Accel Pre-Calculus **Name:** ____________________

2.06 Graphing Sine and Cosine- Vertical Shift Notes

For $y = a\sin[b(\theta - c)] + d$ or $y = a\cos[b(\theta - c)] + d$

Vertical Shift- amount graph moves up or down; 'd' tells you the vertical shift; direction is what you assume

Examples: Identify the transformations that occur on the parent function and graph the function.

1. $y = \sin\left(\theta - \frac{\pi}{4}\right) + 3$

$a = 1$ P.S. right $\pi/4$ units

$b = 1$ $d = 3 \rightarrow$ vertical shift up 3 units.

2. $y = \cos\left(3\theta + \frac{\pi}{2}\right) - 1$

$y = \cos\left[3\left(\theta + \frac{\pi}{6}\right)\right] - 1$ | $a = 3$, $b = 3 \rightarrow$ period $= \frac{2\pi}{3}$ | shift left $\pi/6$ units, $d = -1 \rightarrow$ vertical shift down 1 unit.

2.06 Worksheet: Graphing Sine and Cosine with Amplitude, Period, Phase Shift and Vertical Shift **Date:** ____________

Write the function described:

1. A sine function with amplitude = 15, with a reflection, period = 4π, phase shift right $\frac{\pi}{2}$ and vertical shift down 10.

$a = 15$, $4\pi = \frac{2\pi}{b}$ | $4\pi b = 2\pi$, $b = \frac{2\pi}{4\pi} = \frac{1}{2}$ | $c = \pi/2$, $d = -10$ | $y = -15\sin\left[\frac{1}{2}\left(\theta - \frac{\pi}{2}\right)\right] - 10$

2. A cosine function having an amplitude = $\frac{1}{2}$, period = $\frac{\pi}{3}$, phase shift left $\frac{\pi}{3}$ and vertical shift up 5.

$a = 1/2$, $\frac{\pi}{3} = \frac{2\pi}{b}$ | $b\pi = 6\pi$, $b = 6$ | $c = +\pi/3$, $d = 5$ | $y = \frac{1}{2}\cos\left[6\left(\theta + \frac{\pi}{3}\right)\right] + 5$

3. A cosine function that has been vertically stretched by a factor of 4, has been reflected over the x-axis, has been horizontally compressed to a period of $\frac{2\pi}{3}$, and has been shifted right π units and down 3 units.

$a = 4$, $\frac{2\pi}{3} = \frac{2\pi}{b}$ | $2\pi b = 6\pi$, $b = \frac{6\pi}{2\pi} = 3$ | $c = -\pi$, $d = -3$ | $y = -4\cos[3(\theta - \pi)] - 3$

4. A sine function that has been horizontally stretched to a period of 10, vertically compressed by a factor of $\frac{2}{3}$, shifted left $\frac{5}{2}$, and up 1 unit.

$10 = \frac{2\pi}{b}$, $10b = 2\pi$, $b = \frac{2\pi}{10} = \frac{\pi}{5}$ | $a = 2/3$, $c = +5/2$, $d = 1$ | $y = \frac{2}{3}\sin\left[\frac{\pi}{5}\left(\theta + 5/2\right)\right] + 1$

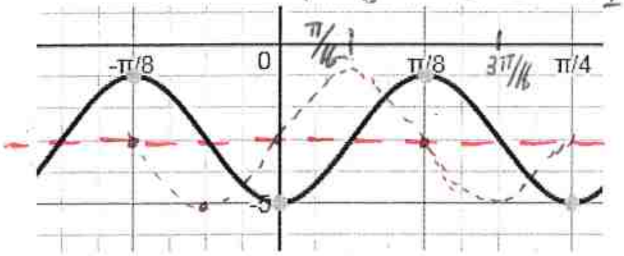
5) $y=-2\cos[8(\theta)]-3$

$y=2\sin[8(\theta-\pi/16)]-3$

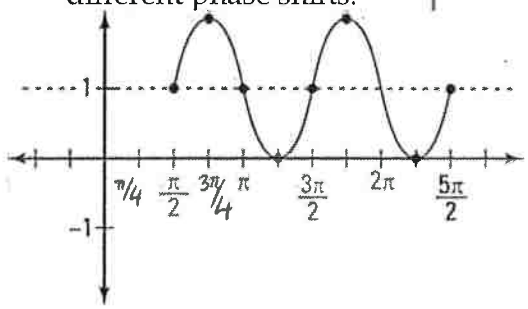
6) a) $y=\sin[2(\theta-\frac{\pi}{2})]+1$

b) $y=\cos[2(\theta-\frac{3\pi}{4})]+1$

5. The wave graphed below. It is **both:** a sine function and a cosine function, with different phase shifts. period $=\pi/4$ | $\frac{\pi}{4}=\frac{2\pi}{b}$ | $b\pi=8\pi$ | $b=8$ | $I=\frac{1}{4}\cdot\frac{\pi}{4}$ | $I=\pi/16$



6. The wave graphed below. It is **both:** a sine function and a cosine function, with different phase shifts. period $=\pi$ | $\pi=\frac{2\pi}{b}$ | $b\pi=2\pi$ | $b=2$



Given $y=a\sin[b(\theta-c)]+d$ and $y=a\cos[b(\theta-c)]+d$. For each function, state the amplitude, period, interval, phase shift (PS), and vertical shift (VS). Sketch the graph and label the axes.

7) $y=-2\sin\frac{\theta}{2}+1$

Amp: 2 Per: ________ Int: ________

PS: none VS: up 1

8) $y=2\cos\left(\theta+\frac{\pi}{2}\right)-3$

Amp: ______ Per: ________ Int: ________

PS: ____________________ VS: ______

9) $y=\frac{1}{2}\sin\left(\frac{\theta}{6}+\frac{\pi}{3}\right)+4$

Amp: ______ Per: ________ Int: ________

PS: ____________________ VS: ______

10) $y=-\cos(6\pi\theta+\pi)-1$

Amp: ______ Per: ________ Int: ________

PS: ____________________ VS: ______

11) $y=3\sin\left(2\theta+\frac{3\pi}{2}\right)+0.5$

Amp: ______ Per: ________ Int: ________

PS: ____________________ VS: ______

12) $y=-\cos\left(\frac{\pi}{4}\theta+\frac{\pi}{2}\right)-1.5$

Amp: ______ Per: ________ Int: ________

PS: ____________________ VS: ______

7) $y = -2\sin\left(\frac{\theta}{2}\right) + 1$

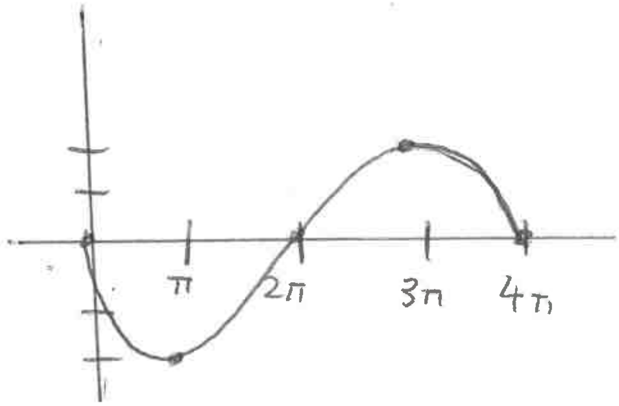
$a = -2 \quad b = 1/2 \quad d = 1$

$\text{period} = \frac{2\pi}{1/2} \rightarrow 2\pi \cdot \frac{2}{1} = 4\pi$

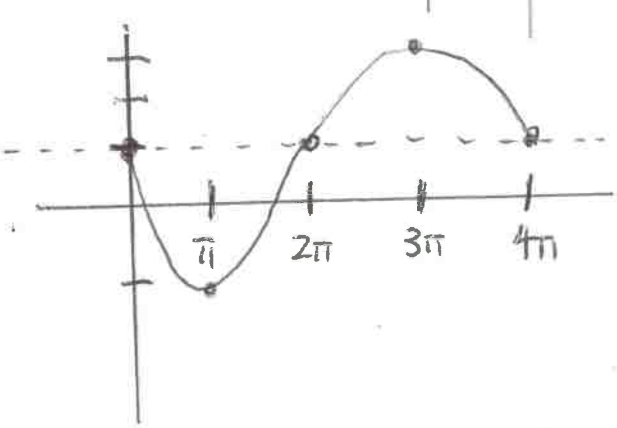
$I = \frac{4\pi}{4} = \pi$

Amp: 2   Per: 4π Int: π

PS none   VS up 1 unit



θ	0	π	2π	3π	4π
$\sin\left(\frac{\theta}{2}\right)$	0	1	0	-1	0
$-2\sin\left(\frac{\theta}{2}\right)$	0	-2	0	2	0

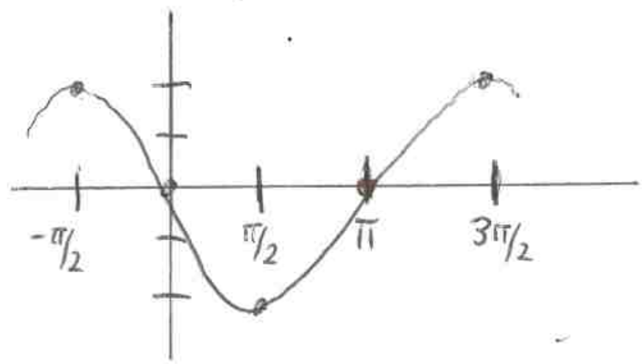


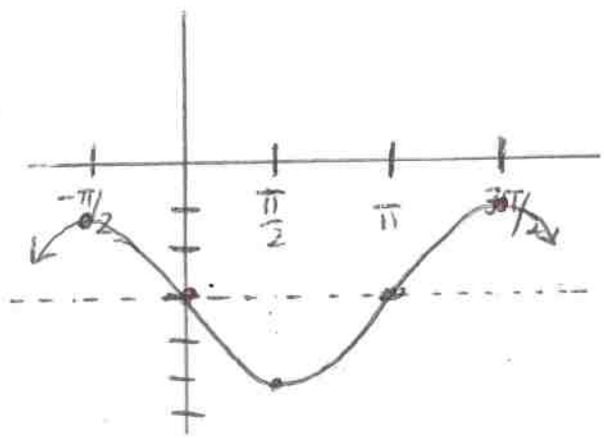
8) $y = 2\cos(\theta + \pi/2) - 3$

$a = 2 \quad b = 1$ P.S. left $\pi/2$ units

$d = -3$ VS: down 3 units   period: 2π

	$\boxed{-\pi/2}$	$\boxed{0}$	$\boxed{\pi/2}$	$\boxed{\pi}$	$\boxed{3\pi/2}$
θ	$0 - \pi/2$	$\pi/2 - \pi/2$	$\pi - \pi/2$	$3\pi/2 - \pi/2$	$2\pi - \pi/2$
$\cos(\theta)$	1	0	-1	0	1
$2\cos(\theta)$	2	0	-2	0	2





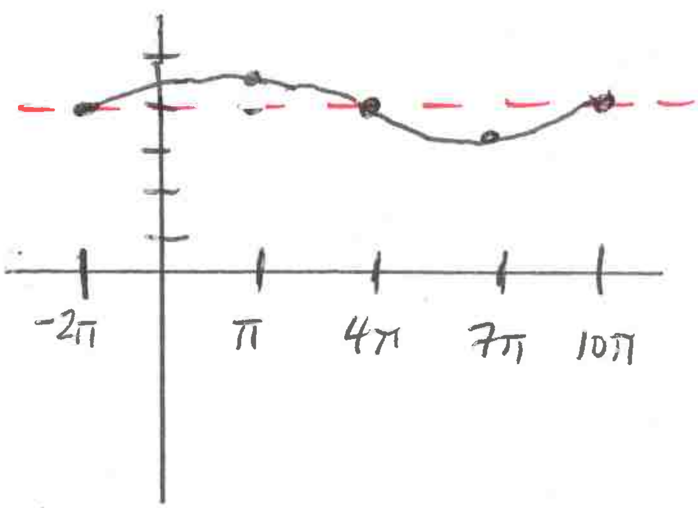
9) $y=\frac{1}{2}\sin\left(\frac{\theta}{6}+\frac{\pi}{3}\right)+4$

$y=\frac{1}{2}\sin\left[\frac{1}{6}(\theta+2\pi)\right]+4$

period $=\frac{2\pi}{b}\rightarrow\frac{2\pi}{1/6}=2\pi\cdot 6=12\pi$

$I=\frac{12\pi}{4}=3\pi$

θ	$0-2\pi$ $\boxed{-2\pi}$	$3\pi-2\pi$ $\boxed{\pi}$	$6\pi-2\pi$ $\boxed{4\pi}$	$9\pi-2\pi$ $\boxed{7\pi}$	$12\pi-2\pi$ $\boxed{10\pi}$
$\sin\left(\frac{\theta}{6}\right)$	0	1	0	−1	0
$\frac{1}{2}\sin\left(\frac{\theta}{6}\right)$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	0



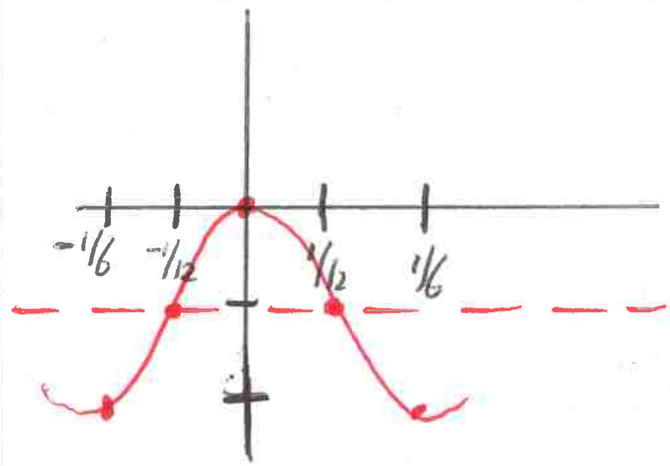
10) $y=-\cos(6\pi\theta+\pi)-1$

$y=-\cos\left[6\pi\left(\theta+\frac{1}{6}\right)\right]-1$

period $=\frac{2\pi}{6\pi}=\frac{1}{3}$ $\quad I=\frac{1}{4}\cdot P\rightarrow\frac{1}{4}\cdot\frac{1}{3}=\frac{1}{12}$

*shift left $\frac{1}{6}$ or $\frac{2}{12}$

θ	$0-\frac{1}{6}$ $\boxed{-1/6}$	$\frac{1}{12}-\frac{2}{12}$ $\boxed{-1/12}$	$\frac{2}{12}-\frac{2}{12}$ $\boxed{0}$	$\frac{3}{12}-\frac{2}{12}$ $\boxed{1/12}$	$\frac{4}{12}-\frac{2}{12}$ $\boxed{1/6}$
$\cos\theta$	1	0	−1	0	1
$-\cos\theta$	−1	0	1	0	−1

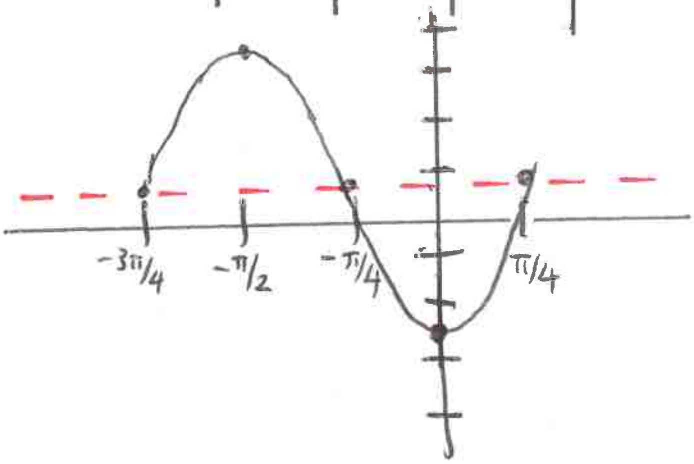


11) $y = 3\sin\left(2\theta + \frac{3\pi}{2}\right) + 0.5$

$y = 3\sin\left[2\left(\theta + \frac{3\pi}{4}\right)\right] + 0.5$

period $= \frac{2\pi}{2} \rightarrow \pi$ $\quad I = \frac{\pi}{4}$

θ	$0 - \frac{3\pi}{4}$ $\boxed{-3\pi/4}$	$\pi/4 - \frac{3\pi}{4}$ $\boxed{-\pi/2}$	$\pi/2 - 3\pi/4$ $\boxed{-\pi/4}$	$3\pi/4 - 3\pi/4$ $\boxed{0}$	$\pi - 3\pi/4$ $\boxed{\pi/4}$
$\sin(2\theta)$	0	1	0	−1	0
$3\sin(2\theta)$	0	3	0	−3	0



12) $y = -\cos\left(\frac{\pi}{4}\theta + \frac{\pi}{2}\right) - 1.5$

$y = -\cos\left[\frac{\pi}{4}(\theta + 2)\right] - 1.5$

period $= \frac{2\pi}{\pi/4} \rightarrow 2\pi \cdot \frac{4}{\pi} = 8$ $\quad I = \frac{8}{4} = 2$

θ	$0 - 2$ $\boxed{-2}$	$2 - 2$ $\boxed{0}$	$4 - 2$ $\boxed{2}$	$6 - 2$ $\boxed{4}$	$8 - 2$ $\boxed{6}$
$\cos(\frac{\pi}{4}\theta)$	1	0	−1	0	1
$-\cos(\frac{\pi}{4}\theta)$	−1	0	1	0	−1