Non-AP Calculus 2.2-2.5 Derivatives Test Review WS #3

No negative exponents in answer.

1. Find $\frac{dy}{dx}$ if $y = 5\sqrt{x}(3 - 7x^3) - \frac{5\pi}{x^4} + 2\pi x^3 - 7\pi^6 + \frac{2(\sqrt[6]{x})}{5} + \frac{2}{\sqrt{x^9}}$

2. Find $\frac{dy}{dx}$ for $y = \frac{9}{\sqrt[5]{(2\pi - 3x^3 - 5x)}}$

3. Find $\frac{dy}{dx}$ for $y = 5\left(\frac{1-9x}{5-x^2}\right)^{13}$ (simplify fully in factored form)

4. If $f(x) = \frac{6-x}{x^2-3}$ find $f'(x)$ (simplify fully). Then write the equation of the line tangent to $f(x)$ at $x = 1$ in point-slope form.

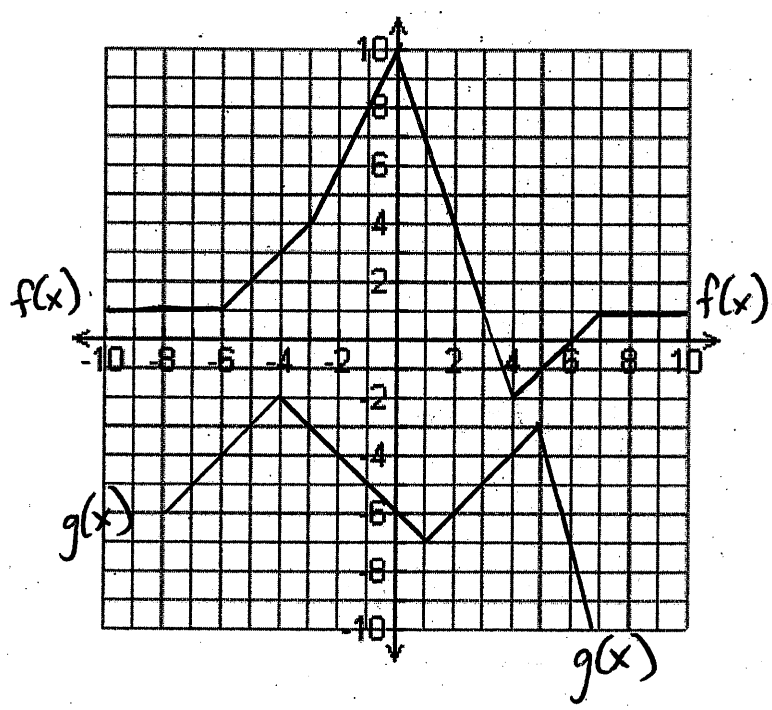
5) Find $\frac{dy}{dx}$ $for\ \ 3y^3x^4 - y\sqrt{y} + 5\pi = 5x$

6) *The graphs of f and g are shown. Let* $h(x) = f(g(x))$. *Let* $p(x) = \frac{g(x)}{f(x)}$. *Let* $q(x) = f(x)g(x)$

a) Find q'(3)

b) Find p'(1)

c) Find h'(-2)



Key

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No negative exponents in answer.

1. Find $\frac{dy}{dx}$ if $y = 5\sqrt{x}(3 - 7x^3) - \frac{5\pi}{x^4} + 2\pi x^3 - 7\pi^6 + \frac{2(\sqrt[6]{x})}{5} + \frac{2}{\sqrt{x^9}}$

$$y = 5x^{1/2}(3-7x^3) - 5\pi x^{-4} + 2\pi x^3 - 7\pi^6 + \tfrac{2}{5}x^{1/6} + 2x^{-9/2}$$

$$y = 15x^{1/2} - 35x^{7/2} - 5\pi x^{-4} + 2\pi x^3 - 7\pi^6 + \tfrac{2}{5}x^{1/6} + 2x^{-9/2}$$

$$y' = 15\cdot\tfrac{1}{2}x^{-1/2} - 35\cdot\tfrac{7}{2}x^{5/2} + 20\pi x^{-5} + 6\pi x^2 - 0 + \tfrac{2}{5}\cdot\tfrac{1}{6}x^{-5/6} + 2\cdot -\tfrac{9}{2}x^{-11/2}$$

$$\boxed{y' = \frac{15}{2x^{1/2}} - \frac{245}{2}x^{5/2} + \frac{20\pi}{x^5} + 6\pi x^2 + \frac{1}{15x^{5/6}} - \frac{9}{x^{11/2}}}$$

2. Find $\frac{dy}{dx}$ for $y = \frac{9}{\sqrt[5]{(2\pi - 3x^3 - 5x)}}$

$$y = \frac{9}{(2\pi - 3x^3 - 5x)^{1/5}}$$

$$y = 9(2\pi - 3x^3 - 5x)^{-1/5}$$

outside: $9(\ )^{-1/5}$
inside: $2\pi - 3x^3 - 5x$
← chain rule

$$\frac{dy}{dx} = \frac{-9}{5}(2\pi - 3x^3 - 5x)^{-6/5}(-9x^2 - 5)$$

$$\boxed{\frac{dy}{dx} = \frac{-9(-9x^2-5)}{5(2\pi - 3x^3 - 5x)^{6/5}}}$$

3. Find $\frac{dy}{dx}$ for $y = 5\left(\frac{1-9x}{5-x^2}\right)^{13}$ (simplify fully in factored form)

*chain rule:
outside: $5(\ )^{13}$
inside: $\frac{1-9x}{5-x^2}$

$$y' = 5\cdot 13\left(\frac{1-9x}{5-x^2}\right)^{12}\cdot\left[\frac{\overbrace{(-9)}^{f'}\overbrace{(5-x^2)}^{g} - \overbrace{(1-9x)}^{f}\overbrace{(-2x)}^{g'}}{\underbrace{(5-x^2)^2}_{g^2}}\right]$$

$$-45 + 9x^2 + 2x - 18x^2$$

$$\frac{-9x^2 + 2x - 45}{(5-x^2)^2}$$

$$\boxed{y' = \frac{65(1-9x)^{12}(-9x^2+2x-45)}{(5-x^2)^{14}}}$$

4. If $f(x) = \frac{6-x}{x^2-3}$ find $f'(x)$ (simplify fully). Then write the equation of the line tangent to $f(x)$ at $x = 1$ in point-slope form.

$$f'(x) = \frac{\overbrace{(-1)}^{f'}\overbrace{(x^2-3)}^{g} - \overbrace{(6-x)}^{f}\overbrace{(2x)}^{g'}}{\underbrace{(x^2-3)^2}_{g^2}}$$

$$f'(x) = \frac{-x^2+3-12x+2x^2}{(x^2-3)^2}$$

$$f'(x) = \frac{x^2-12x+3}{(x^2-3)^2}$$

$$f'(1) = \frac{1-12+3}{(1-3)^2} = \frac{-8}{4}$$

$$f'(1) = -2$$

$$f(1) = \frac{6-1}{1-3} = \frac{5}{-2}$$

point: $(1, -5/2)$ slope: $m = -2$

$$\boxed{y + 5/2 = -2(x-1)}$$

5) Find $\frac{dy}{dx}$ for $\overbrace{3y^3}^{f}\overbrace{x^4}^{g} - y\sqrt{y} + 5\pi = 5x$

* Implicit differentiation
* product rule

$y^1 \cdot y^{1/2} = y^{3/2}$

$\underbrace{9y^2\left(\frac{dy}{dx}\right)}_{f'} \cdot \underbrace{x^4}_{g} + \underbrace{3y^3}_{f} \cdot \underbrace{4x^3}_{g'} - \frac{3}{2}y^{1/2}\left(\frac{dy}{dx}\right) + 0 = 5$

$9x^4y^2\left(\frac{dy}{dx}\right) + 12x^3y^3 - \frac{3}{2}y^{1/2}\left(\frac{dy}{dx}\right) = 5$

$9x^4y^2\left(\frac{dy}{dx}\right) - \frac{3}{2}y^{1/2}\left(\frac{dy}{dx}\right) = 5 - 12x^3y^3$

$\frac{dy}{dx}\left(9x^4y^2 - \frac{3}{2}y^{1/2}\right) = 5 - 12x^3y^3$

$\boxed{\frac{dy}{dx} = \frac{5-12x^3y^3}{9x^4y^2 - \frac{3}{2}y^{1/2}}}$

6) *The graphs of f and g are shown. Let* $h(x) = f(g(x))$. *Let* $p(x) = \frac{g(x)}{f(x)}$. *Let* $q(x) = f(x)g(x)$

a) Find q'(3)

$q(x) = f(x)g(x)$

$q'(x) = f'(x)g(x) + f(x)g'(x)$

$q'(3) = f'(3)g(3) + f(3)g'(3)$

$q'(3) = (-3)(-5) + (1)(1)$

$q'(3) = 15 + 1$

$\boxed{q'(3) = 16}$

$f'(3) = -3$
$f(3) = 1$
$g'(3) = 1$
$g(3) = -5$

b) Find p'(1)

$p(x) = \frac{g(x)}{f(x)}$

$p'(x) = \frac{g'(x)f(x) - g(x)f'(x)}{f(x)^2}$

$p'(1) = \frac{g'(1)f(1) - g(1)f'(1)}{[f(1)]^2}$

$g(1) = -7$
$g'(1) = dne$
$f(1) = 7$
$f'(1) = -3$

$\frac{(dne)(7) - (-7)(-3)}{7^2} = \boxed{\text{does not exist}}$

c) Find h'(-2)

$h(x) = f(g(x))$

$h'(x) = f'[g(x)] \cdot g'(x)$

$h'(-2) = f'[g(-2)] \cdot g'(-2)$

$h'(-2) = f'[-4] \cdot g'(-2)$

$h'(-2) = (1) \cdot (-1)$

$\boxed{h'(-2) = -1}$

$g(-2) = -4$
$g'(-2) = -1$
$f'(-4) = 1$