2.2-2.3 Review WS #1 (Asynchronous Day)

Key

No negative exponents in answer.

1. Find $\frac{dy}{dx}$ if $y = 7x^3(x-1) - \frac{3x^2}{11} + 4\pi x - 5\pi^4 + \sqrt[5]{x^4} + \frac{5}{\sqrt{x^7}}$

$y = 7x^4 - 7x^3 - \frac{3}{11}x^2 + 4\pi x - 5\pi^4 + x^{4/5} + 5x^{-7/2}$

$y' = 28x^3 - 21x^2 - \frac{6}{11}x + 4\pi - 0 + \frac{4}{5}x^{-1/5} - \frac{35}{2}x^{-9/2}$

$$\frac{dy}{dx} = 28x^3 - 21x^2 - \frac{6}{11}x + 4\pi + \frac{4}{5x^{1/5}} - \frac{35}{2x^{9/2}}$$

*power Rule conditions

1) variable in numerator
2) radicals to rationals.
3) no parentheses (expand)

2. If $f(x) = \frac{x+4}{x^2-2}$ find $f'(x)$ (simplify fully). Then write the equation of the line tangent to $f(x)$ at $x = 1$ in point-slope form.

*quotient rule

$f'(x) = \frac{\overbrace{(1)}^{f'}\overbrace{(x^2-2)}^{g} - \overbrace{(x+4)}^{f}\overbrace{(2x)}^{g'}}{\underbrace{(x^2-2)^2}_{g^2}}$

point: $f(1) = \frac{1+4}{1^2-2} = \frac{5}{-1} = -5$

slope: $f'(1) = \frac{1(1^2-2) - (1+4)(2)}{(1-2)^2} \rightarrow \frac{-1-10}{1} = -11$

point: $(1, -5)$

slope: $m = -11$

$y - y_1 = m(x - x_1)$

$y - (-5) = -11(x-1)$

$$y + 5 = -11(x-1)$$

3) Find the derivative of f(x) and then evaluate the slope of the graph at x = 1

$f(x) = (3x^5 - 4\sqrt{x})(2x - 5\pi + 9)$

*product rule

$f(x) = (3x^5 - 4x^{1/2})(2x - 5\pi + 9)$

$f'(x) = \overbrace{(15x^4 - 2x^{-1/2})}^{f'}\overbrace{(2x-5\pi+9)}^{g} + \overbrace{(3x^5 - 4x^{1/2})}^{f}\overbrace{(2)}^{g'}$

$f'(1) = (15 - 2(1))(2 - 5\pi + 9) + (3-4)(2)$

$f'(1) = 13(11 - 5\pi) - 2$

$f'(1) = 143 - 65\pi - 2$

$$f'(1) = 141 - 65\pi$$

3. Particle moves along the x-axis so that its position at time t is given $x(t) = t^3 - 9t^2 + 15t - 7$ where $x(t)$ is in feet per second and $t \geq 0$. Use this to answer the questions below. **Include units with your answers**

a) Find the velocity and acceleration function

$v(t) = 3t^2 - 18t + 15$

$a(t) = 6t - 18$

b) What is its velocity at $t = 2$ seconds?

$v(2) = 3(2)^2 - 18(2) + 15$

$\boxed{v(2) = -9 \text{ ft/s}}$

c) What is its acceleration at t = 4 seconds?

$a(4) = 6(4) - 18 = \boxed{6 \text{ ft/s}^2}$

Avg. velocity $= \frac{\text{change in position}}{\text{change in time}} \rightarrow \frac{x(8) - x(3)}{8 - 3}$

d) Find the average velocity of particle in [3, 8]

$x(8) = 49$

$x(3) = -16$

Avg. velocity $= \frac{49 - (-16)}{8 - 3} = \frac{65}{5}$

$= \boxed{13 \text{ ft/s}}$

e) When is the particle at rest? * set $v(t) = 0$

$v(t) = 3t^2 - 18t + 15$

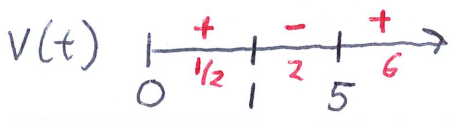
$0 = 3(t^2 - 6t + 5)$

$0 = 3(t - 5)(t - 1)$

$\boxed{t = 1, 5 \text{ seconds}}$ b/c $v(t) = 0$

f) When is the particle moving right? When does particle change directions? (Create Sign Line) Give justification.

$v(t) = 3(t - 5)(t - 1)$



(a) particle moves right $[0, 1), (5, \infty)$ b/c $v(t) > 0$

(b) particle change directions at $t = 1, 5$ seconds b/c $v(t)$ change signs

g) What is displacement of particle from t = 2 to t = 6? Show work.

* displacement = final position − initial position

$x(6) - x(2)$

$= -25 - (-5)$

$= \boxed{-20 \text{ ft}}$

h) What is the total distance of particle from t = 2 to t = 6? Show work.

$x(2) = -5$ → 27

$x(5) = -32$

$x(6) = -25$ → 7

$27 + 7 = \boxed{34 \text{ ft}}$

i) Is the speed increasing or decreasing at t = 4? Justify.

$v(4) = -9 \text{ ft/s}$

$a(4) = 6 \text{ ft/s}^2$

speed is decreasing at $t = 4$ since $v(t)$ and $a(t)$ have opposite signs.

j) Is velocity increasing or decreasing at t = 2? Justify.

↖ this is talking about acceleration.

Since $a(2) = -6 \text{ ft/s}^2$

Velocity is decreasing at $t = 2$ because $a(t) < 0$