AB Calc: 2.2-2.5 Test Review Packet ①

Key

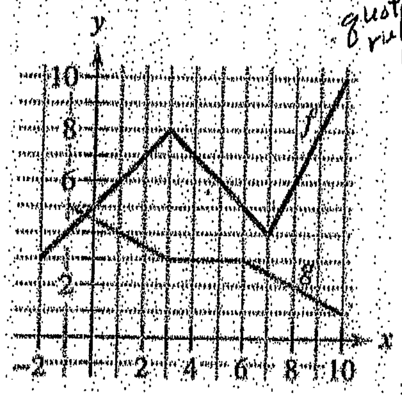
Ch. 2.3 **Product,Quotient Rule** HW Problems Evaluating Derivatives using graphs

Evaluating Derivatives In Exercises 81 and 82, use the graphs of f and g. Let $p(x) = f(x)g(x)$ and $q(x) = f(x)/g(x)$.

*product rule

81. (a) Find $p'(1)$.

(b) Find $q'(4)$.



quotient rule

a) $p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

$p'(1) = f'(1) \cdot g(1) + f(1) \cdot g'(1)$

$= (1) \cdot (4) + (6) \cdot (-\tfrac{1}{2}) = 4 - 3 = \boxed{1}$

b) $q'(x) = \dfrac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x)^2}$

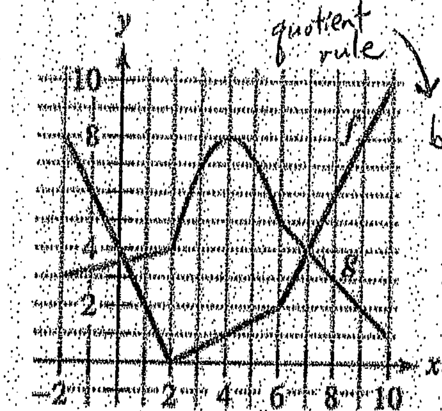
$q'(4) = \dfrac{f'(4)g(4) - f(4)g'(4)}{[g(4)]^2}$

$q'(4) = \dfrac{(-1)(3) - (7)(0)}{3^2}$

$q'(4) = \dfrac{-3}{3^2} = \boxed{-\dfrac{1}{3}}$

82. (a) Find $p'(4)$.

(b) Find $q'(7)$.



a) product Rule → $p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

$p'(4) = f'(4) \cdot g(4) + f(4) \cdot g'(4)$

$= (\tfrac{1}{2}) \cdot (8) + (1)(0) = 4$

$\boxed{p'(4) = 4}$

quotient rule

b) $q'(x) = \dfrac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

$q'(7) = \dfrac{f'(7)g(7) - f(7)g'(7)}{g(7)^2}$

$q'(7) = \dfrac{(2)(4) - 4(-1)}{4^2} = \dfrac{8+4}{16} = \dfrac{12}{16} = \dfrac{3}{4}$ $\boxed{q'(7) = 3/4}$

Using Relationships In Exercises 103–106, use the given information to find $f'(2)$.

$g(2) = 3$ and $g'(2) = -2$

$h(2) = -1$ and $h'(2) = 4$

103. $f(x) = 2g(x) + h(x)$

$f'(x) = 2g'(x) + h'(x)$

$f'(2) = 2g'(2) + h'(2)$

$= 2(-2) + 4 = 0$

$\boxed{f'(2) = 0}$

Apply quotient rule

105. $f(x) = \dfrac{g(x)}{h(x)}$

$f'(x) = \dfrac{g'(x)h(x) - g(x)h'(x)}{h(x)^2}$

$f'(2) = \dfrac{g'(2)h(2) - g(2)h'(2)}{h(2)^2}$

$f'(2) = \dfrac{(-2)(-1) - 3(4)}{(-1)^2}$

$f'(2) = \dfrac{2 - 12}{1} = -10$

$\boxed{f'(2) = -10}$

Ch.2.4 p139

[redacted]

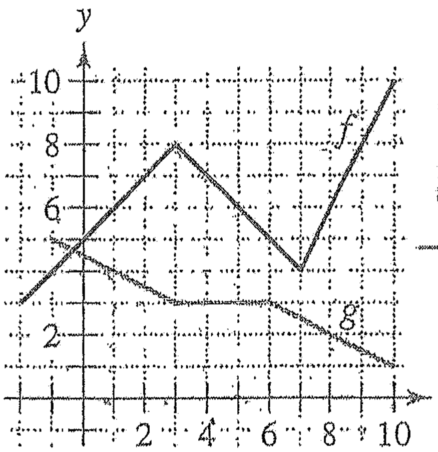
In Exercises 99 the graphs of f and g are shown. Let $h(x) = f(g(x))$ and $s(x) = g(f(x))$. Find each derivative, if it exists. If the derivative does not exist, explain why.

$h(x) = f(g(x))$

$s(x) = g(f(x))$

99. (a) Find $h'(1)$.

(b) Find $s'(5)$.



(a) $h'(x) = f'[g(x)] \cdot g'(x)$

$h'(1) = f'[g(1)] \cdot g'(1)$

$h'(1) = f'[4] \cdot (-\frac{1}{2})$

$= (-1)(-\frac{1}{2})$

$\boxed{h'(1) = \frac{1}{2}}$

$g(1) = 4$

$g'(1) = -\frac{1}{2}$

$f'(4) = -1$

$s'(x) = g'(f(x)) \cdot f'(x)$

$s'(5) = g'(f(5)) \cdot f'(5)$

$s'(5) = g'(6) \cdot (-1)$

$\boxed{s'(5) = \text{DNE}}$

$f(5) = 6$

$f'(5) = -1$

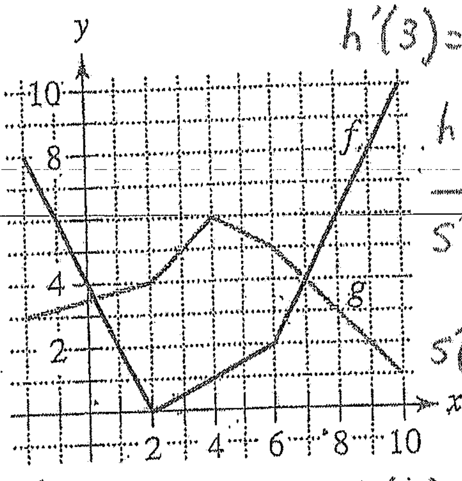
$g'(6) = \text{DNE}$

$h(x) = f(x)g(x)$

$s(x) = \frac{f(x)}{g(x)}$

100. (a) Find $h'(3)$.

(b) Find $s'(9)$.



$h'(x) = f'(x)g(x) + f(x)g'(x)$

$h'(3) = f'(3)g(3) + f(3)g'(3)$

$h'(3) = (\frac{1}{2})(5) + (\frac{1}{2})(1)$

$h'(3) = \frac{5}{2} + \frac{1}{2} = \frac{6}{2} = \boxed{3}$

$f(3) = \frac{1}{2}$

$f'(3) = \frac{1}{2}$

$g(3) = 5$

$g'(3) = 1$

$s'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

$s'(9) = \frac{f'(9)g(9) - f(9)g'(9)}{[g(9)]^2}$

$f(9) = 8$

$f'(9) = 2$

$g(9) = 2$

$g'(9) = -1$

$s'(9) = \frac{(2)(2) - (8)(-1)}{(2)^2} = \frac{4+8}{4} = \frac{12}{4} = 3$ $\boxed{s'(9) = 3}$

③ Key

Ch. 2.4 Chain Rule HW Problems #102, #115

102. Using Relationships Given that $g(5) = -3$, $g'(5) = 6$, $h(5) = 3$, and $h'(5) = -2$, find $f'(5)$ for each of the following, if possible. If it is not possible, state what additional information is required.

Recall: product rule: $\frac{d}{dx} f(x)g(x) = f'(x)(g(x)) + f(x)g'(x)$

quotient rule: $\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

Chain rule: $\frac{d}{dx} f[g(x)] = f'(g(x)) \cdot g'(x)$

(a) $f(x) = g(x)h(x)$ *product rule

$f'(x) = g'(x)h(x) + g(x)h'(x)$

$f'(5) = g'(5)h(5) + g(5)h'(5)$

$f'(5) = 6(3) + (-3)(-2)$

$f'(5) = 18 + 6 = 24$ $\boxed{f'(5) = 24}$

(b) $f(x) = g(h(x))$ *chain rule

$f'(x) = g'[h(x)] \cdot h'(x)$

$f'(5) = g'[h(5)] \cdot h'(5)$

$= g'(3) \cdot h'(5)$

$= g'(3) \cdot -2$

$\boxed{f'(5) = -2g'(3)}$

(c) $f(x) = \dfrac{g(x)}{h(x)}$ *quotient rule

$f'(x) = \dfrac{g'(x)h(x) - g(x)h'(x)}{h(x)^2}$

$f'(5) = \dfrac{g'(5)h(5) - g(5)h'(5)}{h(5)^2}$

$= \dfrac{6(3) - (-3)(-2)}{3^2} = \dfrac{18-6}{9} = \dfrac{12}{9} = 4/3$

$\boxed{f'(5) = 4/3}$

(d) $f(x) = [g(x)]^3$ *chain rule

$f'(x) = 3[g(x)]^2 \cdot g'(x)$

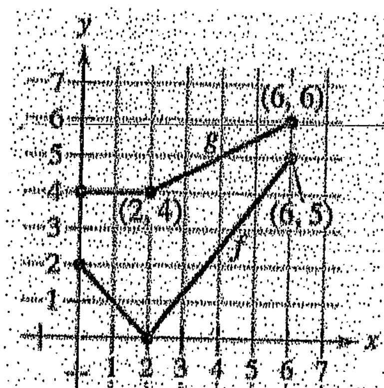
$f'(5) = 3[g(5)]^2 \cdot g'(5)$

$= 3[-3]^2 \cdot 6$

$= 3(9)(6) = 162$

$\boxed{f'(5) = 162}$

115. Think About It Let $r(x) = f(g(x))$ and $s(x) = g(f(x))$, where f and g are shown in the figure. Find (a) $r'(1)$ and (b) $s'(4)$. ← Apply chain rule



a) $r'(x) = f'[g(x)] \cdot g'(x)$

$r'(1) = f'[g(1)] \cdot g'(1)$

$r'(1) = f'[4] \cdot 0$

$r'(1) = \frac{5}{4}(0) = \boxed{0}$

$\boxed{r'(1) = 0}$

b) $s'(x) = g'[f(x)] \cdot f'(x)$

$s'(4) = g'[f(4)] \cdot f'(4)$

$= g'[5/2] \cdot (5/4)$

$= (\frac{1}{2})(5/4) = 5/8$

$\boxed{s'(4) = 5/8}$

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4 Key

Ch.2.5 **Implicit Differentiation** Vertical, Horizontal Tangent Lines HW Problems #57, #58

Vertical and Horizontal Tangent Lines In Exercises 57 and 58, find the points at which the graph of the equation has a vertical or horizontal tangent line. $\left(\frac{dy}{dx}\right)$

*Find **Horizontal Tangent** lines by setting **numerator of derivative** equal to zero, solve for x

*Find **Vertical Tangent** lines by setting **denominator of derivative** equal to zero, solve for x

57. $25x^2 + 16y^2 + 200x - 160y + 400 = 0$ ← Apply implicit differentiation

$50x + 32y\left(\frac{dy}{dx}\right) + 200 - 160\left(\frac{dy}{dx}\right) + 0 = 0$

$32y\left(\frac{dy}{dx}\right) - 160\left(\frac{dy}{dx}\right) = -50x - 200$

$\frac{dy}{dx}\left[32y - 160\right] = -50x - 200$

$\frac{dy}{dx} = \frac{-50x - 200}{32y - 160}$

horiz. tangent: $-50x - 200 = 0$

$-50x = +200$

$\boxed{x = -4}$ plug into equation

$25(-4)^2 + 16y^2 + 200(-4) - 160y + 400 = 0$

$400 + 16y^2 - 800 - 160y + 400 = 0$

$16y^2 - 160y = 0 \quad 16y(y-10) = 0 \quad y = 0, 10$

$\boxed{\text{Horiz. tangents: } (-4,0) \text{ and } (-4,10)}$

vertical tangent: $32y - 160 = 0 \quad 32y = 160 \quad y = 5$

$25x^2 + 16(5)^2 + 200x - 160(5) + 400 = 0$

$25x^2 + 400 + 200x - 800 + 400 = 0$

$25x^2 + 200x = 0$

$25x(x+8) = 0$

$x = 0, -8$

$\boxed{\text{vertical tangents: } (0,5) \text{ and } (-8,5)}$

58. $4x^2 + y^2 - 8x + 4y + 4 = 0$

$8x + 2y\left(\frac{dy}{dx}\right) - 8 + 4\left(\frac{dy}{dx}\right) + 0 = 0$

$2y\left(\frac{dy}{dx}\right) + 4\left(\frac{dy}{dx}\right) = 8 - 8x$

$\frac{dy}{dx}(2y + 4) = 8 - 8x$

$\boxed{\frac{dy}{dx} = \frac{8 - 8x}{2y + 4}}$

horizontal tangent: $8 - 8x = 0 \quad x = 1$

$4(1)^2 + y^2 - 8(1) + 4y + 4 = 0$

$y^2 + 4y = 0$

$y(y+4) = 0$

$y = 0, -4$

$\boxed{\text{horizontal tangents: } (1,0) \text{ and } (1,-4)}$

vertical tangents: $2y + 4 = 0$

$2y = -4 \quad y = -2$

$4x^2 + (-2)^2 - 8x + 4(-2) + 4 = 0$

$4x^2 + 4 - 8x - 8 + 4 = 0$

$4x^2 - 8x = 0$

$4x(x-2) = 0$

$x = 0, 2$

$\boxed{\text{vertical tangents } (0,-2) \text{ and } (2,-2)}$

Key ⑤

AP Calculus AB–5 / BC–5 2000

Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$.

(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

(a) $y^2 + 2xy\dfrac{dy}{dx} - 3x^2y - x^3\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx}(2xy - x^3) = 3x^2y - y^2$

$\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$

$2\begin{cases} 1: \text{implicit differentiation} \\ 1: \text{verifies expression for } \dfrac{dy}{dx} \end{cases}$

(b) When $x = 1$, $y^2 - y = 6$

$y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

$y = 3,\ y = -2$

At $(1,3)$, $\dfrac{dy}{dx} = \dfrac{9-9}{6-1} = 0$

Tangent line equation is $y = 3$

At $(1,-2)$, $\dfrac{dy}{dx} = \dfrac{-6-4}{-4-1} = \dfrac{-10}{-5} = 2$

Tangent line equation is $y + 2 = 2(x-1)$

$4\begin{cases} 1: & y^2 - y = 6 \\ 1: & \text{solves for } y \\ 2: & \text{tangent lines} \end{cases}$

Note: 0/4 if not solving an equation of the form $y^2 - y = k$

(c) Tangent line is vertical when $2xy - x^3 = 0$

$x(2y - x^2) = 0$ gives $x = 0$ or $y = \dfrac{1}{2}x^2$

There is no point on the curve with x-coordinate 0.

When $y = \dfrac{1}{2}x^2$, $\dfrac{1}{4}x^5 - \dfrac{1}{2}x^5 = 6$

$-\dfrac{1}{4}x^5 = 6$

$x = \sqrt[5]{-24}$

$3\begin{cases} 1: & \text{sets denominator of } \dfrac{dy}{dx} \text{ equal to 0} \\ 1: & \text{substitutes } y = \dfrac{1}{2}x^2 \text{ or } x = \pm\sqrt{2y} \\ & \text{into the equation for the curve} \\ 1: & \text{solves for } x\text{-coordinate} \end{cases}$

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6a

Ch. 2 Review (Differentiable Piecewise Functions)

Finding Constants to make a Piecewise Function continuous and differentiable.

2. How to find constants a *and* b so that the function is continuous and differentiable.
   a. You will be given a function like this one:

$$f(x) = \begin{cases} ax + 3 & x \le 1 \\ 3x^2 + x + b & x > 1 \end{cases}$$

3. Practice find constants a and b so that the function is continuous and differentiable.

   a. $f(x) = \begin{cases} 2x^2 + 2x + a & x \le -1 \\ -2bx + 1 & x > -1 \end{cases}$

6b

Ch. 2 Review (Differentiable Piecewise Functions)

Finding Constants to make a Piecewise Function continuous and differentiable.

2. How to find constants a *and* b so that the function is continuous and differentiable.
   a. You will be given a function like this one:

$$f(x) = \begin{cases} ax+3 & x \le 1 \\ 3x^2+x+b & x > 1 \end{cases}$$

continuous - share same y-value (set equations equal) at $x=1$

$ax+3 = 3x^2+x+b$

$a(1)+3 = 3(1)^2+1+b$

$a+3 = 4+b$

$a = 1+b$

$7 = 1+b$

$\boxed{6=b}$

differentiable - share same slope (set derivatives equal) at $x=1$

$f'(x) = \begin{cases} a, & x \le 1 \\ 6x+1, & x<1 \end{cases}$

$a = 6x+1$

$a = 6(1)+1$

$\boxed{a=7}$

3. Practice find constants a and b so that the function is continuous and differentiable.

a. $f(x) = \begin{cases} 2x^2+2x+a & x \le -1 \\ -2bx+1 & x > -1 \end{cases}$

continuous (at $x=-1$)

$2x^2+2x+a = -2bx+1$

$2(-1)^2+2(-1)+a = -2b(-1)+1$

$a = 2b+1$

$a = 2(1)+1$

$\boxed{a=3}$

differentiable at $x=-1$

$f'(x) = \begin{cases} 4x+2 & x \le -1 \\ -2b & x > -1 \end{cases}$

$4x+2 = -2b$

$4(-1)+2 = -2b$

$-2 = -2b$

$\boxed{1=b}$

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2.4 Chain Rule Practice Problems WS #1

Finding a Derivative **In Exercises 7–34, find the derivative of the function.**

Chain Rule: $\frac{d}{dx}[f(g(x)] = f'[g(x)] * g'(x)$

1. $y = (5x - 8)^4$

2) $y = (4x - 1)^3$

3) $y = 5(2 - x^3)^4$

4) $g(x) = 3(4 - 9x)^4$

5) $f(t) = \sqrt{5 - t}$

6) $y = \sqrt[3]{6x^2 + 1}$

7) $f(x) = \sqrt{x^2 - 4x + 2}$

8) $y = 2\sqrt[4]{9 - x^2}$

8

Find the derivative of the function below:

Chain Rule: $\frac{d}{dx}[f(g(x)] = f'[g(x)] * g'(x)$

9) $y = \dfrac{1}{x-2}$

10) $y = \dfrac{1}{\sqrt{3x+5}}$

11) $y = \dfrac{x}{\sqrt{x^2+1}}$

12) $y = \dfrac{x}{\sqrt{x^4+4}}$

13) $g(x) = \left(\dfrac{x+5}{x^2+2}\right)^2$

14) $g(x) = \left(\dfrac{3x^2-2}{2x+3}\right)^3$

Key (9)

2.4 Chain Rule Practice Problems WS #1

Finding a Derivative In Exercises 7–34, find the derivative of the function.

Chain Rule: $\frac{d}{dx}[f(g(x)] = f'[g(x)] * g'(x)$

1. $y = (5x - 8)^4$

outside: $(\ )^4$
inside: $5x-8$

$y' = 4(\quad)^3 \cdot (5)$

$y' = 4(5x-8)^3 \cdot 5$

$\boxed{y' = 20(5x-8)^3}$

2) $y = (4x - 1)^3$

outside: $(\ )^3$
inside: $4x-1$

$y' = 3(4x-1)^2 \cdot (4)$

$\boxed{y' = 12(4x-1)^2}$

3) $y = 5(2 - x^3)^4$

outside: $5(\ )^4$
inside: $2-x^3$

$y' = 5 \cdot 4(\quad)^3 \cdot (-3x^2)$

$y' = 20(2-x^3)^3 \cdot -3x^2$

$\boxed{y' = -60x^2(2-x^3)^3}$

4) $g(x) = 3(4 - 9x)^4$

outside: $3(\ )^4$
inside: $4-9x$

$g'(x) = 3 \cdot 4(\quad)^3 \cdot (-9)$

$g'(x) = 12(4-9x)^3(-9)$

$\boxed{g'(x) = -108(4-9x)^3}$

5) $f(t) = \sqrt{5 - t}$

outside: $(\ )^{1/2}$
inside: $5-t$

$f(t) = (5-t)^{1/2}$

$f'(t) = \frac{1}{2}(\quad)^{-1/2}(-1)$

$f'(t) = \frac{1}{2}(5-t)^{-1/2}(-1)$

$\boxed{f'(t) = \frac{-1}{2(5-t)^{1/2}}}$

6) $y = \sqrt[3]{6x^2 + 1}$

outside: $(\quad)^{1/3}$
inside: $6x^2+1$

$y = (6x^2+1)^{1/3}$

$y' = \frac{1}{3}(\quad)^{-2/3}(12x)$

$y' = \frac{1}{3}(6x^2+1)^{-2/3} \cdot 12x$

$\boxed{y' = \frac{4x}{(6x^2+1)^{2/3}}}$

7) $f(x) = \sqrt{x^2 - 4x + 2}$

outside: $(\ )^{1/2}$
inside: x^2-4x+2

$f(x) = (x^2-4x+2)^{1/2}$

$f'(x) = \frac{1}{2}(\quad)^{-1/2} \cdot (2x-4)$

$f'(x) = \frac{1}{\cancel{2}}(x^2-4x+2)^{-1/2} \cdot \cancel{2}(x-2)$

$\boxed{f'(x) = \frac{x-2}{(x^2-4x+2)^{1/2}}}$

8) $y = 2\sqrt[4]{9 - x^2}$

outside: $2(\ )^{1/4}$
inside: $9-x^2$

$y = 2(9-x^2)^{1/4}$

$y' = 2 \cdot \frac{1}{4}(\quad)^{-3/4} \cdot (-2x)$

$y' = \frac{2}{4}(9-x^2)^{-3/4}(-2x)$

$\boxed{y' = \frac{-x}{(9-x^2)^{3/4}}}$

10

Find the derivative of the function below:

Chain Rule: $\frac{d}{dx}[f(g(x))] = f'[g(x)] * g'(x)$

9) $y = \dfrac{1}{x-2}$

outside: $(\ )^{-1}$
inside: $x-2$

$$y=(x-2)^{-1}$$
$$y'=-1(x-2)^{-2}(1)$$
$$\boxed{y'=\frac{-1}{(x-2)^2}}$$

10) $y = \dfrac{1}{\sqrt{3x+5}}$

outside: $(\ )^{-1/2}$
inside: $3x+5$

$$y=\frac{1}{(3x+5)^{1/2}}$$
$$y=(3x+5)^{-1/2}$$
$$y'=\frac{-1}{2}(\quad)^{-3/2}(3)$$
$$y'=\frac{-1}{2}(3x+5)^{-3/2}(3)$$
$$\boxed{y'=\frac{-3}{2(3x+5)^{3/2}}}$$

11) $y = \dfrac{x}{\sqrt{x^2+1}}$

1) quotient
2) chain
outside: $(\ )^{1/2}$
inside: x^2+1

$$y=\frac{x}{(x^2+1)^{1/2}}$$
$$y'=\frac{(1)(x^2+1)^{1/2}-x\cdot\frac{1}{2}(x^2+1)^{-1/2}(2x)}{\left[(x^2+1)^{1/2}\right]^2}$$
$$y'=\frac{(x^2+1)^{1/2}-\frac{x^2}{(x^2+1)^{1/2}}}{x^2+1}\cdot(x^2+1)^{1/2}$$
$$y'=\frac{x^2+1-x^2}{(x^2+1)(x^2+1)^{1/2}}$$
$$\boxed{y'=\frac{1}{(x^2+1)^{3/2}}}$$

12) $y = \dfrac{x}{\sqrt{x^4+4}}$

1) quotient
2) chain:
outside: $(\ )^{1/2}$
inside: x^4+4

$$y=\frac{x}{(x^4+4)^{1/2}}$$
$$y'=\frac{(1)(x^4+4)^{1/2}-(x)\cdot\frac{1}{2}(x^4+4)^{-1/2}(4x^3)}{\left[(x^4+4)^{1/2}\right]^2}$$
$$y'=\frac{(x^4+4)^{1/2}-\frac{2x^4}{(x^4+4)^{1/2}}}{x^4+4}\cdot(x^4+4)^{1/2}$$
$$y'=\frac{x^4+4-2x^4}{(x^4+4)^{3/2}}$$
$$\boxed{y'=\frac{4-x^4}{(x^4+4)^{3/2}}}$$

13) $g(x) = \left(\dfrac{x+5}{x^2+2}\right)^2$

1) chain
outside: $(\ )^2$
inside: $\dfrac{x+5}{x^2+2}$
2) quotient

$$g'(x)=2\left[\frac{x+5}{x^2+2}\right]^1\left[\frac{(1)(x^2+2)-(x+5)(2x)}{(x^2+2)^2}\right]$$
$$g'(x)=\frac{2(x+5)(x^2+2-2x^2-10x)}{(x^2+2)^3}$$
$$\boxed{g'(x)=\frac{2(x+5)(-x^2-10x+2)}{(x^2+2)^3}}$$

14) $g(x) = \left(\dfrac{3x^2-2}{2x+3}\right)^3$

1) chain
outside: $(\ )^3$
inside: $\dfrac{3x^2-2}{2x+3}$
2) quotient

$$g'(x)=3\left[\frac{3x^2-2}{2x+3}\right]^2\left[\frac{(6x)(2x+3)-(3x^2-2)(2)}{(2x+3)^2}\right]$$
$$g'(x)=\frac{3(3x^2-2)^2(12x^2+18x-6x^2+4)}{(2x+3)^2(2x+3)^2}$$
$$\boxed{g'(x)=\frac{3(3x^2-2)^2(6x^2+18x+4)}{(2x+3)^4}}$$

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AB Calculus Ch. 2 Test Review

1. The average rate of change of $f(x) = 4x - x^2$ on the interval [1,3] is ________.
   a.) -2 b.) -1 c.) 0 d.) 1 e.) 2

2. The instantaneous rate of change of f(x) at the endpoints are : ____________________.

3. Find $\frac{d^2y}{dx^2}$ for $y = \sqrt{(x^2+1)}$.

   a.) $\frac{1}{(x^2+1)^{\frac{3}{2}}}$ b.) $\frac{1}{(x^2+1)^{\frac{1}{2}}}$ c.) $\frac{x}{(x^2+1)^{\frac{1}{2}}}$ d.) $\frac{x}{(x^2+1)^{\frac{3}{2}}}$ e.) $\frac{1}{2(x^2+1)^{\frac{3}{2}}}$

4. If the line tangent to the graph of the function f at the point (1,7) passes through the point (-2,-2), then $f'(1) =$

   a.) - 5 b.) 1 c.) 3 d.) 7 e.) undefined

5. Find a and b such that

$$f(x) = \begin{cases} ax^3 & x \le 3 \\ x^2 + b & x > 3 \end{cases}$$ is differentiable everywhere.

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6. A particle moves along a straight line according to the given equation: $s(t) = \frac{2}{3}t^3 - 2t^2 - 1$, $t \geq 0$.

a) Find when the particle is moving to the left.	b.) Find when the velocity is increasing.
c.) Find when the speed is decreasing.	d.) Where is the particle located when the velocity is zero?
e) Find particle's displacement from t = 0 to t = 3	f) Find particle's distance from t = 0 to t = 3
g)Is the velocity increasing or decreasing at t=2?	h) Find the velocity and position when acceleration is zero.

7. Find $\lim_{h\to 0} \frac{2(-1+h)^3-3(-1+h)-1}{h}$

8. If $f(x) = x^4 - 4x$, evaluate $\lim\limits_{x\to -1} \frac{f(x)-f(-1)}{x+1}$.

a.) -8 b.) 0 c.) 1 d.) 2 e.) 4

9. Find the points at which the graph of the equation has a vertical or horizontal tangent line:

$4x^2 + y^2 - 8x + 4y + 4 = 0$

AB Calculus Ch. 2 Test Review Session

1. The average rate of change of $f(x)=4x-x^2$ on the interval [1,3] is________.

   a.) -2 b.) -1 $\boxed{\text{c.) } 0}$ d.) 1 e.) 2

$f(1)=4-1=3$

$f(3)=4(3)-9=3$

$\frac{f(3)-f(1)}{3-1}=\frac{3-3}{2}=\frac{0}{2}$

2. The instantaneous rate of change of f(x) at the endpoints are : ____________

$f'(x)=4-2x$

$f'(1)=4-2(1)=2$

$f'(3)=4-2(3)=-2$

$\boxed{f'(1)=2 \quad f'(3)=-2}$

3. Find $\frac{d^2y}{dx^2}$ for $y=\sqrt{(x^2+1)}$. $=(x^2+1)^{1/2}$ $\quad f'(x)=\frac{1}{2}(x^2+1)^{-1/2}(2x)=\frac{x}{\sqrt{x^2+1}}$

$\boxed{\text{a.)}}$ $\frac{1}{(x^2+1)^{\frac{3}{2}}}$ b.) $\frac{1}{(x^2+1)^{\frac{1}{2}}}$ c.) $\frac{x}{(x^2+1)^{\frac{1}{2}}}$ d.) $\frac{x}{(x^2+1)^{\frac{3}{2}}}$ e.) $\frac{1}{2(x^2+1)^{\frac{3}{2}}}$

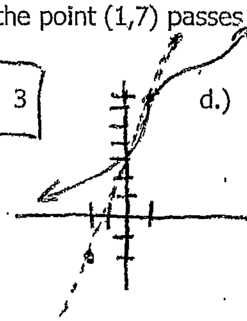
$f'(x)=\frac{x}{\sqrt{x^2+1}}=\frac{x}{(x^2+1)^{1/2}}$

$f''(x)=\frac{1(x^2+1)^{1/2}-x\cdot\frac{1}{2}(x^2+1)^{-1/2}(2x)}{x^2+1}$

$=\frac{\sqrt{x^2+1}-\frac{x^2}{\sqrt{x^2+1}}}{x^2+1}\cdot\frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}$ $\rightarrow$ $\frac{x^2+1-x^2}{(x^2+1)^{3/2}}=\boxed{\frac{1}{(x^2+1)^{3/2}}}$

4. If the line tangent to the graph of the function f at the point (1,7) passes through the point (-2,-2), then $f'(1)=$

   a.) -5 b.) 1 $\boxed{\text{c.) } 3}$ d.) 7 e.) undefined



Find slope : $m=\frac{-2-7}{-2-1}=\frac{-9}{-3}=3$

5. Find a and b such that

$$f(x)=\begin{cases} ax^3 & x\le 3 \\ x^2+b & x>3 \end{cases}$$ is differentiable everywhere.

↳ continuous and differentiable

$\boxed{\text{at } x=3}$

$ax^3=x^2+b$

$a(3)^3=3^2+b$

$27a=9+b$

$27(2/9)=9+b$

$6=9+b$

$\boxed{-3=b}$

at $x=3$

$3ax^2=2x$

$3a(9)=2(3)$

$\boxed{a=2/9}$

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$v(t) = 2t^2 - 4t$

$a(t) = 4t - 4$

$v(t)$: $-$ on $(0,2)$, $+$ after 2

$0 = 2t(t-2)$

$t = 0, 2$

6. A particle moves along a straight line according to the given equation: $s(t) = \frac{2}{3}t^3 - 2t^2 - 1$, $t \geq 0$.

a) Find when the particle is moving to the left.

$0 < t < 2$

$\boxed{(0,2)}$

b.) Find when the velocity is increasing.

$\boxed{a(t) > 0}$

$a(t)$: $-$ on $(0,1)$, $+$ after 1

$\boxed{(1, \infty)}$

$0 = 4t - 4 \quad 0 = 4(t-1)$

c.) Find when the speed is decreasing.

opposite signs for $v(t)$ and $a(t)$

$1 < t < 2$

$\boxed{(1,2)}$

d.) Where is the particle located when the velocity is zero?

$t = 0, 2$

$s(2) = \frac{2}{3}(2)^3 - 2(2)^2 - 1$

$= \frac{16}{3} - 8 - 1 = \frac{16}{3} - \frac{27}{3} = \frac{-11}{3}$

$\boxed{s(0) = -1 \quad s(2) = -11/3}$

e) Find particle's displacement from t = 0 to t = 3

$s(0) = -1$

$s(3) = -1$

displacement $= 0$

f) Find particle's distance from t = 0 to t = 3

$s(0) = -1$

$s(2) = -11/3$

$s(3) = -1$

$8/3$, $8/3$

$= \frac{16}{3} \approx 5.3\overline{3}$

g) Is the velocity increasing or decreasing at t=2?

$a(2) = 8 - 4 = 4$

since $a(2) > 0$ velocity is increasing at $t = 2$.

h) Find the velocity and position when acceleration is zero.

$0 = 4t - 4$

$1 = t$

$v(1) = 2(1)^2 - 4(1) = -2$

$s(1) = -7/3$

$\lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad f'(-1) = \frac{f(-1+h) - f(-1)}{h}$

7. Find $\lim_{h \to 0} \frac{2(-1+h)^3 - 3(-1+h) - 1}{h}$

Find $f'(-1)$

$f(x) = 2x^3 - 3x \qquad f'(x) = 6x^2 - 3$

$f'(-1) = 6(-1)^2 - 3 = 6 - 3 = 3$

$\boxed{f'(-1) = 3}$

8. If $f(x) = x^4 - 4x$, evaluate $\lim\limits_{x \to -1} \frac{f(x) - f(-1)}{x+1}$. Find $f'(-1)$

$\lim\limits_{x \to c} \frac{f(x) - f(c)}{x - c}$

$f'(x) = 4x^3 - 4$

$f'(-1) = 4(-1)^3 - 4 = \boxed{-8}$

$\boxed{\text{a.)} \ -8}$ b.) 0 c.) 1 d.) 2 e.) 4

9. Find the points at which the graph of the equation has a vertical or horizontal tangent line:

$4x^2 + y^2 - 8x + 4y + 4 = 0$

$8x + 2y\left(\frac{dy}{dx}\right) - 8 + 4\left(\frac{dy}{dx}\right) = 0$

$\frac{dy}{dx}(2y + 4) = 8 - 8x$

$\frac{dy}{dx} = \frac{2(4-4x)}{2(y+2)} = \frac{4-4x}{y+2}$

horizontal tangent

set numerator $f'(x) = 0$

$4 - 4x = 0 \quad 4 = 4x$

$x = 1$

$4(1)^2 + y^2 - 8(1) + 4y + 4 = 0$

$y^2 + 4y = 0 \quad y(y+4) = 0$

$y = 0, -4$

$\boxed{(1,0) \text{ and } (1,-4)}$

vertical tangent

set denominator $f'(x) = 0$

$y + 2 = 0 \quad y = -2$

$4x^2 + 4 - 8x - 8 + 4 = 0$

$4x^2 - 8x = 0$

$4x(x-2) = 0 \quad x = 0, 2$

$\boxed{(0,-2) \text{ and } (2,-2)}$

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Chapter 2 Derivatives Test Review #2

1. Given $x^2y + y^2 = 2x$
   a. Find dy/dx

   b. Find a point on the graph where there is a horizontal tangent

   c. Find the points on the graph where there is a vertical tangent

2. Find d^2y/dx^2 for $\frac{x+2}{3-x}$

3. Find dy/dx for $\left(\frac{3x+1}{1-x^2}\right)^3$

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Chapter 2 Derivatives Test Review #2

Key

1. Given $x^2y + y^2 = 2x$

a. Find dy/dx

$$2xy + x^2\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = 2$$

$$x^2\frac{dy}{dx} + 2y\frac{dy}{dx} = 2 - 2xy$$

$$\frac{dy}{dx}\left(x^2 + 2y\right) = 2 - 2xy$$

$$\boxed{\frac{dy}{dx} = \frac{2-2xy}{x^2+2y}}$$

b. Find the points on the graph where there is a horizontal tangent

* set numerator of derivative = 0

$2 - 2xy = 0$

$2 = 2xy$

$xy = 1$

$x = 1,\ y = 1$

* check with equation $x^2y + y^2 = 2x$

$(1)^2(1) + (1)^2 = 2(1)$ ✓

$\boxed{(1,1)}$

c. Find the points on the graph where there is a vertical tangent

* set denominator of derivative = 0

$x^2 + 2y = 0$

$0^2 + 2(0) = 0$

$(-2)^2 + 2(-2) = 0$

* check with equation

$0^2(0) + 0^2 = 2(0)$ ✓

$(-2)^2(-2) + (-2)^2 = 2(-2)$ ✓

$\boxed{(0,0) \text{ and } (-2,-2)}$

2. Find d^2y/dx^2 for $\frac{x+2}{3-x}$ $\qquad \frac{f'g - fg'}{g^2}$

$$\frac{dy}{dx} = \frac{(1)(3-x) - (x+2)(-1)}{(3-x)^2}$$

$$= \frac{3 - x + x + 2}{(3-x)^2} = \frac{5}{(3-x)^2}$$

$$\frac{dy}{dx} = 5(3-x)^{-2}$$

$$\frac{d^2y}{dx^2} = (-2)(5)(3-x)^{-3}(-1) = \boxed{\frac{10}{(3-x)^3}}$$

OR $\quad \frac{0\cancel{(3-x)^2} - 5(2)(3-x)'(-1)}{(3-x)^4} = \boxed{\frac{10}{(3-x)^3}}$

3. Find dy/dx for $\left(\frac{3x+1}{1-x^2}\right)^3$

* chain (first)
* quotient

$$\frac{dy}{dx} = 3\left(\frac{3x+1}{1-x^2}\right)^2\left[\frac{3(1-x^2) - (3x+1)(-2x)}{(1-x^2)^2}\right] = \frac{3(3x+1)^2(3x^2+2x+3)}{(1-x^2)^2(1-x^2)^2}$$

$3 - 3x^2 + 6x^2 + 2x$

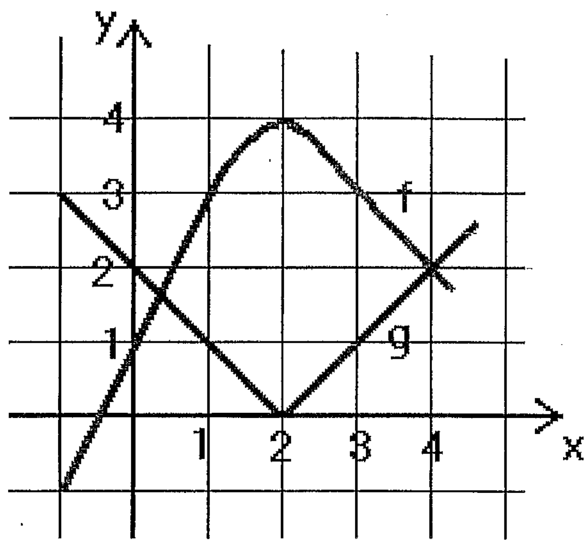
$$\boxed{\frac{dy}{dx} = \frac{3(3x+1)^2(3x^2+2x+3)}{(1-x^2)^4}}$$

AB Calculus Ch. 2 Derivatives   Morning Test Review

(17)

1. Let h(t) = f [g(t)]. Find h'(1)

2. Let z(t) = f(t)g(t). Find z'(3)



3.

Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.

(a) Show that $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$.

4. Find $\frac{d^2y}{dx^2}$ for $f(x) = \frac{x-1}{x+2}$

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5. The position of the particle traveling along a straight line is $x(t) = t^3 - 9t^2 + 15t + 3$ for $t \geq 0$

a) Find when the particle is moving to the left.	b) Is speed increasing or decreasing at t = 2?
c) Find avg. acceleration in interval [0, 2]	d) Where is the particle located when the velocity is zero?
e) Find particle's displacement from t = 0 to t = 3	f) Find particle's distance from t = 0 to t = 3
g) Is the velocity increasing or decreasing at t=2?	h) Find the velocity and position when acceleration is zero.

6. Find $\frac{dy}{dx}$ for $f(x) = \sqrt{x}(x^3-7)^4$.

7. Find $\frac{dy}{dx}$ for $f(x) = \frac{\sqrt{2-x^2}}{4+3x}$

AB Calculus Ch. 2 Derivatives   Morning Test Review

Key

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*chain Rule $f'[g(t)]\cdot g'(t)$

1. Let h(t) = f [g(t)]. Find h'(1)

$h'(t) = f'[g(t)]\cdot g'(t)$

$h'(1) = f'[g(1)]\cdot g'(1)$

$= f'[1]\cdot g'(1)$

$= (1)(-1) = \boxed{-1}$

2. Let z(t) = f(t)g(t). Find z'(3)

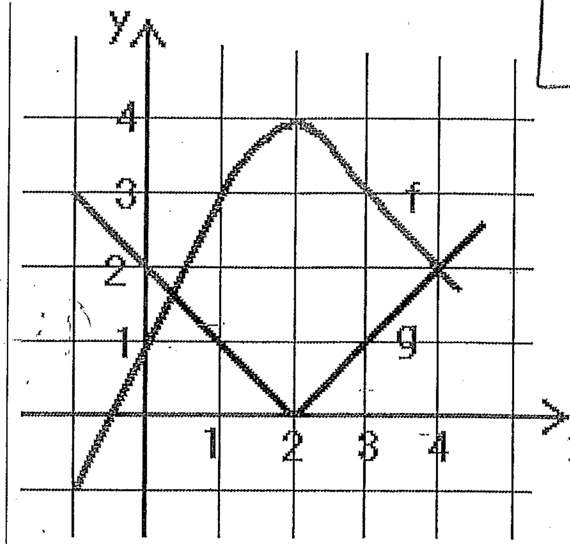
$z'(t) = f'(t)g(t) + f(t)g'(t)$

$z'(3) = f'(3)g(3) + f(3)g'(3)$

$= (-1)(1) + (3)(1)$

$= -1 + 3$

$= \boxed{2}$



3. *implicit, product Rule

Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{4x - 2xy}{x^2 + y^2 + 1}$.

$6y^2\frac{dy}{dx} + 12xy + 6x^2\left(\frac{dy}{dx}\right) - 24x + 6\frac{dy}{dx} = 0$

$\frac{dy}{dx}(6y^2 + 6x^2 + 6) = 24x - 12xy$

$\frac{dy}{dx} = \frac{24x - 12xy}{6y^2 + 6x^2 + 6} = \boxed{\frac{4x - 2xy}{x^2 + y^2 + 1}}$

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4. Find $\frac{d^2y}{dx^2}$ for $f(x) = \frac{x-1}{x+2}$

$$\frac{dy}{dx} = \frac{(1)(x+2)-(x-1)(1)}{(x+2)^2}$$

$$= \frac{x+2-x+1}{(x+2)^2} = \frac{3}{(x+2)^2}$$

$$\frac{dy}{dx} = 3(x+2)^{-2}$$

$$\frac{d^2y}{dx^2} = 3(-2)(x+2)^{-3}(1) = \boxed{\frac{-6}{(x+2)^3}}$$

$v(t) = 3t^2 - 18t + 15 \rightarrow 3(t^2-6t+5) \rightarrow 3(t-5)(t-1)$

$a(t) = 6t - 18$

5. The position of the particle traveling along a straight line is $x(t) = t^3 - 9t^2 + 15t + 3$ for $t \geq 0$

a) Find when the particle is moving to the left.

$+ \quad - \quad +$

$0 \quad 1 \quad 5$

$\boxed{(1,5) \text{ since } v(t) < 0}$

b) Is speed increasing or decreasing at t = 2?

Since $v(2) < 0$ and $a(2) < 0$ have same signs, speed increasing at $t = 2$.

c) Find avg. acceleration in in interval [0, 2]

$$= \frac{v(2)-v(0)}{2-0} = \frac{-9-15}{2-0} = \frac{-24}{2}$$

$v(2) = -9$

$v(0) = 15$

$= \boxed{-12 \text{ units}/s^2}$

d) Where is the particle located when the velocity is zero?

$v(t) = 0$ at $t = 1, 5$

$\boxed{s(1) = 10 \quad s(5) = -22}$

e) Find particle's displacement from t = 0 to t = 3

$s(3) = -6$

$s(0) = 3$

$s(3) - s(0)$

$-6 - 3 = \boxed{-9}$

f) Find particle's distance from t = 0 to t = 3

$s(0) = 3$

$s(1) = 10$

$s(3) = -6$

> 7

> 16

$7 + 16 = \boxed{23}$

g) Is the velocity increasing or decreasing at t=2?

*Is $a(t) > 0$ or $a(t) < 0$

$a(2) = -6$, so since $a(2) < 0$, velocity decreasing at $t = 2$.

h) Find the velocity and position when acceleration is zero.

$a(t) = 6t - 18$

$0 = 6(t-3)$

$t = 3$

$\boxed{s(3) = -6 \quad v(3) = -12}$

6. Find $\frac{dy}{dx}$ for $f(x) = \sqrt{x}(x^3-7)^4$ $\quad (x)^{1/2}(x^3-7)^4$

product, chain (first)

$f'g + fg'$

$$\frac{1}{2}(x)^{-1/2}(x^3-7)^4 + x^{1/2} \cdot 4(x^3-7)^3(3x^2)$$

$$= \boxed{\frac{(x^3-7)^4}{2\sqrt{x}} + 12x^{5/2}(x^3-7)^3}$$

7. Find $\frac{dy}{dx}$ for $f(x) = \frac{\sqrt{2-x^2}}{4+3x} = \frac{(2-x^2)^{1/2}}{4+3x}$

* quotient (first)

* chain

$$f'(x) = \frac{\frac{1}{2}(2-x^2)^{-1/2}(-2x)(4+3x) - (2-x^2)^{1/2}(3)}{(4+3x)^2}$$

$$f'(x) = \frac{\frac{-x(4+3x)}{\sqrt{2-x^2}} - 3\sqrt{2-x^2}}{(4+3x)^2} \cdot \frac{(2-x^2)^{1/2}}{(2-x^2)^{1/2}}$$

$$= \frac{-4x-3x^2-3(2-x^2)}{(4+3x)^2(2-x^2)^{1/2}} = \frac{-4x-3x^2-6+3x^2}{(4+3x)^2(2-x^2)^{1/2}}$$

$$= \boxed{\frac{-4x-6}{(4+3x)^2(2-x^2)^{1/2}}}$$

A.P. Calculus AB 2.2-2.5 Review Session Problems (WS #4)

1) Consider the curve given by $x^2 - xy + y^2 = 4$.
   a) Find the two points on the curve at $x = 0$

   b) Find $\frac{dy}{dx}$ by differentiating implicitly.

   c) Use $\frac{dy}{dx}$ to find the slope of the lines tangent to the curve at the points found in part **a.**

   d) Write the equations of the line tangent to the curve at the points above.

   e) Set up and write the equations (don't solve) in order to find vertical and horizontal tangents to this graph curve.

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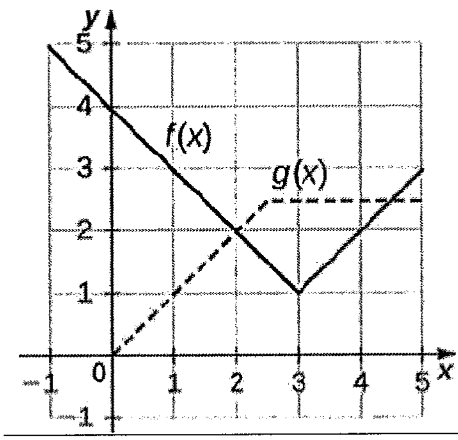
2) If g is differentiable everywhere and $g(x) = \begin{cases} 6x^3 - 8x^2 + 8, & x < -2 \\ ax + b, & x \geq -2 \end{cases}$, find a and b
(Involve derivatives in your work)

3)



a) If $h(x) = f(x) \cdot g(x)$, find $h'(2)$

b) If $w(x) = f(g(x))$, find $w'(3)$

c) $If\ z(x) = 2[f(x)]^3 \quad find\ z'(0)$

d) If $k(x) = f(f(x))$, find $k'(4)$

e) If $p(x) = \frac{g(x)}{f(x)}$ find $p'(1)$

4) Find $\frac{dy}{dx}$ for $y = 2\left(\frac{5x^3-2}{3-x^2}\right)^7$ (Write derivative as a simplified rational expression)

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5) A particle moves along a straight line according to the given equation: $x(t) = \frac{t^4}{4} - t^3 - 2t^2 + 1$, for all real numbers in **meters per minute**

a) Find the velocity and acceleration function	b) Find when the particle changes direction (justify with because statement)
c) Determine interval when particle is moving left (justify with because statement)	d) Determine interval when particle is moving left (justify with because statement)
e) Find the average velocity of particle in interval [1, 3] (show work, include units)	f) Find the average acceleration of particle in interval [1, 3] (show work, include units)
g) Find particle's displacement from t = 1 to t = 6 (Show your work)	h) Find particle's distance from t = 1 to t = 6 (Show your work)
i) At t = 2, is the speed increasing or decreasing? Provide justification for your answer.	j) At t = 5, is the velocity increasing or decreasing? Provide justification for your answer.

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Key

A.P. Calculus AB 2.2-2.5 Review Session Problems (WS #4)

1) Consider the curve given by $x^2 - xy + y^2 = 4$.

a) Find the two points on the curve at $x = 0$

$(0)^2 - (0)y + y^2 = 4$

$y^2 = 4$

$\sqrt{y^2} = \pm\sqrt{4}$

$y = 2, -2$

$\boxed{\text{The 2 points are } (0,2) \text{ and } (0,-2)}$

b) Find $\frac{dy}{dx}$ by differentiating implicitly.

*product rule

$x^2 - (\overbrace{x}^{f}\overbrace{y}^{g}) + y^2 = 4$

$2x - \left(\overbrace{(1)}^{f'}\overbrace{(y)}^{g} + \overbrace{(x)}^{f}\overbrace{(1)\left(\frac{dy}{dx}\right)}^{g'}\right) + 2y\left(\frac{dy}{dx}\right) = 0$

$2x - y - x\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = 0$

$-x\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = -2x + y$

$\frac{dy}{dx}(-x + 2y) = -2x + y$

$\boxed{\frac{dy}{dx} = \frac{-2x+y}{-x+2y}}$

c) Use $\frac{dy}{dx}$ to find the slope of the lines tangent to the curve at the points found in part a.

i) point: $(0,2)$

slope: $\left.\frac{dy}{dx}\right|_{(0,2)} = \frac{-2(0)+2}{-0+2(2)} = \frac{2}{4}$

$\boxed{\text{slope: } m = 1/2}$

$\boxed{y - 2 = \frac{1}{2}(x - 0)}$

ii) point: $(0,-2)$

slope: $\left.\frac{dy}{dx}\right|_{(0,-2)} = \frac{-2(0)-2}{-0+2(-2)} = \frac{-2}{-4}$

$\boxed{m = 1/2}$

$\boxed{y + 2 = \frac{1}{2}(x - 0)}$

d) Write the equation of the line tangent to the curve at the points above

$\boxed{y - 2 = \frac{1}{2}(x - 0)}$ $\boxed{y + 2 = \frac{1}{2}(x - 0)}$

e) Detail how you would set up (don't solve) in order to find vertical and horizontal tangent:

horizontal tangent: set numerator of $\frac{dy}{dx} = 0 \rightarrow \boxed{-2x + y = 0}$

vertical tangent: set denominator of $\frac{dy}{dx} = 0 \rightarrow \boxed{-x + 2y = 0}$

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2) If g is differentiable everywhere and $g(x)=\begin{cases}6x^3-8x^2+8, & x<-2\\ ax+b, & x\ge -2\end{cases}$, find a and b

(Involve derivatives in your work)

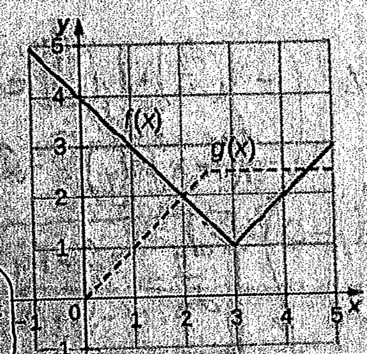
continuous property: set equations equal (at $x=-2$)

$$6x^3-8x^2+8=ax+b$$
$$6(-2)^3-8(-2)^2+8=a(-2)+b$$
$$-72=-2a+b$$
$$-72=-2(104)+b$$
$$\boxed{136=b}$$

differentiable property: set derivatives equal (at $x=-2$)

$$18x^2-16x=a$$
$$18(-2)^2-16(-2)=a$$
$$\boxed{a=104}$$

3)



a) If $h(x)=f(x)\cdot g(x)$, find $h'(2)$ *product rule

$$h'(x)=f'(x)g(x)+f(x)g'(x)$$
$$h'(2)=f'(2)g(2)+f(2)g'(2)$$
$$h'(2)=(-1)(2)+(2)(1)$$
$$\boxed{h'(2)=0}$$

b) If $w(x)=f(g(x))$, find $w'(3)$ *chain rule

$$w'(x)=f'[g(x)]\cdot g'(x)$$
$$w'(3)=f'[g(3)]\cdot g'(3)$$
$$w'(3)=f'[2.5]\cdot g'(3)$$
$$w'(3)=(-1)(0)$$
$$\boxed{w'(3)=0}$$

c) If $z(x)=2[f(x)]^3$ find $z'(0)$

*chain rule
out: $2[\ \ ]^3$
in: $f(x)$

$$z'(x)=6(\ \ )^2\cdot f'(x)$$
$$z'(x)=6[f(x)]^2\cdot f'(x)$$
$$z'(0)=6[f(0)]^2\cdot f'(0)$$
$$z'(0)=6[4]^2\cdot(-1)$$
$$\boxed{z'(0)=-96}$$

d) If $k(x)=f(f(x))$, find $k'(4)$ chain rule

$$k'(x)=f'[f(x)]\cdot f'(x)$$
$$k'(4)=f'[f(4)]\cdot f'(4)$$
$$k'(4)=f'[2]\cdot f'(4)$$
$$k'(4)=(-1)(1)$$
$$\boxed{k'(4)=-1}$$

e) If $p(x)=\frac{g(x)}{f(x)}$ find $p'(1)$ *quotient rule

$$p'(x)=\frac{g'(x)f(x)-g(x)f'(x)}{[f(x)]^2}$$
$$p'(1)=\frac{g'(1)f(1)-g(1)f'(1)}{[f(1)]^2}$$
$$p'(1)=\frac{(1)(3)-(1)(-1)}{3^2}$$
$$p'(1)=\frac{3+1}{9}=\boxed{\frac{4}{9}}$$

4) Find $\frac{dy}{dx}$ for $y=2\left(\frac{5x^3-2}{3-x^2}\right)^7$ (Write derivative as a simplified rational expression)

*(1) chain rule
(2) quotient rule

out: $2[\ \ ]^7$
in: $\frac{5x^3-2}{3-x^2}$
quotient rule

$45x^2-15x^4+10x^4-4x$

$$y'=14\left[\frac{5x^3-2}{3-x^2}\right]^6\cdot\left[\frac{\overset{f'}{15x^2}\,\overset{g}{(3-x^2)}-\overset{f}{(5x^3-2)}\,\overset{g'}{(-2x)}}{\underset{g^2}{(3-x^2)^2}}\right]$$
$$y'=14\left[\frac{5x^3-2}{3-x^2}\right]^6\left[\frac{-5x^4+45x^2-4x}{(3-x^2)^2}\right]$$
$$\boxed{y'=\frac{14(5x^3-2)^6(-5x^4+45x^2-4x)}{(3-x^2)^8}}$$

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5) A particle moves along a straight line according to the given equation: $x(t) = \frac{t^4}{4} - t^3 - 2t^2 + 1$, for all real numbers in **meters per minute**

a) Find the velocity and acceleration function

$v(t) = t^3 - 3t^2 - 4t$

$a(t) = 3t^2 - 6t - 4$

b.) Find when the particle changes direction (justify with because statement)

$0 = t(t^2 - 3t - 4)$

$0 = t(t-4)(t+1)$

$t = 0, -1, 4$

$v(t)$

− + − +

-2 -1 -1/2 0 1 4 5

Direction change at $t = -1, 0, 4$ b/c $v(t)$ change signs

c) Determine interval when particle is moving left (justify with because statement)

particle moves left $(-\infty, -1) \cup (0, 4)$ b/c $v(t) < 0$

d) Determine interval when particle is moving left (justify with because statement)

particle moves right $(-1, 0) \cup (4, \infty)$ b/c $v(t) > 0$

e) Find the average velocity of particle in interval [1, 3] (show work, include units)

Avg. velocity $= \frac{\text{change in position}}{\text{change in time}} \rightarrow \frac{x(3) - x(1)}{3 - 1}$

$x(3) = -23.75$

$x(1) = -1.75$

$\frac{-23.75 - (-1.75)}{3 - 1} = \boxed{-11 \text{ meters/min}}$

f) Find the average acceleration of particle in interval [1, 3] (show work, include units)

Avg. acceleration $= \frac{\text{change in velocity}}{\text{change in time}} \rightarrow \frac{v(3) - v(1)}{3 - 1}$

$v(3) = -12$

$v(1) = -6$

$\frac{-12 - (-6)}{3 - 1} = \boxed{-3 \text{ meters/min}^2}$

g) Find particle's displacement from t = 1 to t = 6 (Show your work)

displacement = final position − initial position

displacement $= x(6) - x(1)$

$= 37 - (-1.75)$

$x(6) = 37$

$x(1) = -1.75$

$\boxed{\text{displacement is } 38.75 \text{ m}}$

h) Find particle's distance from t = 1 to t = 6 (Show your work) * count the distance of endpoints to locations of direction change

(* direction change at $t = 4$)

$x(1) = -1.75$

$x(4) = -31$

$x(6) = 37$

$x(t)$: -31, -1.75, 37

29.25

68

$\boxed{\text{Total distance is } 29.25 + 68 = 97.25 \text{ m}}$

i) At t = 2, is the speed increasing or decreasing? Provide justification for your answer.

$v(2) = -12$ $a(2) = -4$

$\boxed{\text{Speed is increasing since } v(t) \text{ and } a(t) \text{ have same signs at } t = 2}$

j) At t = 5, is the velocity increasing or decreasing? Provide justification for your answer.

$a(5) = 41$ * This phrasing is describing acceleration, not velocity

$\boxed{\text{Velocity is increasing at } t = 5 \text{ since acceleration is positive}}$

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Ch. 2.2-2.5 Test Topics:

1) Extended Implicit Differentiation Problem (pg. 5 in packet) 2.5 Implicit Differentiation FRQ problem

2) Graph Problems Miscellaneous Ch 2 Review WS (pgs. 1-4)

3) Derivative problems involving (Power/Product/Quotient/Chain/Implicit)

4) PVA Particle Motion Problem (similar to one on 2.2-2.3 quiz)

5) Differentiable Piecewise Problem (pg. 6a-6b in packet) Differentiable Piecewise Function WS