Key

*Power Rule Conditions
i) Variable in numerator
ii) Radicals → rational exponents
iii) Expand parentheses

Ch. 2.2-2.3 Morning Quiz Review

1. Find $\frac{dy}{dx}$ if $y = 7x^3(\sqrt{x} - 1) - \frac{2x^2}{11} + 4\pi x - 5\pi^4 + \sqrt[3]{x} + \frac{5}{2\sqrt{x^7}}$

$$y = 7x^3(x^{1/2} - 1) - \frac{2}{11}x^2 + 4\pi x - 5\pi^4 + x^{1/3} + \frac{5}{2}x^{-7/2}$$

$$y = 7x^{\overset{-2/2}{7/2}} - 7x^3 - \frac{2}{11}x^2 + 4\pi x - 5\pi^4 + x^{1/3} + \frac{5}{2}x^{\overset{-2/2}{-7/2}}$$

$$\frac{dy}{dx} = \frac{7}{2}\cdot 7x^{5/2} - 7\cdot 3x^2 - 2\left(\frac{2}{11}x\right) + 4\pi - 0 + \frac{1}{3}x^{-2/3} - \frac{7}{2}\left(\frac{5}{2}x^{-9/2}\right)$$

$$\boxed{\frac{dy}{dx} = \frac{49}{2}x^{5/2} - 21x^2 - \frac{4}{11}x + 4\pi + \frac{1}{3x^{2/3}} - \frac{35}{4x^{9/2}}}$$

2. If $f(x) = \frac{x^2}{x-1}$ find $f'(x)$. Then write the equation of the line tangent to $f(x)$ at $x = -1$ in point-slope form.

*quotient rule: $\frac{f'g - fg'}{g^2}$

$$f(x) = \frac{\overbrace{x^2}^{f}}{\underbrace{x-1}_{g}}$$

$$f'(x) = \frac{\overbrace{2x}^{f'}\overbrace{(x-1)}^{g} - \overbrace{x^2}^{f}\overbrace{(1)}^{g'}}{\underbrace{(x-1)^2}_{g^2}}$$

$$f'(x) = \frac{2x^2 - 2x - x^2}{(x-1)^2}$$

$$f'(x) = \frac{x^2 - 2x}{(x-1)^2}$$

*find point at $x = -1$

$$f(-1) = \frac{(-1)^2}{-1-1} = \frac{1}{-2} = \frac{-1}{2}$$

*find slope at $x = -1$

$$f'(-1) = \frac{(-1)^2 - 2(-1)}{(-1-1)^2} = \frac{3}{2^2} = \frac{3}{4}$$

point: $(-1, -\frac{1}{2})$

slope: $m = \frac{3}{4}$

$$y - y_1 = m(x - x_1)$$

$$y - (-\tfrac{1}{2}) = \tfrac{3}{4}(x - (-1))$$

$$\boxed{y + \tfrac{1}{2} = \tfrac{3}{4}(x+1)}$$

3. Find the derivative of f(x) if $f(x) = (x^3 - 2\sqrt{x^5})(2x - 5\pi^3 + 7)$

*product rule: $f'g + fg'$

$$f(x) = \underbrace{(x^3 - 2x^{5/2})}_{f}\underbrace{(2x - 5\pi^3 + 7)}_{g}$$

$$f'(x) = \overbrace{\left(3x^2 - \tfrac{5}{2}\cdot 2x^{3/2}\right)}^{f'}\overbrace{(2x - 5\pi^3 + 7)}^{g} + \overbrace{(x^3 - 2x^{5/2})}^{f}\overbrace{(2)}^{g'}$$

$$\boxed{f'(x) = (3x^2 - 5x^{3/2})(2x - 5\pi^3 + 7) + (x^3 - 2x^{5/2})(2)}$$

4. A particle moves along the x-axis so that at times $t \geq 0$, its position is given by

$x(t) = t^3 - 3t^2 - 9t + 2$ (in meters)

a) Find the velocity and acceleration function

$\boxed{v(t) = 3t^2 - 6t - 9}$ $\boxed{a(t) = 6t - 6}$

b) What is its velocity at t = 2 seconds?

$v(2) = 3(2)^2 - 6(2) - 9 = 12 - 12 - 9$

$\boxed{v(2) = -9 \text{ m/s}}$

c) What is its acceleration at t = 4 seconds?

$a(4) = 6(4) - 6$

$\boxed{a(4) = 18 \text{ m/s}^2}$

d) At what times does the particle change directions? Justify

*set $v(t) = 0$ to find first particle at rest

$0 = 3(t^2 - 2t - 3)$

$0 = 3(t-3)(t+1)$

$t = 3, -1$

v(t): 0 — (−) — 3 — (+) — 4

change direction at $t = 3$ since $v(t)$ change signs

e) At t = 0, is the particle moving to the right or to the left? Justify.

$v(0) = 3(0)^2 - 6(0) - 9$

Since $v(0) < 0$, particle is moving left at $t = 0$

f) Find the average velocity of particle in [1, 3]

$\text{Avg. velocity} = \dfrac{\text{change in position}}{\text{change in time}} \rightarrow \dfrac{x(3) - x(1)}{3 - 1}$

$x(3) = -25$

$x(1) = -9$

$\text{Avg. velocity} = \dfrac{-25 - (-9)}{3 - 1} = \boxed{-8 \text{ m/s}}$

g) What is displacement of particle from t = 1 to t = 4? Show work.

*displacement = final position − initial position

$= x(4) - x(1)$

$= -18 - (-9)$

$= \boxed{-9 \text{ meters}}$

h) What is the total distance of particle from t = 1 to t = 4? Show work.

*we include t=3 because there's a change of direction at t=3.

v(t): 0 — (−) — 3 — (+) —

$x(1) = -9$

$x(3) = -25$

$x(4) = -18$

$x(1) \to x(3)$: 16

$x(3) \to x(4)$: 7

Total distance is 23 meters

i) Is velocity increasing or decreasing at t = 2? Justify.

*Just look at the sign of acceleration.

$a(2) = 6 \text{ m/s}^2$. Therefore velocity is increasing since $a(t) > 0$

j) Is the speed increasing or decreasing at t = 4? Justify

*compare signs of $v(t)$ and $a(t)$.

$v(4) = 15$

$a(4) = 18$

speed is increasing at $t = 4$ since $v(t)$ and $a(t)$ have same signs at $t = 4$