

Ch. 2.2-2.4 Vertical Motion Quiz Review Problems

1) The Position function of a particle is given by: $s(t) = 3(6t - 2)^2 + 5$

a.) Find the average velocity of the particle in feet per second over the interval $[0, 1]$

b.) Find the equation for the instantaneous velocity of the particle at any given point.

c) Find the equation for the acceleration of the particle at any given point.

d) Find the velocity and the acceleration at time $t = 2$.

2) A ball is dropped from the top of a building that is 900 feet tall.

$$s(t) = -25t^2 + v_0t + s_0$$

a.) How long after being released does it take the ball to impact the ground?

b.) What is the velocity of the ball at the time of impact?

c.) What is the average velocity of the ball over the interval $[1, 4]$

- 3) A ball is thrown straight down from the top of a 600-foot building with an initial velocity of -30 feet per second.

$$s(t) = -16t^2 + v_0t + s_0$$

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| a) Determine the position and velocity equations for the ball. | b) Find the instantaneous velocity when $t=1$ and $t=3$. |
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- c) Find the time required for the ball to reach the ground .

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| d) Find the velocity of the ball at impact. | e) Find the average velocity on the interval $[1 , 5]$ |
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- 4) A ball is thrown off of a 320 ft. tall building with an initial velocity of 96 ft/sec

$$s(t) = -16t^2 + v_0t + s_0$$

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| a) What is the maximum height achieved by the ball? | b) What is the velocity of the ball at the instant when the ball hits the ground? Give units. |
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| c) What is the velocity of the ball when it is 128 feet above the ground? | d) What is the average velocity of the ball from the time it is released until it hits the ground? |
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Ch. 2.2-2.4 Vertical Motion Quiz Review Problems

Key

1) The Position function of a particle is given by: $s(t) = 3(6t - 2)^2 + 5$

a.) Find the average velocity of the particle over the interval $[0, 1]$.

$$s(1) = 53 \quad s(0) = 17 \quad \left| \quad \text{Avg. velocity} = \frac{\text{change in position}}{\text{change in time}} = \frac{s(1) - s(0)}{1 - 0} = \frac{53 - 17}{1} = \boxed{36 \text{ ft/s}}$$

b.) Find the equation for the instantaneous velocity of the particle at any given point.

$$v(t) = 3 \cdot 2(6t - 2)'(6) + 0$$

$$v(t) = 36(6t - 2) = 216t - 72$$

c) Find the equation for the acceleration of the particle at any given point.

$$a(t) = 216$$

d) Find the velocity and the acceleration at time $t = 2$.

$$v(2) = 360 \text{ ft/s} \quad a(2) = 216 \text{ ft/s}^2$$

2) A ball is dropped from the top of a building that is 900 feet tall.

$$s(t) = -25t^2 + v_0t + s_0 \quad s(t) = -25t^2 + 0t + 900$$

$$v_0 = 0$$

$$s_0 = 900$$

$$\boxed{s(t) = -25t^2 + 900}$$

a.) How long after being released does it take the ball to impact the ground?

$$\begin{aligned} & \text{*set } s(t) = 0 \\ & 0 = -25t^2 + 900 \\ & 25t^2 = 900 \end{aligned} \quad \left| \quad \begin{aligned} & t^2 = 36 \\ & t = 6, t = -6 \end{aligned} \right. \quad \boxed{t = 6 \text{ seconds}}$$

b.) What is the velocity of the ball at the time of impact?

$$v(t) = -50t$$

$$v(6) = -50(6) = \boxed{-300 \text{ ft/s}}$$

c.) What is the average velocity of the ball over the interval $[1, 4]$

$$\text{Avg. velocity} = \frac{s(4) - s(1)}{4 - 1} = \frac{500 - 875}{4 - 1} = \frac{-375}{3} = \boxed{-125 \text{ ft/s}}$$

- 3) A ball is thrown straight down from the top of a 600-foot building with an initial velocity of -30 feet per second. $s_0 = 600$
 $v_0 = -30$

$$s(t) = -16t^2 + v_0t + s_0 \quad \boxed{s(t) = -16t^2 - 30t + 600}$$

- a) Determine the position and velocity equations for the ball.

$$s(t) = -16t^2 - 30t + 600$$

$$v(t) = -32t - 30$$

- b) Find the instantaneous velocity when $t=1$ and $t=3$.

$$v(1) = -32(1) - 30 = -62 \text{ ft/s}$$

$$v(3) = -32(3) - 30 = -126 \text{ ft/s}$$

- c) Find the time required for the ball to reach the ground. *set $s(t) = 0$

$$0 = -16t^2 - 30t + 600$$

$$= -2(8t^2 - 15t + 300)$$

*quadratic formula $\boxed{t \approx 5.25 \text{ secs.}}$

- d) Find the velocity of the ball at impact.

$$v(5.25) = -32(5.25) - 30$$

$$\boxed{v(5.25) = -198 \text{ ft/s}}$$

- e) Find the average velocity on the interval $[1, 5]$

$$s(1) = 554$$

$$s(5) = 50$$

$$\text{Avg. velocity} = \frac{s(5) - s(1)}{5 - 1}$$

$$= \frac{50 - 554}{5 - 1}$$

$$\boxed{-126 \text{ ft/s}}$$

- 4) A ball is thrown off of a 320 ft. tall building with an initial velocity of 96 ft/sec

$$s(t) = -16t^2 + v_0t + s_0 \quad s(t) = -16t^2 + 96t + 320$$

- a) What is the maximum height achieved by the ball?

$$v(t) = -32t + 96 \quad \boxed{s(3) = 464 \text{ ft}}$$

$$0 = -32t + 96$$

$$32t = 96$$

$$t = 3$$

- b) What is the velocity of the ball at the instant when the ball hits the ground? Give units. (set $s(t) = 0$)

$$0 = -16t^2 + 96t + 320$$

$$0 = -16(t^2 - 6t - 20)$$

$$t = -2.38, 8.385$$

$$\boxed{t = 8.385 \text{ secs}}$$

- c) What is the velocity of the ball when it is 128 feet above the ground?

$$128 = -16t^2 + 96t + 320$$

$$0 = -16t^2 + 96t + 192$$

$$t = -1.58, \boxed{t = 7.583}$$

$$v(7.583) = \boxed{-146.66 \text{ ft/s}}$$

- d) What is the average velocity of the ball from the time it is released until it hits the ground? $[0, 8.385]$

$$\text{Avg. velocity} = \frac{s(8.385) - s(0)}{8.385 - 0}$$

$$= \frac{0 - 320}{8.385 - 0}$$

$$= \frac{0 - 320}{8.385}$$

$$= \boxed{-38.163 \text{ ft/s}}$$