

1) Use the Limit Definition of a derivative to find $G'(x)$ if $G(x) = 3x^2 - 4x + 5$

2) Use the Alternative definition of the derivative to find $H'(2)$ if $H(x) = \sqrt{5 - x}$

3) Use the Limit Definition of a Derivative to find $H'(x)$ if $H(x) = \sqrt{x - 3}$

4) Use the Limit Definition of a derivative to find $f'(3)$ if $f(x) = \frac{5}{x-2}$

5) Use either general or alternative method above to find the equation of the tangent line to $f(x) = 3x - 4x^2$ at $x = -1$.

Key

1) Use the Limit Definition of a derivative to find $G'(x)$ if $G(x) = 3x^2 - 4x + 5$

$$G'(x) = \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h}$$

$$g(x+h) = 3(x+h)^2 - 4(x+h) + 5$$

$$G'(x) = \lim_{h \rightarrow 0} \frac{3(x+h)^2 - 4(x+h) + 5 - (3x^2 - 4x + 5)}{h}$$

$$G'(x) = \lim_{h \rightarrow 0} \frac{3(x^2 + 2xh + h^2) - 4x - 4h + 5 - 3x^2 + 4x - 5}{h}$$

$$G'(x) = \lim_{h \rightarrow 0} \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{4x} - 4h + 5 - \cancel{3x^2} + \cancel{4x} - 5}{h}$$

$$G'(x) = \lim_{h \rightarrow 0} \frac{k(6x + 3h - 4)}{k}$$

$$G'(x) = 6x - 4$$

2) Use the Alternative definition of the derivative to find $H'(2)$ if $H(x) = \sqrt{5-x}$

$$h'(c) = \lim_{x \rightarrow c} \frac{h(x) - h(c)}{x - c}$$

$$h'(2) = \lim_{x \rightarrow 2} \frac{h(x) - h(2)}{x - 2}$$

$$h'(2) = \lim_{x \rightarrow 2} \frac{\sqrt{5-x} - \sqrt{3}}{x - 2}$$

$$h(2) = \sqrt{5-2} = \sqrt{3}$$

$$h'(2) = \lim_{x \rightarrow 2} \frac{\sqrt{5-x} - \sqrt{3}}{x - 2} \cdot \frac{\sqrt{5-x} + \sqrt{3}}{\sqrt{5-x} + \sqrt{3}}$$

$$= \lim_{x \rightarrow 2} \frac{(5-x) - 3}{(x-2)(\sqrt{5-x} + \sqrt{3})}$$

$$= \lim_{x \rightarrow 2} \frac{5-x-3}{(x-2)(\sqrt{5-x} + \sqrt{3})}$$

$$\lim_{x \rightarrow 2} \frac{(2-x) \cdot (-1)}{(x-2)(\sqrt{5-x} + \sqrt{3})}$$

$$\lim_{x \rightarrow 2} \frac{-1}{\sqrt{5-2} + \sqrt{3}} = \frac{-1}{2\sqrt{3}}$$

$$H'(2) = \frac{-1}{2\sqrt{3}}$$

3) Use the Limit Definition of a Derivative to find $H'(x)$ if $H(x) = \sqrt{x-3}$

$$h'(x) = \lim_{h \rightarrow 0} \frac{h(x+h) - h(x)}{h}$$

$$h'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{x+h-3} - \sqrt{x-3}}{h}$$

$$h'(x) = \lim_{h \rightarrow 0} \frac{(\sqrt{x+h-3} - \sqrt{x-3}) \cdot (\sqrt{x+h-3} + \sqrt{x-3})}{h(\sqrt{x+h-3} + \sqrt{x-3})}$$

$$h'(x) = \lim_{h \rightarrow 0} \frac{x+h-3 - (x-3)}{h(\sqrt{x+h-3} + \sqrt{x-3})}$$

$$h(x+h) = \sqrt{x+h-3}$$

$$h'(x) = \lim_{h \rightarrow 0} \frac{x+h-3 - x+3}{h(\sqrt{x+h-3} + \sqrt{x-3})}$$

$$h'(x) = \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h-3} + \sqrt{x-3}}$$

$$= \frac{1}{\sqrt{x-3} + \sqrt{x-3}}$$

$$h'(x) = \frac{1}{2\sqrt{x-3}}$$

4) Use the Limit Definition of a derivative to find $f'(3)$ if $f(x) = \frac{5}{x-2}$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x+h) = \frac{5}{x+h-2}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{5}{x+h-2} - \frac{5}{x-2}}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{5(x-2) - 5(x+h-2)}{h(x+h-2)(x-2)}$$

$$\lim_{h \rightarrow 0} \frac{5x - 10 - 5x - 5h + 10}{h(x+h-2)(x-2)}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{-5h}{h(x+h-2)(x-2)}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{-5}{(x-2)(x-2)} = \frac{-5}{(x-2)^2}$$

$$f'(x) = \frac{-5}{(x-2)^2} \quad f'(3) = \frac{-5}{(3-2)^2} = -5$$

5) Use either general or alternative method above to find the equation of the tangent line to $f(x) = 3x - 4x^2$ at $x = -1$.

General Method

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{3(x+h) - 4(x+h)^2 - (3x - 4x^2)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{3x + 3h - 4(x^2 + 2xh + h^2) - 3x + 4x^2}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{3x + 3h - 4x^2 - 8xh - 4h^2 - 3x + 4x^2}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{h(3 - 8x - 4h)}{h} = 3 - 8x - 4(0)$$

$$f'(x) = 3 - 8x$$

$$f(x) = 3x - 4x^2$$

$$f(-1) = 3(-1) - 4(-1)^2 = -3 - 4 = -7$$

point: $(-1, -7)$

slope: $m = 11$

$$f'(-1) = 3 - 8(-1) = 11$$

$$y + 7 = 11(x + 1)$$

$$f(-1) = 3(-1) - 4(-1)^2 = -7$$

Alternative Method $f'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$

$$f'(-1) = \lim_{x \rightarrow -1} \frac{f(x) - f(-1)}{x - (-1)}$$

$$f'(-1) = \lim_{x \rightarrow -1} \frac{3x - 4x^2 - (-7)}{x + 1}$$

$$= \lim_{x \rightarrow -1} \frac{-4x^2 + 3x + 7}{x + 1}$$

$$= \lim_{x \rightarrow -1} \frac{-1(4x^2 - 3x - 7)}{x + 1}$$

$$= \lim_{x \rightarrow -1} \frac{-1(4x-7)(x+1)}{(x+1)}$$

$$= \lim_{x \rightarrow -1} -1(4x-7) = 11$$

$$f'(-1) = 11$$

