2.15 Applications of Sinusoidal Functions Day 1

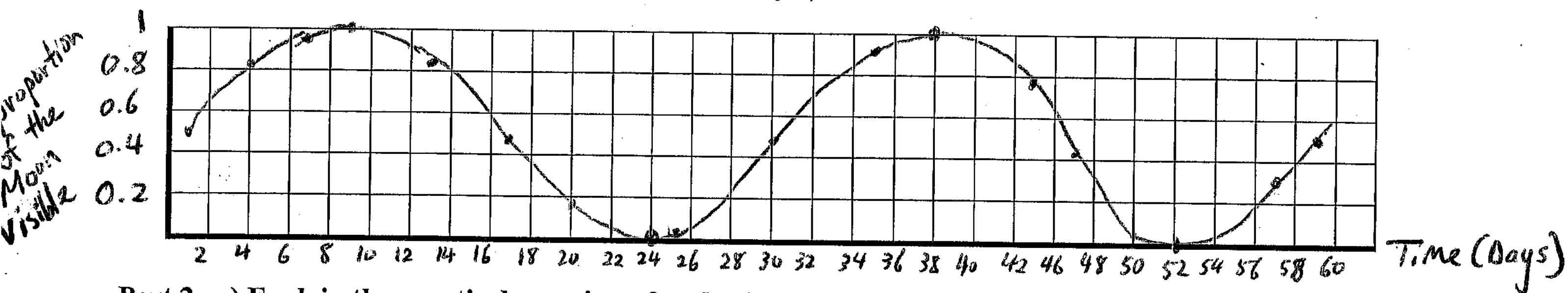
Date: ____________________

Problem 1: Moon Watch

As an avid sky-watcher, you know that the moon is always half illuminated by the sun and that how much of the moon we can see illuminated depends on where it is in its orbit around Earth. For the first months of the year, you watched the moon and kept data for the amount of the moon that was visible each day.

Day of Year	1	4	7	9	13	17	20	24	27	30	35	38	43	47	50	52	57	59
Proportion of Moon Visible	0.5	0.81	0.98	1	0.82	0.46	0.17	0	0.16	0.5	0.92	1	0.79	0.41	0.12	0	0.33	0.5

Part 1. Graph this relationship. (Hint: scale the x-axis by 2)



Part 2. a) Explain the practical meaning of each of the following characteristics.
b) Find the value of each characteristic for this application.

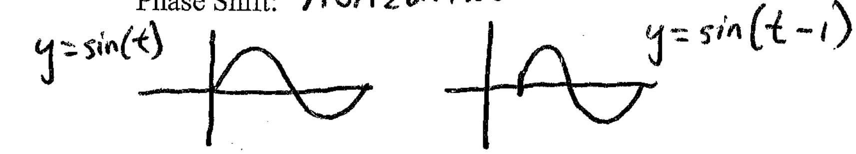
Sinusoidal Function: smooth curve with a repeating oscillating pattern around the midline. (comes from the word "sine")

Period: time period needed to complete a full cycle.
$P = 30 - 1 = 29$ days

Vertical Shift: midline shift (midline = average value). midline is $y = 0.5$
$d = 0.5$

Amplitude: distance from midline to the peak (or to minimum)
$a = 0.5$

Phase Shift: horizontal shift



Part 3. **Write the equation of the sinusoidal function that models the visibility of the moon.**

$a = 0.5$

period $= 29$

period $= \frac{2\pi}{b}$

$29 = \frac{2\pi}{b}$

$29 b = 2\pi$

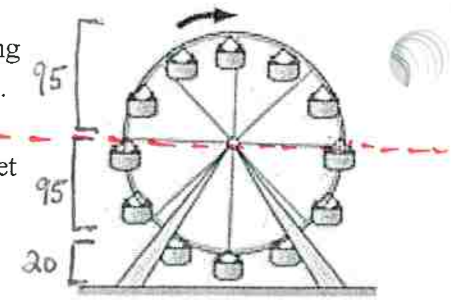
$b = \frac{2\pi}{29}$

$y = a\sin[b(x-c)] + d$

$y = 0.5\sin\left[\frac{2\pi}{29}(x-1)\right] + 0.5$

Problem 2: A Day at the Amusement Park

There are many rides at the amusement park whose movement can be modeled using trigonometric functions. The Ferris wheel is a good example of periodic movement.



You want to ride the Ferris wheel. It has a radius of 95 feet and is suspended 20 feet above the ground. The wheel rotates at a rate of 2 revolutions every 20 minutes.

Part 1.

a) What is the period of the function that models the movement of the Ferris wheel?

1 revolution = 10 mins. $p = 10$ mins. period $= \frac{2\pi}{b}$ | $10 = \frac{2\pi}{b}$ | $b = \frac{2\pi}{10}$

$10b = 2\pi$ | $b = \pi/5$

b) What is midline of the function? What is the practical meaning of midline?

$d = 20 + 95 = 115$ $y = 115$ | midline is the average height during the ride.

$d = 115$

c) What is amplitude of the function? What is the practical meaning of amplitude?

Amplitude is 95 ft. This is the height above/below the midline (avg. height of ferris wheel)

d) Write the equation of a sinusoidal function that models your height above the ground over time.

You enter the ride at the lowest point (20 ft) and exit at the same location.

$a = -95$ $c = 0$

$b = \pi/5$ $d = 115$

e) Graph the function:

period = 10

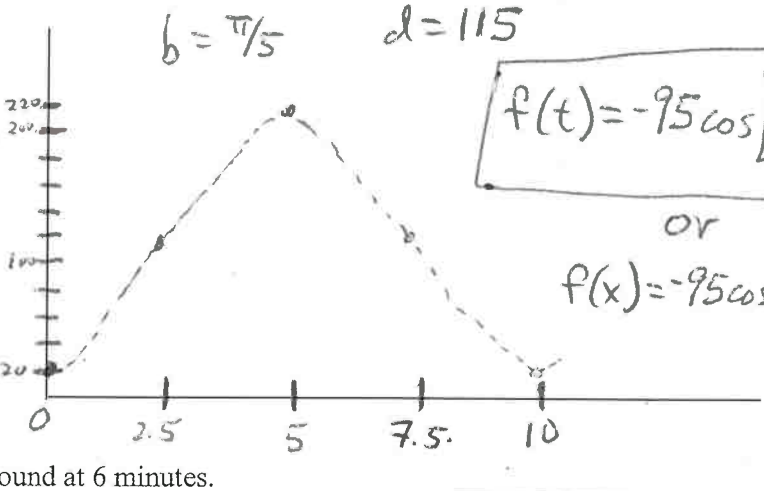
$I = \frac{1}{4} \cdot P = \frac{1}{4}(10) = \frac{5}{2} = 2.5$

(time, height)

(0, 20) | (5, 210)

(2.5, 115) | (7.5, 115)

(10, 20)



$$\boxed{f(t) = -95\cos\left[\pi/5(t)\right] + 115}$$

or

$$f(x) = -95\cos\left(\frac{\pi}{5}x\right) + 115$$

Part 2.

a) Determine your height above the ground at 6 minutes.

$f(6) = -95\cos\left(\frac{\pi}{5}(6)\right) + 115 \approx 191.85 \approx \boxed{192 \text{ ft.}}$

b) Suppose the Ferris wheel moved faster and completed a revolution in 5 minutes. How would the function change? (Rewrite the equation reflecting this change).

$p = 5$ | $5 = \frac{2\pi}{b}$ | $5b = 2\pi$ | $b = \frac{2\pi}{5}$ | $y = -95\cos\left(\frac{2\pi}{5}t\right) + 115$

c) If the radius of the Ferris wheel remained the same, but the height of the wheel was raised 15 feet higher, how would the function change? (Rewrite the equation reflecting this change).

midline would change

$d = 115 + 15 = 130$

$y = -95\cos\left(\frac{\pi}{5}t\right) + 130$

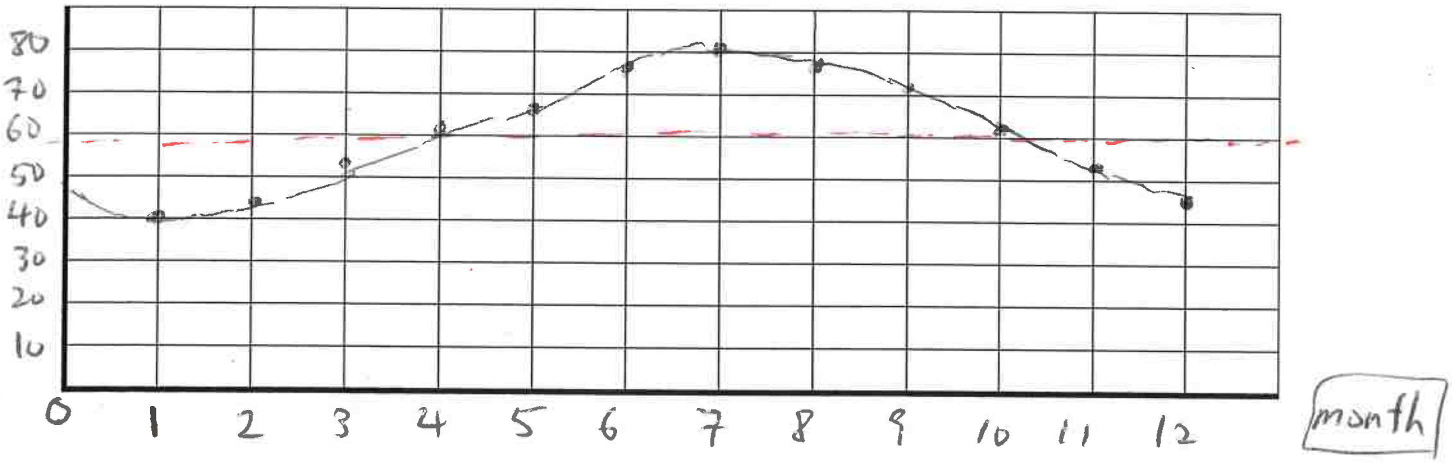
Homework: Global Warming?

Scientists are continually monitoring the average temperatures across the globe to determine if Earth is experiencing climate change. One statistic scientists use to describe the climate of an area is average monthly temperature. The average monthly temperature of a region is the mean of its average high and low temperatures.

The table below shows the average monthly temperature (°F) in Atlanta from January to December.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
40°	44°	53°	61°	68°	76°	80°	78°	72°	62°	53°	45°

Part 1. Graph this relationship. Use January as month 1 of the year, meaning that for January, $t = 1$.



Part 2.

a) What is the period of the function that models the monthly temperatures in Atlanta? Find b.

period = 12 $\quad 12 = \frac{2\pi}{b} \quad | \quad 12b = 2\pi \quad b = \frac{2\pi}{12} = \frac{\pi}{6}$

b) What is the vertical shift of this model? Find ~~k~~ (d)

$d = 60$

c) What is the amplitude of this model? Find a.

$a = 20$

d) What is the phase shift of this model? Find c.

$c = 1$

e) Write the equation of a sinusoidal function that models the average monthly temperatures.

$$y = -20\cos\left[\frac{\pi}{6}(x-1)\right] + 60$$

f) Use the equation to predict the temperature in October.

$y = -20\cos\left[\frac{\pi}{6}(10-1)\right] + 60 \quad \boxed{y \approx 60}$