Ch. 2.1 Exercise Problems → Rates of Change and the Derivative

p. 168-171 #17-29 odd

$$* \; f'(c) = \lim_{x\to c} \frac{f(x)-f(c)}{x-c}$$ ← use the alternative limit definition of derivative

17) Find rate of change at the indicated value

$f(x) = 5x-2$

a) $\boxed{c=0}$ $\quad f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0} \to \lim_{x\to 0} \frac{5x-2-(-2)}{x-0} \to \frac{5x}{x} \to \boxed{5}$

$\boxed{f'(0)=5}$

b) $\boxed{c=2}$ $\quad f'(2) = \lim_{x\to 2} \frac{f(x)-f(2)}{x-2} \to \lim_{x\to 2} \frac{5x-2-8}{x-2} \to \lim_{x\to 2} \frac{5x-10}{x-2}$

$\lim_{x\to 2} \frac{5(x-2)}{x-2} \to \boxed{5}$

19) $f(x) = \frac{x^2}{x+3}$

a) $\boxed{c=0}$ $\quad f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0} \to \lim_{x\to 0} \frac{\frac{x^2}{x+3} - 0}{x-0}$

$\lim_{x\to 0} \frac{x^2}{x+3} \cdot \frac{1}{x} \to \lim_{x\to 0} \frac{x}{x+3} \to \frac{0}{3} = \boxed{0}$

b) $\boxed{c=1}$ $\quad f'(1) = \lim_{x\to 1} \frac{f(x)-f(1)}{x-1} \to \lim_{x\to 1} \frac{\frac{x^2}{x+3} - \frac{1}{4}}{x-1} \to \frac{\frac{4x^2}{4(x+3)} - \frac{x+3}{4(x+3)}}{x-1}$

$\lim_{x\to 1} \frac{4x^2-x-3}{4(x+3)} \cdot \frac{1}{x-1}$

$\lim_{x\to 1} \frac{(4x+3)\cancel{(x-1)}}{4(x+3)\cancel{(x-1)}} \to \frac{7}{4(4)} \to \boxed{\frac{7}{16}}$

Find derivative of function at the given value

21) $f(x)=2x+3$ at $c=1$ $\qquad f'(c)=\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$

$$f'(1)=\lim_{x\to 1}\frac{f(x)-f(1)}{x-1} \to f'(1)=\lim_{x\to 1}\frac{2x+3-(5)}{x-1}$$

$$\lim_{x\to 1}\frac{2x-2}{x-1} \to \lim_{x\to 1}\frac{2(x-1)}{(x-1)} = \boxed{2} \qquad \boxed{f'(1)=2}$$

23) $f(x)=x^2-2$ at $c=0$

$$f'(0)=\lim_{x\to 0}\frac{f(x)-f(0)}{x-0} \to \lim_{x\to 0}\frac{x^2-2-(-2)}{x-0} \to \lim_{x\to 0}\frac{x^2}{x} \to 0$$

$$\boxed{f'(0)=0}$$

25) $f(x)=3x^2+x+5$ at $c=-1$

$$f'(-1)=\lim_{x\to -1}\frac{f(x)-f(-1)}{x-(-1)} \to \lim_{x\to -1}\frac{3x^2+x+5-(7)}{(x+1)} \to \lim_{x\to -1}\frac{3x^2+x-2}{x+1} \to \frac{0}{0}$$

$$\lim_{x\to -1}\frac{(3x-2)(\cancel{x+1})}{(\cancel{x+1})} \to -5 \qquad \boxed{f'(-1)=-5}$$

27) $f(x)=\sqrt{x}$ at $c=4$

$$f'(4)=\lim_{x\to 4}\frac{f(x)-f(4)}{x-4} \to \lim_{x\to 4}\frac{\sqrt{x}-\sqrt{4}}{x-4}\cdot\frac{(\sqrt{x}+\sqrt{4})}{(\sqrt{x}+\sqrt{4})}$$

$$f'(4)=\lim_{x\to 4}\frac{(\cancel{x-4})(1)}{(\cancel{x-4})(\sqrt{x}+\sqrt{4})} \to \frac{1}{\sqrt{4}+\sqrt{4}} = \frac{1}{2\sqrt{4}} = \frac{1}{4} \qquad \boxed{f'(4)=\frac{1}{4}}$$

29) $f(x)=\frac{2-5x}{1+x}$ at $c=0$

$$f'(0)=\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}$$

$$\lim_{x\to 0}\frac{\frac{2-5x}{1+x}-2}{x-0}$$

$$\lim_{x\to 0}\frac{\frac{2-5x}{1+x}-\frac{2(1+x)}{1+x}}{x}$$

$$\lim_{x\to 0}\frac{2-5x-2-2x}{1+x}\cdot\frac{1}{x}$$

$$\lim_{x\to 0}\frac{-7\cancel{x}}{1+x}\cdot\frac{1}{\cancel{x}} \to -7$$

$$\boxed{f'(0)=-7}$$