

Non-AP Calculus 2.2-2.4 Derivatives Quiz Review WS #4

No negative exponents in answer.

1. Find $\frac{dy}{dx}$ if $y = 5x^3(\sqrt{x} - 1) - \frac{2x^2}{15} + 4\pi - 5\pi^3x + \sqrt[5]{x^2} + \frac{2}{\sqrt{x^9}}$

2. Find $\frac{dy}{dx}$ for $y = \sqrt[5]{3x^7 - 2x + 3\pi}$

3. Find $g'(x)$ for $g(x) = x^2(x^4 - 7)^3$ (simplify fully in factored form)

4. Find $\frac{dy}{dx}$ for $y = \left(\frac{1+x^3}{2-x^2}\right)^7$ (simplify fully in factored form)

5. If $f(x) = \frac{2-x}{x^2-2}$ find $f'(x)$ (simplify fully). Then write the equation of the line tangent to $f(x)$ at $x = 1$ in point-slope form.

6) Find $\frac{dy}{dx}$ for $f(x) = \frac{2x^3 - 7x}{1 - 5x^2}$

Power Rule: $\frac{d}{dx}[x^n] = n * x^{n-1}$

Product Rule: $\frac{d}{dx}[f(x)g(x)] = f'g + fg'$

Chain Rule: $\frac{d}{dx} f[g(x)] = f'[g(x)] \times g'(x)$

Point-slope form $y - y_1 = m(x - x_1)$

Quotient Rule: $\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{f'g - fg'}{g^2}$

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Key

No negative exponents in answer.

1. Find $\frac{dy}{dx}$ if $y = 5x^3(\sqrt{x} - 1) - \frac{2x^2}{15} + 4\pi - 5\pi^3x + \sqrt[5]{x^2} + \frac{2}{\sqrt{x^9}}$

$$y = 5x^3(x^{1/2} - 1)$$

$$y = 5x^{7/2} - 5x^3 - \frac{2}{15}x^2 + 4\pi - 5\pi^3x + x^{2/5} + 2x^{-9/2}$$

$$y' = 5 \cdot \frac{7}{2}x^{5/2} - 15x^2 - \frac{2}{15} \cdot 2x + 0 - 5\pi^3 + \frac{2}{5}x^{-3/5} + 2 \cdot \frac{-9}{2}x^{-11/2}$$

$$y' = \frac{35}{2}x^{5/2} - 15x^2 - \frac{4}{15}x - 5\pi^3 + \frac{2}{5x^{3/5}} - \frac{9}{x^{11/2}}$$

2. Find $\frac{dy}{dx}$ for $y = \sqrt[5]{3x^7 - 2x + 3\pi}$

*chain rule $y = (3x^7 - 2x + 3\pi)^{1/5}$

outside: $()^{1/5}$

$$y' = \frac{1}{5} ()^{-4/5} \cdot (21x^6 - 2)$$

inside: $3x^7 - 2x + 3\pi$

$$y' = \frac{1}{5} (3x^7 - 2x + 3\pi)^{-4/5} (21x^6 - 2)$$

$$y' = \frac{21x^6 - 2}{5(3x^7 - 2x + 3\pi)^{4/5}}$$

3. Find $g'(x)$ for $g(x) = x^2(x^4 - 7)^3$ (simplify fully in factored form)

* product rule: $f'g + fg'$
 * chain rule: outside: $()^3$
 inside: $x^4 - 7$

$$g'(x) = \frac{f'}{2x} \cdot \frac{g}{(x^4-7)^3} + \frac{f}{x^2} \cdot \frac{g'}{3(x^4-7)^2 \cdot (4x^3)}$$

$$g'(x) = 2x(x^4-7)^3 + 12x^5(x^4-7)^2$$

$$g'(x) = 2x(x^4-7)^2 [x^4-7 + 6x^4]$$

$$g'(x) = 2x(x^4-7)^2 (7x^4-7)$$

4. Find $\frac{dy}{dx}$ for $y = \left(\frac{1+x^3}{2-x^2}\right)^7$ (simplify fully in factored form)

*chain rule:

outside: $()^7$

inside: $\frac{1+x^3}{2-x^2}$

$$\frac{dy}{dx} = 7 \left(\frac{1+x^3}{2-x^2}\right)^6 \cdot \left[\frac{\frac{f'}{3x^2} \cdot \frac{g}{(2-x^2)} - \frac{f}{(1+x^3)} \cdot \frac{g'}{(-2x)}}{(2-x^2)^2} \right] \rightarrow \frac{6x^2 - 3x^4 + 2x + 2x^4}{(2-x^2)^2}$$

$$\frac{dy}{dx} = \frac{7(1+x^3)^6(-1x^4+6x^2+2x)}{(2-x^2)^8}$$

5. If $f(x) = \frac{2-x}{x^2-2}$ find $f'(x)$ (simplify fully). Then write the equation of the line tangent to $f(x)$ at $x = 1$ in point-slope form. **quotient rule*

$$f'(x) = \frac{\overbrace{(-1)(x^2-2)}^{f'g} - \overbrace{(2-x)(2x)}^{fg'}}{\underbrace{(x^2-2)^2}_{g^2}}$$

$$f(x) = \frac{2-x}{x^2-2}$$

$$f'(x) = \frac{x^2-4x+2}{(x^2-2)^2}$$

$$f(1) = \frac{2-1}{1^2-2} = \frac{1}{-1} = -1$$

$$f'(1) = \frac{1-4+2}{(1-2)^2} = \frac{-1}{1} = -1$$

$$f(1) = -1$$

point: $(1, -1)$

slope: $m = -1$

$$y - y_1 = m(x - x_1)$$

$$y - (-1) = -1(x - 1)$$

$$\boxed{y + 1 = -1(x - 1)}$$

$$f'(x) = \frac{-x^2 + 2 - 4x + 2x^2}{(x^2-2)^2}$$

$$f'(x) = \frac{1x^2 - 4x + 2}{(x^2-2)^2}$$

6) Find $\frac{dy}{dx}$ for $f(x) = \frac{2x^3 - 7x}{1 - 5x^2}$

$$f'(x) = \frac{\overbrace{(6x^2-7)(1-5x^2)}^{f'g} - \overbrace{(2x^3-7x)(-10x)}^{fg'}}{\underbrace{(1-5x^2)^2}_{g^2}}$$

$$f'(x) = \frac{-10x^4 - 29x^2 - 7}{(1-5x^2)^2}$$

$$f'(x) = \frac{6x^2 - 30x^4 - 7 + 35x^2 + 20x^4 - 70x^2}{(1-5x^2)^2}$$

Power Rule: $\frac{d}{dx} [x^n] = n * x^{n-1}$

Point-slope form $y - y_1 = m(x - x_1)$

Product Rule: $\frac{d}{dx} [f(x)g(x)] = f'g + fg'$

Quotient Rule: $\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{f'g - fg'}{g^2}$

Chain Rule: $\frac{d}{dx} f[g(x)] = f'[g(x)] * g'(x)$