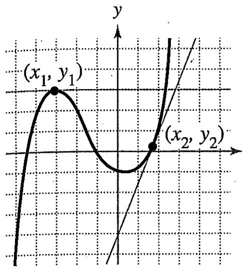
2.1 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Estimating Slope **In Exercises 1 and 2, estimate the slope of the graph at the points (x_1, y_1) and (x_2, y_2).**

1.



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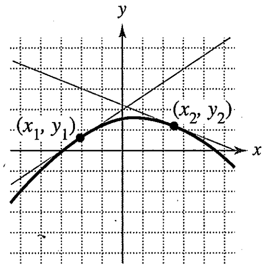
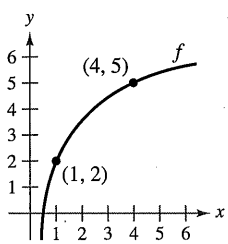


Slopes of Secant Lines **In Exercises 3 and 4, use the graph shown in the figure. To print an enlarged copy of the graph, go to *MathGraphs.com*.**



3. Identify or sketch each of the quantities on the figure.

(a) $f(1)$ and $f(4)$ (b) $f(4) - f(1)$

(c) $y = \dfrac{f(4) - f(1)}{4 - 1}(x - 1) + f(1)$

4. Insert the proper inequality symbol (< or >) between the given quantities.

(a) $\dfrac{f(4) - f(1)}{4 - 1}$ ▒ $\dfrac{f(4) - f(3)}{4 - 3}$

(b) $\dfrac{f(4) - f(1)}{4 - 1}$ ▒ $f'(1)$

Finding the Slope of a Tangent Line **In Exercises 5–10, find the slope of the tangent line to the graph of the function at the given point.**

5. $f(x) = 3 - 5x, \quad (-1, 8)$ **6.** $g(x) = \frac{3}{2}x + 1, \quad (-2, -2)$

7. $g(x) = x^2 - 9, \quad (2, -5)$ **8.** $f(x) = 5 - x^2, \quad (3, -4)$

9. $f(t) = 3t - t^2, \quad (0, 0)$ **10.** $h(t) = t^2 + 4t, \quad (1, 5)$

Finding the Derivative by the Limit Process **In Exercises 11–24, find the derivative of the function by the limit process.**

11. $f(x) = 7$ **12.** $g(x) = -3$

13. $f(x) = -10x$ **14.** $f(x) = 7x - 3$

15. $h(s) = 3 + \frac{2}{3}s$ **16.** $f(x) = 5 - \frac{2}{3}x$

17. $f(x) = x^2 + x - 3$ **18.** $f(x) = x^2 - 5$

19. $f(x) = x^3 - 12x$ **20.** $f(x) = x^3 + x^2$

21. $f(x) = \dfrac{1}{x - 1}$ **22.** $f(x) = \dfrac{1}{x^2}$

23. $f(x) = \sqrt{x + 4}$ **24.** $f(x) = \dfrac{4}{\sqrt{x}}$

Finding an Equation of a Tangent Line **In Exercises 25–32, (a) find an equation of the tangent line to the graph of f at the given point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the *derivative* feature of a graphing utility to confirm your results.**

25. $f(x) = x^2 + 3, \quad (-1, 4)$ **26.** $f(x) = x^2 + 2x - 1, \quad (1, 2)$

27. $f(x) = x^3, \quad (2, 8)$ **28.** $f(x) = x^3 + 1, \quad (-1, 0)$

29. $f(x) = \sqrt{x}, \quad (1, 1)$ **30.** $f(x) = \sqrt{x - 1}, \quad (5, 2)$

31. $f(x) = x + \dfrac{4}{x}, \quad (-4, -5)$ **32.** $f(x) = \dfrac{6}{x + 2}, \quad (0, 3)$

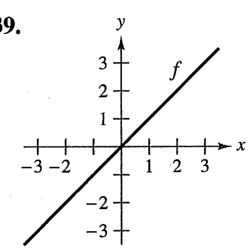
Finding an Equation of a Tangent Line **In Exercises 33–38, find an equation of the line that is tangent to the graph of f and parallel to the given line.**

	Function	**Line**
33.	$f(x) = x^2$	$2x - y + 1 = 0$
34.	$f(x) = 2x^2$	$4x + y + 3 = 0$
35.	$f(x) = x^3$	$3x - y + 1 = 0$
36.	$f(x) = x^3 + 2$	$3x - y - 4 = 0$
37.	$f(x) = \dfrac{1}{\sqrt{x}}$	$x + 2y - 6 = 0$
38.	$f(x) = \dfrac{1}{\sqrt{x - 1}}$	$x + 2y + 7 = 0$

WRITING ABOUT CONCEPTS

Sketching a Derivative **In Exercises 39–44, sketch the graph of f'. Explain how you found your answer.**

39.



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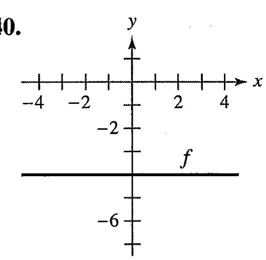

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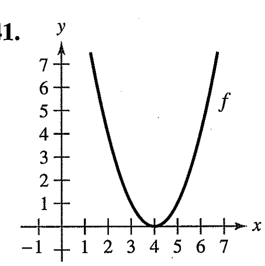

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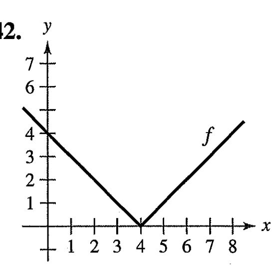
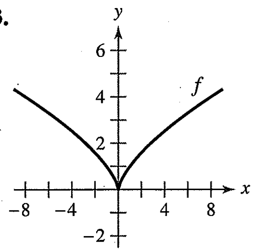


WRITING ABOUT CONCEPTS (continued)

43.



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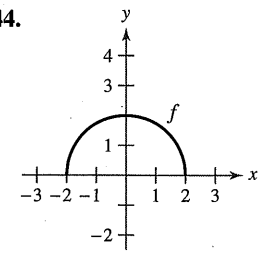


45. Sketching a Graph Sketch a graph of a function whose derivative is always negative. Explain how you found the answer.

46. Sketching a Graph Sketch a graph of a function whose derivative is always positive. Explain how you found the answer.

47. Using a Tangent Line The tangent line to the graph of $y = g(x)$ at the point $(4, 5)$ passes through the point $(7, 0)$. Find $g(4)$ and $g'(4)$.

48. Using a Tangent Line The tangent line to the graph of $y = h(x)$ at the point $(-1, 4)$ passes through the point $(3, 6)$. Find $h(-1)$ and $h'(-1)$.

Working Backwards In Exercises 49–52, the limit represents $f'(c)$ for a function f and a number c. Find f and c.

49. $\lim_{\Delta x \to 0} \frac{[5 - 3(1 + \Delta x)] - 2}{\Delta x}$

50. $\lim_{\Delta x \to 0} \frac{(-2 + \Delta x)^3 + 8}{\Delta x}$

51. $\lim_{x \to 6} \frac{-x^2 + 36}{x - 6}$

52. $\lim_{x \to 9} \frac{2\sqrt{x} - 6}{x - 9}$

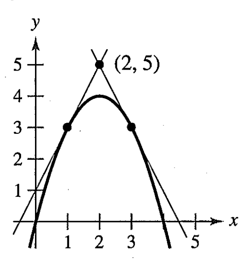
Writing a Function Using Derivatives In Exercises 53 and 54, identify a function f that has the given characteristics. Then sketch the function.

53. $f(0) = 2; f'(x) = -3$ for $-\infty < x < \infty$

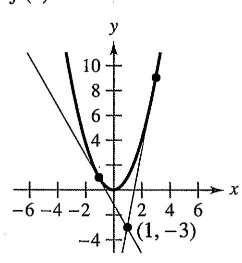
54. $f(0) = 4; f'(0) = 0; f'(x) < 0$ for $x < 0; f'(x) > 0$ for $x > 0$

Finding an Equation of a Tangent Line In Exercises 55 and 56, find equations of the two tangent lines to the graph of f that pass through the indicated point.

55. $f(x) = 4x - x^2$



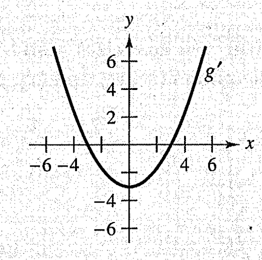
56. $f(x) = x^2$



57. Graphical Reasoning Use a graphing utility to graph each function and its tangent lines at $x = -1$, $x = 0$, and $x = 1$. Based on the results, determine whether the slopes of tangent lines to the graph of a function at different values of x are always distinct.

(a) $f(x) = x^2$ (b) $g(x) = x^3$

58. HOW DO YOU SEE IT? The figure shows the graph of g'.



(a) $g'(0) = \square$ (b) $g'(3) = \square$

(c) What can you conclude about the graph of g knowing that $g'(1) = -\frac{8}{3}$?

(d) What can you conclude about the graph of g knowing that $g'(-4) = \frac{7}{3}$?

(e) Is $g(6) - g(4)$ positive or negative? Explain.

(f) Is it possible to find $g(2)$ from the graph? Explain.

59. Graphical Reasoning Consider the function $f(x) = \frac{1}{2}x^2$.

(a) Use a graphing utility to graph the function and estimate the values of $f'(0)$, $f'\left(\frac{1}{2}\right)$, $f'(1)$, and $f'(2)$.

(b) Use your results from part (a) to determine the values of $f'\left(-\frac{1}{2}\right)$, $f'(-1)$, and $f'(-2)$.

(c) Sketch a possible graph of f'.

(d) Use the definition of derivative to find $f'(x)$.

60. Graphical Reasoning Consider the function $f(x) = \frac{1}{3}x^3$.

(a) Use a graphing utility to graph the function and estimate the values of $f'(0)$, $f'\left(\frac{1}{2}\right)$, $f'(1)$, $f'(2)$, and $f'(3)$.

(b) Use your results from part (a) to determine the values of $f'\left(-\frac{1}{2}\right)$, $f'(-1)$, $f'(-2)$, and $f'(-3)$.

(c) Sketch a possible graph of f'.

(d) Use the definition of derivative to find $f'(x)$.

Graphical Reasoning In Exercises 61 and 62, use a graphing utility to graph the functions f and g in the same viewing window, where

$$g(x) = \frac{f(x + 0.01) - f(x)}{0.01}.$$

Label the graphs and describe the relationship between them.

61. $f(x) = 2x - x^2$

62. $f(x) = 3\sqrt{x}$

Approximating a Derivative In Exercises 63 and 64, evaluate $f(2)$ and $f(2.1)$ and use the results to approximate $f'(2)$.

63. $f(x) = x(4 - x)$

64. $f(x) = \frac{1}{4}x^3$

Using the Alternative Form of the Derivative In Exercises 65–74, use the alternative form of the derivative to find the derivative at $x = c$ (if it exists).

65. $f(x) = x^2 - 5, \quad c = 3$

66. $g(x) = x^2 - x, \quad c = 1$

67. $f(x) = x^3 + 2x^2 + 1, \quad c = -2$

68. $f(x) = x^3 + 6x, \quad c = 2$

69. $g(x) = \sqrt{|x|}, \quad c = 0$

70. $f(x) = 3/x, \quad c = 4$

71. $f(x) = (x - 6)^{2/3}, \quad c = 6$

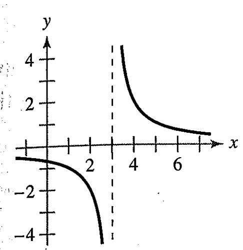
72. $g(x) = (x + 3)^{1/3}, \quad c = -3$

73. $h(x) = |x + 7|, \quad c = -7$

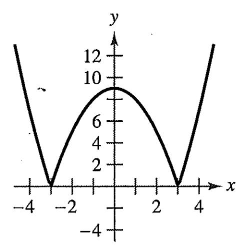
74. $f(x) = |x - 6|, \quad c = 6$

Determining Differentiability **In Exercises 75–80, describe the x-values at which f is differentiable.**

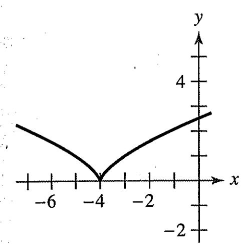
75. $f(x) = \dfrac{2}{x - 3}$



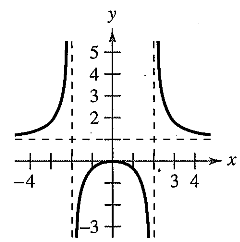
76. $f(x) = |x^2 - 9|$



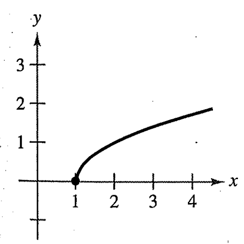
77. $f(x) = (x + 4)^{2/3}$



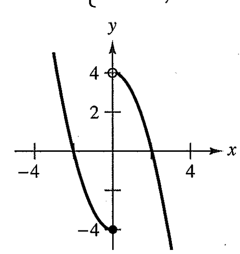
78. $f(x) = \dfrac{x^2}{x^2 - 4}$



79. $f(x) = \sqrt{x - 1}$



80. $f(x) = \begin{cases} x^2 - 4, & x \le 0 \\ 4 - x^2, & x > 0 \end{cases}$



Graphical Reasoning **In Exercises 81–84, use a graphing utility to graph the function and find the x-values at which f is differentiable.**

81. $f(x) = |x - 5|$

82. $f(x) = \dfrac{4x}{x - 3}$

83. $f(x) = x^{2/5}$

84. $f(x) = \begin{cases} x^3 - 3x^2 + 3x, & x \le 1 \\ x^2 - 2x, & x > 1 \end{cases}$

Determining Differentiability **In Exercises 85–88, find the derivatives from the left and from the right at $x = 1$ (if they exist). Is the function differentiable at $x = 1$?**

85. $f(x) = |x - 1|$

86. $f(x) = \sqrt{1 - x^2}$

87. $f(x) = \begin{cases} (x - 1)^3, & x \le 1 \\ (x - 1)^2, & x > 1 \end{cases}$

88. $f(x) = \begin{cases} x, & x \le 1 \\ x^2, & x > 1 \end{cases}$

Determining Differentiability **In Exercises 89 and 90, determine whether the function is differentiable at $x = 2$.**

89. $f(x) = \begin{cases} x^2 + 1, & x \le 2 \\ 4x - 3, & x > 2 \end{cases}$

90. $f(x) = \begin{cases} \frac{1}{2}x + 1, & x < 2 \\ \sqrt{2x}, & x \ge 2 \end{cases}$

91. **Graphical Reasoning** A line with slope m passes through the point $(0, 4)$ and has the equation $y = mx + 4$.

(a) Write the distance d between the line and the point $(3, 1)$ as a function of m.

(b) Use a graphing utility to graph the function d in part (a). Based on the graph, is the function differentiable at every value of m? If not, where is it not differentiable?

92. **Conjecture** Consider the functions $f(x) = x^2$ and $g(x) = x^3$.

(a) Graph f and f' on the same set of axes.

(b) Graph g and g' on the same set of axes.

(c) Identify a pattern between f and g and their respective derivatives. Use the pattern to make a conjecture about $h'(x)$ if $h(x) = x^n$, where n is an integer and $n \ge 2$.

(d) Find $f'(x)$ if $f(x) = x^4$. Compare the result with the conjecture in part (c). Is this a proof of your conjecture? Explain.

True or False? **In Exercises 93–96, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

93. The slope of the tangent line to the differentiable function f at the point $(2, f(2))$ is

$$\frac{f(2 + \Delta x) - f(2)}{\Delta x}.$$

94. If a function is continuous at a point, then it is differentiable at that point.

95. If a function has derivatives from both the right and the left at a point, then it is differentiable at that point.

96. If a function is differentiable at a point, then it is continuous at that point.

97. **Differentiability and Continuity** Let

$$f(x) = \begin{cases} x \sin \dfrac{1}{x}, & x \ne 0 \\ 0, & x = 0 \end{cases}$$

and

$$g(x) = \begin{cases} x^2 \sin \dfrac{1}{x}, & x \ne 0 \\ 0, & x = 0 \end{cases}.$$

Show that f is continuous, but not differentiable, at $x = 0$. Show that g is differentiable at 0, and find $g'(0)$.

98. **Writing** Use a graphing utility to graph the two functions $f(x) = x^2 + 1$ and $g(x) = |x| + 1$ in the same viewing window. Use the *zoom* and *trace* features to analyze the graphs near the point $(0, 1)$. What do you observe? Which function is differentiable at this point? Write a short paragraph describing the geometric significance of differentiability at a point.