Non-AP Calculus 2.1 **Definition of Derivative Quiz Review WS #2**

1) Use the General Limit Definition of a derivative to find f'(x) if $f(x) = 4x^2 + 6x - 3$

2) Use the Alternative definition of the derivative to find $G'(2)$ if $G(x) = \sqrt{2x - 1}$

3) Use the Limit Definition of a Derivative to find f'(x) if $f(x) = \sqrt{8 - x}$

4) Use the General Limit Definition of a derivative to find $f'(2)$ if $f(x) = \frac{9}{5-x}$

5) Use either general or alternative method above to find the equation of the tangent line to $f(x) = 2 - 5x - 3x^2$ at $x = 2$.

1) Use the General Limit Definition of a derivative to find f'(x) if $f(x) = 4x^2 + 6x - 3$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x+h) = 4(x+h)^2 + 6(x+h) - 3$$

$$f'(x) = \lim_{h\to 0} \frac{\overbrace{4(x+h)^2 + 6(x+h) - 3}^{f(x+h)} - \overbrace{(4x^2+6x-3)}^{f(x)}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{4(x^2+2xh+h^2)+6x+6h-3-4x^2-6x+3}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{4x^2}+8xh+4h^2+\cancel{6x}+6h-\cancel{3}-\cancel{4x^2}-\cancel{6x}+\cancel{3}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{8xh+4h^2+6h}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(8x+4h+6)}{\cancel{h}}$$

$$\boxed{f'(x) = 8x+6}$$

2) Use the Alternative definition of the derivative to find $G'(2)$ if $G(x) = \sqrt{2x-1}$

$$G'(c) = \lim_{x\to c} \frac{G(x) - G(c)}{x - c}$$

$$G(2) = \sqrt{2(2)-1} = \sqrt{3}$$

$$G'(2) = \lim_{x\to 2} \frac{G(x) - G(2)}{x-2}$$

$$G'(2) = \lim_{x\to 2} \frac{\sqrt{2x-1} - \sqrt{3}}{x-2}$$

$$G'(2) = \lim_{x\to 2} \frac{(\sqrt{2x-1} - \sqrt{3})}{(x-2)} \cdot \frac{(\sqrt{2x-1} + \sqrt{3})}{(\sqrt{2x-1} + \sqrt{3})}$$

$$G'(2) = \lim_{x\to 2} \frac{2x-1-3}{(x-2)(\sqrt{2x-1}+\sqrt{3})} = \lim_{x\to 2} \frac{2x-4}{(x-2)(\sqrt{2x-1}+\sqrt{3})}$$

$$= \lim_{x\to 2} \frac{2\cancel{(x-2)}}{\cancel{(x-2)}(\sqrt{2x-1}+\sqrt{3})} = \frac{2}{\sqrt{3}+\sqrt{3}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\boxed{G'(2) = \frac{1}{\sqrt{3}}}$$

3) Use the Limit Definition of a Derivative to find f'(x) if $f(x) = \sqrt{8-x}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x+h) = \sqrt{8-(x+h)} = \sqrt{8-x-h}$$

$$f'(x) = \lim_{h\to 0} \frac{(\sqrt{8-x-h} - \sqrt{8-x})}{h} \cdot \frac{(\sqrt{8-x-h} + \sqrt{8-x})}{(\sqrt{8-x-h} + \sqrt{8-x})}$$

$$f'(x) = \lim_{h\to 0} \frac{8-x-h-(8-x)}{h(\sqrt{8-x-h} + \sqrt{8-x})}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{8}-\cancel{x}-h-\cancel{8}+\cancel{x}}{h(\sqrt{8-x-h} + \sqrt{8-x})}$$

$$f'(x) = \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}(\sqrt{8-x-h} + \sqrt{8-x})}$$

$$f'(x) = \lim_{h\to 0} \frac{-1}{\sqrt{8-x-h} + \sqrt{8-x}}$$

$$= \frac{-1}{\sqrt{8-x} + \sqrt{8-x}}$$

$$\boxed{f'(x) = \frac{-1}{2\sqrt{8-x}}}$$

4) Use the General Limit Definition of a derivative to find f '(2) if $f(x) = \frac{9}{5-x}$ *Find $f'(x)$ first!!

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$f(x+h) = \frac{9}{5-(x+h)}$

$f(x+h) = \frac{9}{5-x-h}$

$f'(x) = \lim_{h\to 0} \frac{\left(\frac{9}{5-x-h} - \frac{9}{5-x}\right)}{h}$ $(5-x-h)(5-x)$ $(5-x-h)(5-\cancel{x})$

$f'(x) = \lim_{h\to 0} \frac{9(5-x)-9(5-x-h)}{h(5-x-h)(5-x)}$

$f'(x) = \lim_{h\to 0} \frac{\cancel{45}-\cancel{9x}-\cancel{45}+\cancel{9x}+9h}{h(5-x-h)(5-x)}$

$= \lim_{h\to 0} \frac{9\cancel{h}}{\cancel{h}(5-x-h)(5-x)}$

$f'(x) = \lim_{h\to 0} \frac{9}{(5-x-h)(5-x)} = \frac{9}{(5-x)(5-x)}$

$\boxed{f'(x) = \frac{9}{(5-x)^2}}$

$f'(2) = \frac{9}{(5-2)^2} = \frac{9}{3^2} = \boxed{1}$

$\boxed{f'(2)=1}$

5) Use either general or alternative method above to find the equation of the tangent line to $f(x) = 2 - 5x - 3x^2$ at $x = 2$.

$f(x+h) = 2-5(x+h)-3(x+h)^2$

General Method: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$f'(x) = \lim_{h\to 0} \frac{2-5(x+h)-3(x+h)^2-(2-5x-3x^2)}{h}$

$f'(x) = \lim_{h\to 0} \frac{2-5x-5h-3(x^2+2xh+h^2)-2+5x+3x^2}{h}$

$f'(x) = \lim_{h\to 0} \frac{\cancel{2}-\cancel{5x}-5h-\cancel{3x^2}-6xh-3h^2-\cancel{2}+\cancel{5x}+\cancel{3x^2}}{h}$

$f'(x) = \lim_{h\to 0} \frac{-5h-6xh-3h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(-5-6x-3h)}{\cancel{h}}$

$f'(x) = \lim_{h\to 0} -5-6x-3h = -5-6x$

$\boxed{f'(x) = -5-6x}$

point: $f(2) = 2-5(2)-3(2)^2 = -20$

slope: $f'(2) = -5-6(2) = -17$

Alternative Method: $f'(c) = \lim_{x\to c} \frac{f(x)-f(c)}{x-c}$

$f'(2) = \lim_{x\to 2} \frac{f(x)-f(2)}{x-2}$ ← $f(2)=-20$

$f'(2) = \lim_{x\to 2} \frac{2-5x-3x^2-(-20)}{x-2}$

$f'(2) = \lim_{x\to 2} \frac{-3x^2-5x+22}{x-2}$

$f'(2) = \lim_{x\to 2} \frac{-(3x^2+5x-22)}{x-2}$ $11 \times -66 \; -6 \; 3 \; 5 \; 3$

$f'(2) = \lim_{x\to 2} \frac{-(3x+11)\cancel{(x-2)}}{\cancel{(x-2)}}$

$f'(2) = -(3(2)+11)$

$\boxed{f'(2) = -17}$

point: $(2,-20)$

slope: $m = -17$

$y - y_1 = m(x-x_1)$

$\boxed{y+20 = -17(x-2)}$