# Non-AP Calculus 2.2-2.5 Derivatives Test Review WS #4

No negative exponents in answer.

1. Find $\frac{dy}{dx}$ if $y = 2\sqrt{x}\left(\pi^6 - \sqrt[3]{x}\right) - \frac{5\pi}{x^3} + 5\pi x^3 - 9\pi^6 x + \frac{2(\sqrt[7]{x})}{3} + \frac{2}{\sqrt{x^5}}$

2. Find $\frac{dy}{dx}$ for $y = \frac{5}{\sqrt{(7\pi - 3x^4 - 2x)^7}}$

3. Find $\frac{dy}{dx}$ for $y = 5\left(\frac{1-9x}{5x-x^2}\right)^7$ (simplify fully in factored form)

4. If $f(x) = \frac{9-x}{x^2-5}$ find $f'(x)$ (simplify fully). Then write the equation of the line tangent to $f(x)$ at $x = 1$ in point-slope form.

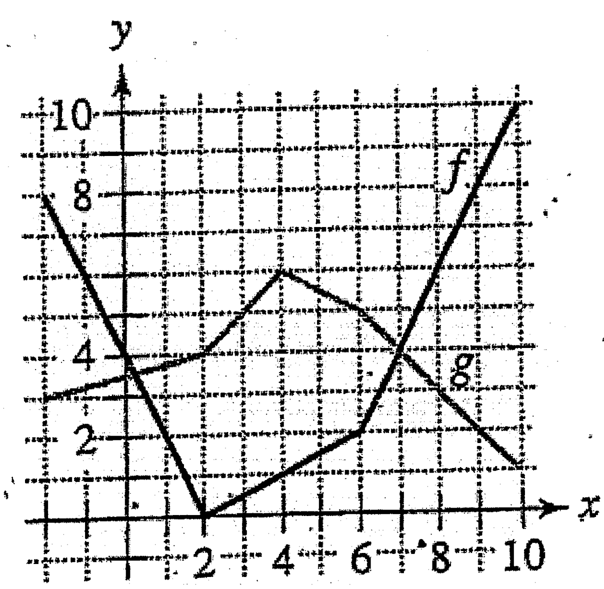
5) Find $\frac{dy}{dx}$ $for\ \ 2xy - y + 5\pi x = 12$

6) $The\ graphs\ of\ f\ and\ g\ are\ shown. Let\ h(x) = f\big(g(x)\big).\ \ Let\ p(x) = \frac{g(x)}{f(x)}. Let\ q(x) = f(x)g(x)$

a) Find q'(3)

b) Find p'(5)

c) Find h'(3)



# Non-AP Calculus 2.2-2.5 Derivatives Test Review WS #4

Key

No negative exponents in answer.

1. Find $\frac{dy}{dx}$ if $y = 2\sqrt{x}\left(\pi^6 - \sqrt[3]{x}\right) - \frac{5\pi}{x^3} + 5\pi x^3 - 9\pi^6 x + \frac{2(\sqrt[7]{x})}{3} + \frac{2}{\sqrt{x^5}}$

$$y = 2x^{1/2}\left(\pi^6 - x^{1/3}\right) - \frac{5\pi}{x^3} + 5\pi x^3 - 9\pi^6 x + \frac{2x^{1/7}}{3} + \frac{2}{x^{5/2}}$$

$$y = 2\pi^6 x^{1/2} - 2x^{5/6} - 5\pi x^{-3} + 5\pi x^3 - 9\pi^6 x + \frac{2}{3}x^{1/7} + 2x^{-5/2}$$

$$y' = 2\pi^6 \cdot \frac{1}{2}x^{-1/2} - \frac{10}{6}x^{-1/6} + 15\pi x^{-4} + 15\pi x^2 - 9\pi^6 + \frac{2}{21}x^{-6/7} - \frac{10}{2}x^{-7/2}$$

$$\boxed{y' = \frac{\pi^6}{x^{1/2}} - \frac{5}{3x^{1/6}} + \frac{15\pi}{x^4} + 15\pi x^2 - 9\pi^6 + \frac{2}{21x^{6/7}} - \frac{5}{x^{7/2}}}$$

2. Find $\frac{dy}{dx}$ for $y = \frac{5}{\sqrt{(7\pi - 3x^4 - 2x)^7}}$

$$y = \frac{5}{(7\pi - 3x^4 - 2x)^{7/2}}$$

$$y = 5(7\pi - 3x^4 - 2x)^{-7/2}$$

chain rule:

outside: $5(\ )^{-7/2}$

inside: $7\pi - 3x^4 - 2x$

$$y' = \frac{-35}{2}(7\pi - 3x^4 - 2x)^{-9/2}(-12x^3 - 2)$$

$$\boxed{y' = \frac{-35(-12x^3 - 2)}{2(7\pi - 3x^4 - 2x)^{9/2}}} \rightarrow \frac{-35(2)(-6x^2 - 1)}{2(7\pi - 3x^4 - 2x)^{9/2}} \rightarrow \frac{-35(-6x^2 - 1)}{(7\pi - 3x^4 - 2x)^{9/2}}$$

3. Find $\frac{dy}{dx}$ for $y = 5\left(\frac{1-9x}{5x - x^2}\right)^7$ (simplify fully in factored form)

chain rule:

outside: $5(\ )^7$

inside: $\frac{\overset{f}{1-9x}}{\underbrace{5x - x^2}_{g}}$

$$y' = 35\left(\frac{1-9x}{5x-x^2}\right)^6 \left[\frac{\overbrace{(-9)}^{f'}\overbrace{(5x-x^2)}^{g} - \overbrace{(1-9x)}^{f}\overbrace{(5-2x)}^{g'}}{\underbrace{(5x-x^2)^2}_{g^2}}\right]$$

$-45x + 9x^2 - 5 + 2x + 45x - 18x^2$

$-9x^2 + 2x - 5$

$$\boxed{y' = \frac{35(1-9x)^6(-9x^2 + 2x - 5)}{(5x - x^2)^8}}$$

4. If $f(x) = \frac{9-x}{x^2-5}$ find $f'(x)$ (simplify fully). Then write the equation of the line tangent to $f(x)$ at $x = 1$ in point-slope form.

$$f'(x) = \frac{(-1)(x^2-5) - (9-x)(2x)}{(x^2-5)^2}$$

$$f'(x) = \frac{-x^2 + 5 - 18x + 2x^2}{(x^2-5)^2}$$

$$f'(x) = \frac{x^2 - 18x + 5}{(x^2-5)^2}$$

$$f(1) = \frac{9-1}{1^2-5} = \frac{8}{-4} = -2$$

$$f'(1) = \frac{1 - 18 + 5}{(1-5)^2} = \frac{-12}{(-4)^2} = \frac{-12}{16} = \frac{-3}{4}$$

point: $(1, -2)$

slope: $m = -3/4$

$$\boxed{y + 2 = \frac{-3}{4}(x - 1)}$$

5) Find $\frac{dy}{dx}$ for $\overbrace{2x}^{f}\overbrace{y}^{g} - y + 5\pi x = 12$

$$\overbrace{(2)}^{f'}\overbrace{(y)}^{g} + \overbrace{(2x)}^{f}\cdot\overbrace{1\left(\frac{dy}{dx}\right)}^{g'} - 1\left(\frac{dy}{dx}\right) + 5\pi = 0$$

$$2y + 2x\left(\frac{dy}{dx}\right) - 1\left(\frac{dy}{dx}\right) + 5\pi = 0$$

$$2x\left(\frac{dy}{dx}\right) - 1\left(\frac{dy}{dx}\right) = -5\pi - 2y$$

$$\frac{dy}{dx}(2x-1) = -5\pi - 2y$$

$$\boxed{\frac{dy}{dx} = \frac{-5\pi - 2y}{2x-1}}$$

6) *The graphs of $f$ and $g$ are shown. Let $h(x) = f(g(x))$. Let $p(x) = \frac{g(x)}{f(x)}$. Let $q(x) = f(x)g(x)$*

a) Find q'(3)

$$q(x) = f(x)g(x)$$

$$q'(x) = f'(x)g(x) + f(x)g'(x)$$

$$q'(3) = f'(3)g(3) + f(3)g'(3)$$

$$q'(3) = \left(\tfrac{1}{2}\right)(5) + \left(\tfrac{1}{2}\right)(1)$$

$$q'(3) = \frac{5}{2} + \frac{1}{2} = \frac{6}{2}$$

$$\boxed{q'(3) = 3}$$

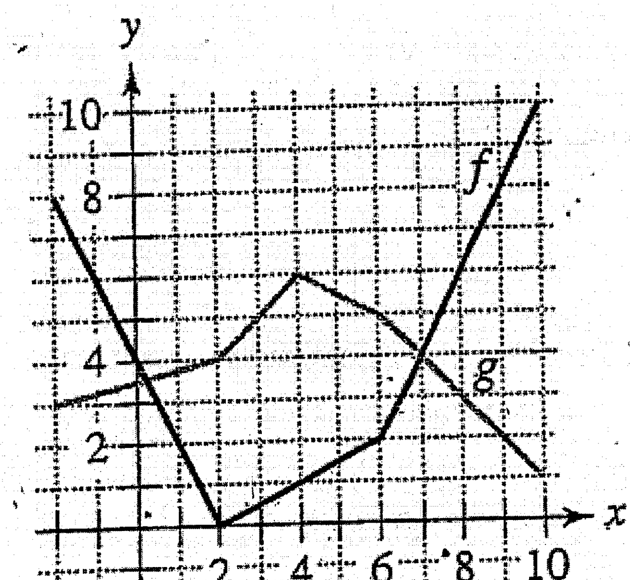
b) Find p'(5)

$$p(x) = \frac{g(x)}{f(x)}$$

$$p'(x) = \frac{g'(x)f(x) - g(x)f'(x)}{[f(x)]^2}$$

$$p'(5) = \frac{g'(5)f(5) - g(5)f'(5)}{[f(5)]^2}$$

$$p'(5) = \frac{\left(\frac{-1}{2}\right)\left(\frac{3}{2}\right) - \left(\frac{11}{2}\right)\left(\frac{1}{2}\right)}{\left(\frac{3}{2}\right)^2} = \boxed{\frac{-14}{9}}$$




c) Find h'(3)

$$h(x) = f[g(x)]$$

$$h'(x) = f'[g(x)]\cdot g'(x)$$

$$h'(3) = f'[g(3)]\cdot g'(3)$$

$$h'(3) = f'[5]\cdot g'(3)$$

$$h'(3) = \left(\tfrac{1}{2}\right)\cdot(1)$$

$$\boxed{h'(3) = \tfrac{1}{2}}$$