Non-AP Calculus 2.2-2.4 Derivatives Quiz Review WS #1

No negative exponents in answer.

1. Find $\frac{dy}{dx}$ if $y = 7x^3(x-1) - \frac{3x^2}{11} + 4\pi x - 5\pi^4 + \sqrt[5]{x^4} + \frac{5}{\sqrt{x^7}}$

2. Find $\frac{dy}{dx}$ for $y = \sqrt[4]{5x^3 - 2x + 9\pi}$

3. Find g'(x) for $g(x) = x^3(x^2-7)^5$ (simplify fully in factored form)

4. Find $\frac{dy}{dx}$ for $y = \left(\frac{x^3}{1+x^2}\right)^5$ (simplify fully in factored form)

5. If $f(x) = \frac{x+4}{x^2-2}$ find $f'(x)$ (simplify fully). Then write the equation of the line tangent to $f(x)$ at $x = 1$ in point-slope form.

6) Find $\frac{dy}{dx}$ for $f(x) = \dfrac{2x^3 - 4x + 3}{1 - 5x^2}$

Power Rule:	$\frac{d}{dx}[x^n] = n * x^{n-1}$	**Point-slope form**	$y - y_1 = m(x - x_1)$
Product Rule:	$\frac{d}{dx}[f(x)g(x)] = f'g + fg'$	**Quotient Rule:**	$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'g - fg'}{g^2}$
Chain Rule:	$\frac{d}{dx} f[g(x)] = f'[g(x)] \times g'(x)$		

Solution Key

Non-AP Calculus 2.2-2.4 Quiz Review WS #1

No negative exponents in answer.

1. Find $\frac{dy}{dx}$ if $y = 7x^3(x-1) - \frac{3x^2}{11} + 4\pi x - 5\pi^4 + \sqrt[5]{x^4} + \frac{5}{\sqrt{x^7}}$

$y = 7x^4 - 7x^3 - \frac{3}{11}x^2 + 4\pi x - 5\pi^4 + x^{4/5} + 5x^{-7/2}$

$y' = 28x^3 - 21x^2 - \frac{3}{11}\cdot 2x + 4\pi - 0 + \frac{4}{5}x^{-1/5} + 5\cdot -\frac{7}{2}x^{-9/2}$

$$\boxed{\frac{dy}{dx} = 28x^3 - 21x^2 - \frac{6}{11}x + 4\pi + \frac{4}{5x^{1/5}} - \frac{35}{2x^{9/2}}}$$

2. Find $\frac{dy}{dx}$ for $y = \sqrt[4]{5x^3 - 2x + 9\pi}$ * chain rule

outside: $(\ )^{1/4}$
inside: $5x^3 - 2x + 9\pi$

$y = (5x^3 - 2x + 9\pi)^{1/4}$

$y' = \frac{1}{4}(5x^3 - 2x + 9\pi)^{-3/4}\cdot(15x^2 - 2)$

$$\boxed{\frac{dy}{dx} = \frac{15x^2 - 2}{4(5x^3 - 2x + 9\pi)^{3/4}}}$$

* 1) product rule $f'g + fg'$
* 2) chain rule → outside: $(\ )^5$
inside: $x^2 - 7$

3. Find g'(x) for $g(x) = x^3(x^2 - 7)^5$

$g'(x) = \underbrace{3x^2\cdot(x^2-7)^5}_{x^2(x^2-7)^4} + \underbrace{x^3\cdot 5(x^2-7)^4(2x)}_{x^2(x^2-7)^4}$

$g'(x) = x^2(x^2-7)^4\left[3(x^2-7) + 10x^2\right]$

$3x^2 - 21 + 10x^2$

$$\boxed{g'(x) = x^2(x^2-7)^4(13x^2 - 21)}$$

* 1) chain rule → outside: $(\ )^5$
inside $\frac{x^3}{1+x^2}$
* 2) quotient rule

4. Find $\frac{dy}{dx}$ for $y = \left(\frac{x^3}{1+x^2}\right)^5$

$y' = 5\left(\frac{x^3}{1+x^2}\right)^4 \cdot \left[\frac{\overbrace{3x^2}^{f'}\overbrace{(1+x^2)}^{g} - \overbrace{x^3}^{f}\overbrace{(2x)}^{g'}}{\underbrace{(1+x^2)^2}_{g^2}}\right]$

$y' = 5\left(\frac{x^3}{1+x^2}\right)^4\left[\frac{3x^2 + 3x^4 - 2x^4}{(1+x^2)^2}\right]$

$$\boxed{y' = \frac{5x^{12}(x^4 + 3x^2)}{(1+x^2)^6}} = \frac{5x^{12}\cdot x^2(x^2+3)}{(1+x^2)^6}$$

$$\boxed{\frac{dy}{dx} = \frac{5x^{14}(x^2+3)}{(1+x^2)^6}}$$

5. If $f(x) = \frac{x+4}{x^2-2}$ find $f'(x)$ (simplify fully). Then write the equation of the line tangent to $f(x)$ at $x = 1$ in point-slope form.

quotient rule:

$$f'(x) = \frac{(1)(x^2-2)-(x+4)(2x)}{(x^2-2)^2}$$

$$f'(x) = \frac{x^2-2-2x^2-8x}{(x^2-2)^2}$$

$$f'(x) = \frac{-x^2-8x-2}{(x^2-2)^2}$$

point: $f(1) = \frac{1+4}{1^2-2} = \frac{5}{-1} = -5$

slope: $f'(1) = \frac{-1^2-8(1)-2}{(1^2-2)^2} = \frac{-11}{1} = -11$

point: $(1,-5)$

Slope: $m = -11$

$$\boxed{y+5 = -11(x-1)}$$

6) Find $\frac{dy}{dx}$ for $f(x) = \frac{2x^3-4x+3}{1-5x^2}$

$$f'(x) = \frac{\overbrace{(6x^2-4)}^{f'}\overbrace{(1-5x^2)}^{g} - \overbrace{(2x^3-4x+3)}^{f}\overbrace{(-10x)}^{g'}}{\underbrace{(1-5x^2)^2}_{g^2}}$$

$$f'(x) = \frac{6x^2-30x^4-4+20x^2+20x^4-40x^2+30x}{(1-5x^2)^2}$$

$$\boxed{f'(x) = \frac{-10x^4-14x^2+30x-4}{(1-5x^2)^2}}$$

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