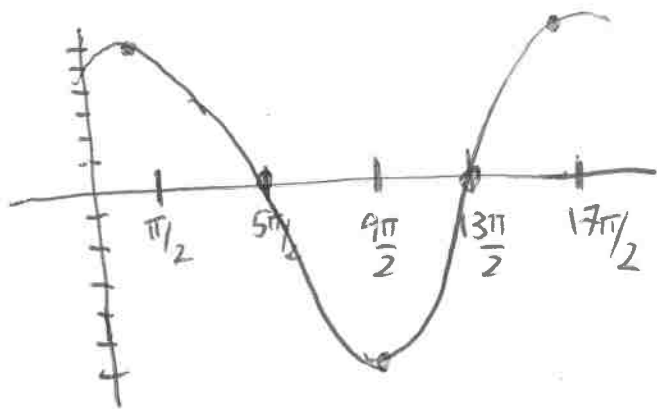$$y = 6\cos\left[\frac{1}{4}\left(\theta - \frac{\pi}{2}\right)\right]$$

Amp: __6__   Per: __8π__   PS: right π/2

$$period = \frac{2\pi}{b} = \frac{2\pi}{1/4} = 2\pi \cdot 4 = 8\pi$$

$$I = \frac{8\pi}{4} = 2\pi$$

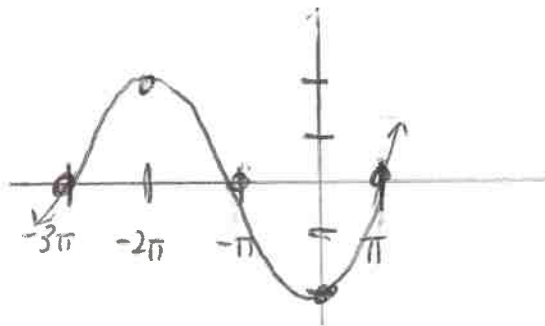| θ	0 +π/2	2π +π/2	4π +π/2	6π +π/2	8π +π/2
	π/2	5π/2	9π/2	13π/2	17π/2
$\cos\left(\frac{\theta}{4}\right)$	1	0	-1	0	1
$6\cos\left(\frac{\theta}{4}\right)$	6	0	-6	0	6

10)

$$y = 2\sin\left(\frac{\theta}{2} + \frac{3\pi}{2}\right)$$

$$y = 2\sin\left[\frac{1}{2}\left(\theta + 3\pi\right)\right]$$

a = 2   b = ½   P.S. __shift left 3π__

$$period = \frac{2\pi}{1/2} = 2\pi \cdot \frac{2}{1} = 4\pi \qquad I = \frac{4\pi}{4} = \pi$$

θ	0 -3π	π -3π	2π -3π	3π -3π	4π -3π
	-3π	-2π	-π	0	π
sin(θ)	0	1	0	-1	0
2sin(θ)	0	2	0	-2	0

11)

$$y = -3\cos(3\pi\theta + \pi)$$
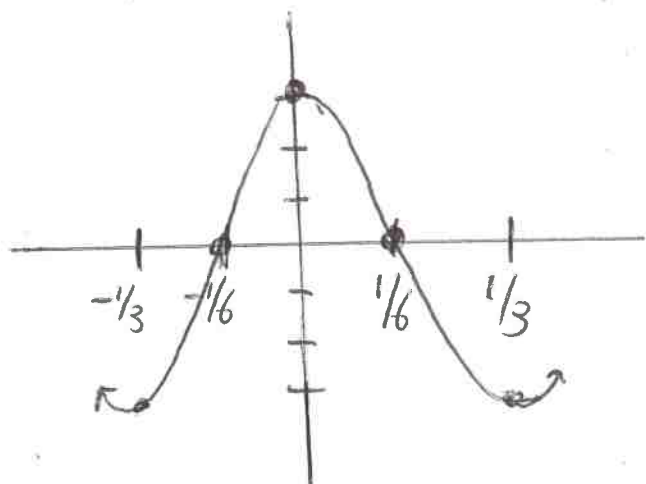
$$y = -3\cos\left[3\pi\left(\theta + \frac{1}{3}\right)\right]$$

$a = -3 \quad b = 3\pi \quad$ P.S. left $\frac{1}{3}$

period $= \frac{2\pi}{b} \rightarrow \frac{2\pi}{3\pi} = \frac{2}{3} \qquad I = \frac{1}{4} \cdot P$

$$I = \frac{1}{4} \cdot \frac{2}{3} = \frac{2}{12} = \frac{1}{6}$$

θ	$\boxed{-1/3}$ $0 - 1/3$	$\boxed{-1/6}$ $1/6\ ^{-2/6}$	$\boxed{0}$ $2/6\ ^{-2/6}$	$\boxed{+1/6}$ $3/6\ ^{-2/6}$	$\boxed{1/3}$ $4/6\ ^{-2/6}$
$\cos\theta$	1	0	-1	0	1
$-3\cos\theta$	-3	0	3	0	-3

$-1/3 \quad -1/6 \quad 1/6 \quad 1/3$

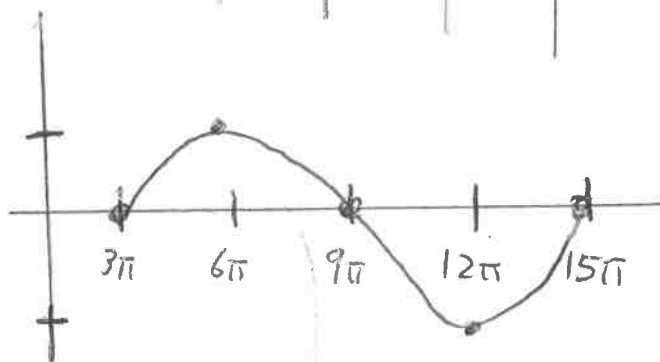# 12)

$$y = \sin\left(\frac{\theta}{6} - \frac{\pi}{2}\right)$$

$$y = \sin\left[\frac{1}{6}(\theta - 3\pi)\right]$$

$a = 1 \quad b = \frac{1}{6} \quad$ P.S. right 3π units

period $= \frac{2\pi}{1/6} \rightarrow 2\pi \cdot \frac{6}{1} = 12\pi \quad I = \frac{12\pi}{4} = 3\pi$

θ	$\boxed{3\pi}$ $0 + 3\pi$	$\boxed{6\pi}$ $3\pi + 3\pi$	$\boxed{9\pi}$ $6\pi + 3\pi$	$\boxed{12\pi}$ $9\pi + 3\pi$	$\boxed{15\pi}$ $12\pi + 3\pi$
$\sin\theta$	0	1	0	-1	0
$1\sin\theta$	0	1	0	-1	0

$3\pi \quad 6\pi \quad 9\pi \quad 12\pi \quad 15\pi$

7) $y = \frac{5}{3}\cos\left(2\theta + \frac{\pi}{2}\right)$

Amp: _____ Per: _____ PS: _____

8) $y = 2.5\sin(\theta - \pi)$

Amp: _____ Per: _____ PS: _____

9) $y = 6\cos\left(\frac{\theta}{4} - \frac{\pi}{8}\right)$

Amp: _____ Per: _____ PS: _____

10) $y = 2\sin\left(\frac{\theta}{2} + \frac{3\pi}{2}\right)$

Amp: _____ Per: _____ PS: _____

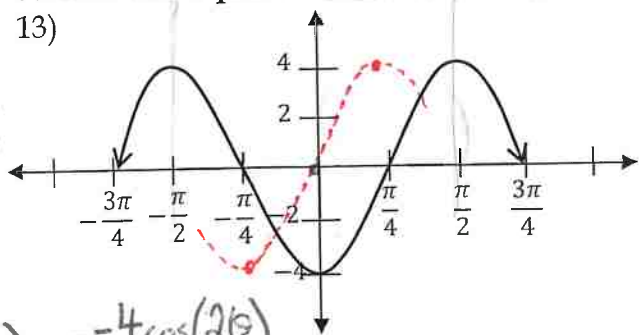11) $y = -3\cos(3\pi\theta + \pi)$

Amp: _____ Per: _____ PS: _____

12) $y = \sin\left(\frac{\theta}{6} - \frac{\pi}{2}\right)$

Amp: _____ Per: _____ PS: _____

Write a sine equation _and_ a cosine equation for each graph below:

$a = -4$

$period = \pi$

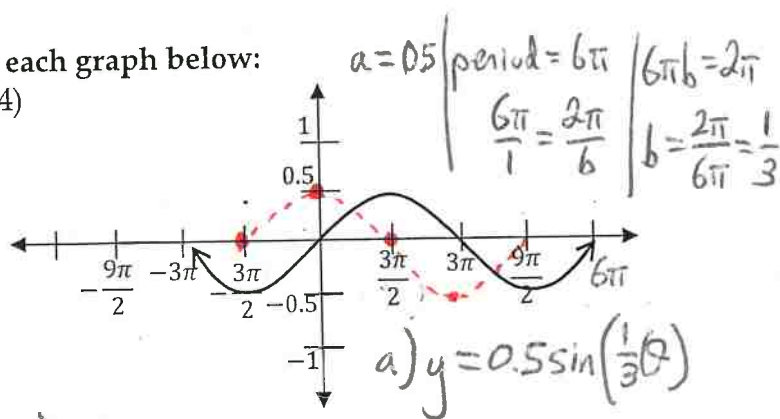$\frac{\pi}{1} = \frac{2\pi}{b}$

$b\pi = 2\pi$

$b = 2$

13)

a) $y = -4\cos(2\theta)$

b) phase shift right $\pi/4$

$a = 4 \quad b = 2$

$y = 4\sin\left[2\left(\theta - \frac{\pi}{4}\right)\right]$

14)

$a = 0.5 \quad period = 6\pi \quad 6\pi b = 2\pi$

$\frac{6\pi}{1} = \frac{2\pi}{b} \quad b = \frac{2\pi}{6\pi} = \frac{1}{3}$

a) $y = 0.5\sin\left(\frac{1}{3}\theta\right)$

b) shift cosine graph $\frac{3\pi}{2}$ to the right

$y = 0.5\cos\left[\frac{1}{3}\left(\theta - \frac{3\pi}{2}\right)\right]$

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