Non-AP Calculus 2.1 Definition of Derivative Practice Worksheet

1) Use the Limit Definition of a derivative to find f'(x) if f(x) = $2x^2 - 3x + 1$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

2) Use the Alternative definition of the derivative to find $f'(2)$ if $f(x) = \sqrt{2 - x}$

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

3) Use the Limit Definition of a Derivative to find f'(x) if $f(x) = \sqrt{2x - 1}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

4) Use the Limit Definition of a derivative to find $f'(3)$ if $f(x) = \frac{2}{5-x}$

5) Use either general or alternative method above to find the equation of the tangent line to $f(x) = 2x - 3x^2$ at $x = -1$. $y - y_1 = m(x - x_1)$

Key

Non-AP Calculus 2.1 **Definition of Derivative Practice Worksheet**

1) Use the Limit Definition of a derivative to find f'(x) if f(x) = $2x^2 - 3x + 1$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ $f(\;) = 2(\;)^2 - 3(\;) + 1$

$f'(x) = \lim_{h\to 0} \frac{2(x+h)^2 - 3(x+h) + 1 - (2x^2 - 3x + 1)}{h}$

$f'(x) = \lim_{h\to 0} \frac{2(x^2+2xh+h^2) - 3x - 3h + 1 - 2x^2 + 3x - 1}{h}$

$f'(x) = \lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{3x} - 3h + \cancel{1} - \cancel{2x^2} + \cancel{3x} - \cancel{1}}{h}$

$f'(x) = \lim_{h\to 0} \frac{4xh + 2h^2 - 3h}{h}$

$f'(x) = \lim_{h\to 0} \frac{\cancel{h}(4x + 2h - 3)}{\cancel{h}} = 4x + 2(0) - 3$

$\boxed{f'(x) = 4x - 3}$

2) Use the Alternative definition of the derivative to find $H'(2)$ if $H(x) = \sqrt{3-x}$

$f'(c) = \lim_{x\to c} \frac{f(x) - f(c)}{x - c}$ $c = 2$ $h(2) = \sqrt{3-2} = 1$

$h'(2) = \lim_{x\to 2} \frac{h(x) - h(2)}{x - 2}$

$h'(2) = \lim_{x\to 2} \frac{\sqrt{3-x} - 1}{x - 2}$

$h'(2) = \lim_{x\to 2} \frac{(\sqrt{3-x} - 1)}{(x-2)} \cdot \frac{(\sqrt{3-x} + 1)}{(\sqrt{3-x} + 1)}$

$h'(2) = \lim_{x\to 2} \frac{3 - x - 1}{(x-2)(\sqrt{3-x} + 1)}$

$h'(2) = \lim_{x\to 2} \frac{\cancel{(2-x)}(-1)}{\cancel{(x-2)}(\sqrt{3-x} + 1)}$

$h'(2) = \frac{-1}{\sqrt{3-2} + 1} = \frac{-1}{1+1} = \boxed{-\frac{1}{2}}$

3) Use the Limit Definition of a Derivative to find f'(x) if f(x) = $\sqrt{2x-1}$

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$f(\;) = \sqrt{2(\;) - 1}$

$f(x+h) = \sqrt{2(x+h) - 1}$

$f'(x) = \lim_{h\to 0} \frac{\sqrt{2x+2h-1} - \sqrt{2x-1}}{h}$

$f'(x) = \lim_{h\to 0} \frac{\sqrt{2x+2h-1} - \sqrt{2x-1}}{h} \cdot \frac{(\sqrt{2x+2h-1} + \sqrt{2x-1})}{\sqrt{2x+2h-1} + \sqrt{2x-1}}$

$f'(x) = \lim_{h\to 0} \frac{2x + 2h - 1 - (2x - 1)}{h[\sqrt{2x+2h-1} + \sqrt{2x-1}]}$

$f'(x) = \lim_{h\to 0} \frac{\cancel{2x} + 2h - \cancel{1} - \cancel{2x} + \cancel{1}}{h[\sqrt{2x+2h-1} + \sqrt{2x-1}]}$

$f'(x) = \lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}(\sqrt{2x+2h-1} + \sqrt{2x-1})}$

$= \frac{2}{\sqrt{2x-1} + \sqrt{2x-1}} = \frac{2}{2\sqrt{2x-1}} = \boxed{\frac{1}{\sqrt{2x-1}}}$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f(\ ) = \frac{2}{5-(\ )}$$

4) Use the Limit Definition of a derivative to find f'(3) if f(x) = $\frac{2}{5-x}$

$$f(x+h) = \frac{2}{5-(x+h)}$$

$$f'(x) = \lim_{h\to 0}\frac{\frac{2}{5-(x+h)} - \frac{2}{5-x}}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{\frac{2}{5-x-h} - \frac{2}{5-x}}{h}\cdot(5-x)(5-x-h)$$

$$\lim_{h\to 0}\frac{2(5-x)-2(5-x-h)}{h(5-x)(5-x-h)}$$

$$\lim_{h\to 0}\frac{\cancel{10}-\cancel{2x}-\cancel{10}+\cancel{2x}+2h}{h(5-x)(5-x-h)}$$

$$\lim_{h\to 0}\frac{2\cancel{h}}{\cancel{h}(5-x)(5-x-h)}$$

$$f'(x) = \lim_{h\to 0}\frac{2}{(5-x)(5-x-h)} = \frac{2}{(5-x)(5-x)}$$

$$\boxed{f'(x) = \frac{2}{(5-x)^2}}$$

$$f'(3) = \frac{2}{(5-3)^2} = \frac{2}{(2)^2}$$

$$\boxed{f'(3) = \frac{1}{2}}$$

$f(-1) = 2(-1)-3 = -5$

5) Use either general or alternative method above to find the equation of the tangent line to $f(x) = 2x - 3x^2$ at $x = -1$. $y - y_1 = m(x - x_1)$

$$f(\ ) = 2(\ ) - 3(\ )^2$$

$$f(x+h) = 2(x+h) - 3(x+h)^2$$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{2(x+h)-3(x+h)^2-(2x-3x^2)}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{2x+2h-3(x^2+2xh+h^2)-2x+3x^2}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{\cancel{2x}+2h-\cancel{3x^2}-6xh-3h^2-\cancel{2x}+\cancel{3x^2}}{h}$$

$$\lim_{h\to 0}\frac{2h-6xh-3h^2}{h} \rightarrow \lim_{h\to 0}\frac{\cancel{h}(2-6x-3h)}{\cancel{h}}$$

$$f'(x) = 2-6x-0$$

$$\boxed{f'(x) = 2-6x}$$

$$f'(-1) = 2-6(-1)$$
$$= 2+6$$
$$f'(-1) = 8$$

Point: $f(-1) = -2-3 = -5$

slope: $m = 8$

Alt. method $f'(c) = \lim_{x\to c}\frac{f(x)-f(c)}{x-c}$

$$f'(-1) = \lim_{x\to -1}\frac{f(x)-f(-1)}{x-(-1)}$$

$$f'(-1) = \lim_{x\to -1}\frac{2x-3x^2-(-5)}{x+1} = \frac{2x-3x^2+5}{x+1}$$

$$= \lim_{x\to -1}\frac{-1(3x^2-2x-5)}{x+1}$$

$$f'(-1) = \lim_{x\to -1}\frac{-1(3x-5)\cancel{(x+1)}}{\cancel{(x+1)}}$$

$$f'(-1) = -1(-3-5) = 8$$

point: $(-1, -5)$

slope: $m = 8$

$$\boxed{y+5 = 8(x+1)}$$