A.P. Calculus AB Ch 2.1 Definition of Derivative Quiz Review

1. Use the general definition of the derivative to find $\frac{d}{dx}G(x)$ if $G(x) = 3x^2 - 2x + 5$

2. Use the alternative definition of the derivative to find $H'(1)$ if $H(x) = \sqrt{3 - x}$

3. Use either of above methods to find the equation of the tangent line to $f(x) = 2 - 3x^2$ at $x = 2$.

Key

1. Use the general definition of the derivative to find $\frac{d}{dx}G(x)$ if $G(x) = 3x^2 - 2x + 5$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$g(\ ) = 3(\ )^2 - 2(\ ) + 5$

$g(x+h) = 3(x+h)^2 - 2(x+h) + 5$

$$g'(x) = \lim_{h\to 0}\frac{g(x+h)-g(x)}{h}$$

$$\lim_{h\to 0}\frac{3(x+h)^2-2(x+h)+5-(3x^2-2x+5)}{h}$$

$$\lim_{h\to 0}\frac{3(x^2+2xh+h^2)-2x-2h+5-3x^2+2x-5}{h}$$

$$\lim_{h\to 0}\frac{\cancel{3x^2}+6xh+3h^2\cancel{-2x}-2h\cancel{+5}\cancel{-3x^2}\cancel{+2x}\cancel{-5}}{h}$$

$$\lim_{h\to 0}\frac{6xh+3h^2-2h}{h}$$

$$\lim_{h\to 0}\frac{\cancel{h}(6x+3h-2)}{\cancel{h}}$$

$$6x+3(0)-2$$

$$\boxed{G'(x) = 6x-2}$$

2. Use the alternative definition of the derivative to find $H'(1)$ if $H(x) = \sqrt{3-x}$

Alt. definition:

$$f'(c) = \lim_{x\to c}\frac{f(x)-f(c)}{x-c}$$

$$H'(1) = \lim_{x\to 1}\frac{H(x)-H(1)}{x-1}$$

$$H'(1) = \lim_{x\to 1}\frac{\sqrt{3-x}-\sqrt{2}}{x-1}$$

$H(1) = \sqrt{3-1} = \sqrt{2}$

$$H'(1) = \lim_{x\to 1}\frac{(\sqrt{3-x}-\sqrt{2})}{x-1}\cdot\frac{(\sqrt{3-x}+\sqrt{2})}{(\sqrt{3-x}+\sqrt{2})}$$

$$H'(1) = \lim_{x\to 1}\frac{3-x-2}{(x-1)(\sqrt{3-x}+\sqrt{2})}$$

$$H'(1) = \lim_{x\to 1}\frac{(1-\cancel{x})\,-1}{(\cancel{x-1})(\sqrt{3-x}+\sqrt{2})}$$

$$H'(1) = \lim_{x\to 1}\frac{-1}{(\sqrt{3-x}+\sqrt{2})}$$

$$H'(1) = \lim_{x\to 1}\frac{-1}{\sqrt{3-1}+\sqrt{2}}$$

$$= \frac{-1}{\sqrt{2}+\sqrt{2}}$$

$$\boxed{H'(1) = \frac{-1}{2\sqrt{2}}}$$

3. Use either of above methods to find the equation of the tangent line to $f(x) = 2 - 3x^2$ at $x = 2$.

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$f(\ ) = 2-3(\ )^2$

$f(x+h) = 2-3(x+h)^2$

$$f'(x) = \lim_{h\to 0}\frac{2-3(x+h)^2-(2-3x^2)}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{2-3(x^2+2xh+h^2)-2+3x^2}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{\cancel{2}\cancel{-3x^2}-6xh-3h^2\cancel{-2}\cancel{+3x^2}}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{\cancel{h}(-6x-3h)}{\cancel{h}} = -6x-3(0) = -6x$$

$f'(x) = -6x$

point: $f(2) = 2-3(2)^2 = 2-12 = -10$

slope: $f'(2) = -6(2) = -12$

point: $(2, -10)$

slope: $m = -12$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y + 10 = -12(x-2)}$$