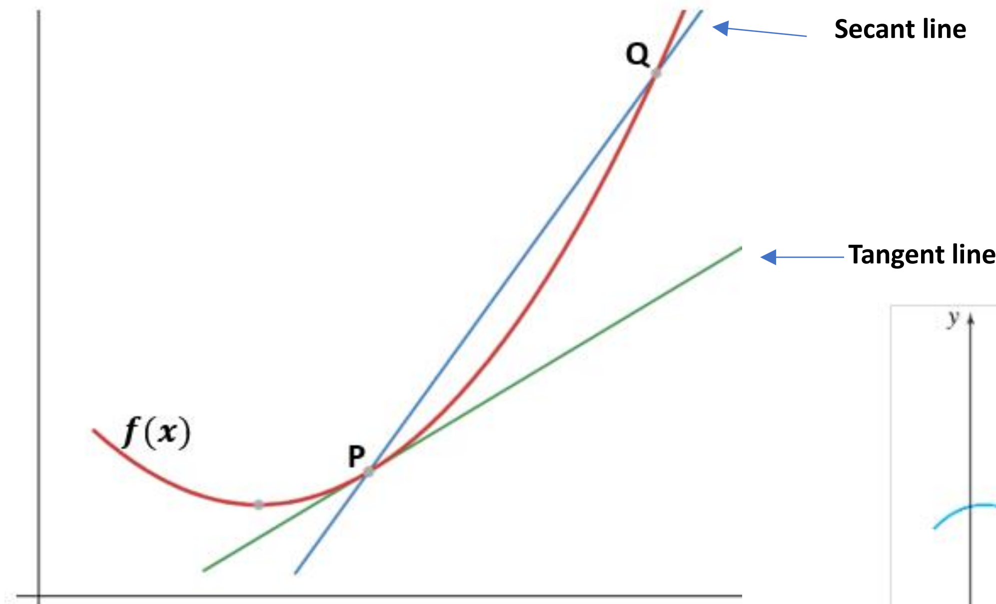
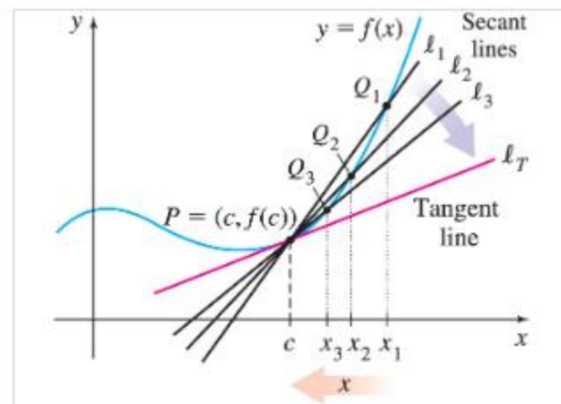
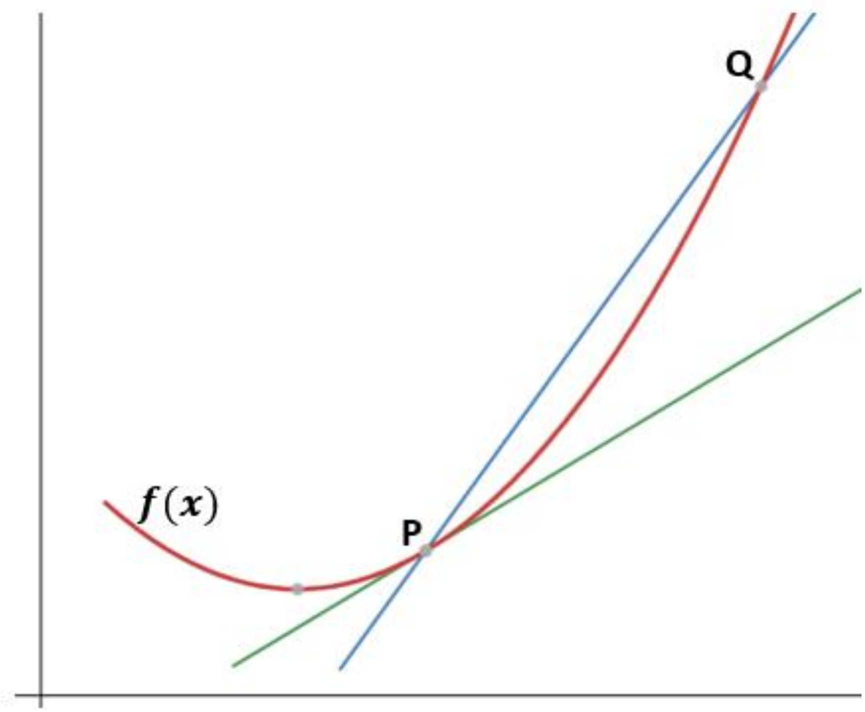
AP Calculus – 2.2 Notes - Limit Definition of a Derivative

Goal: To discover a formula to calculate the slope (steepness) of all tangent lines to a curved graph.







General Limit Definition of the Derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Alternate Limit Definition of a derivative

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

<u>*f '(x) is "f prime of x"*:</u> This is the notation for the derivative function.

<u>Derivative</u> is the slope (steepness) of a curve at a single point

*The <u>derivative function</u> is a **slope-finding formula** for a curved graph, where the slope is of the curve is ever-changing.

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Alternate Limit Definition of a derivative:

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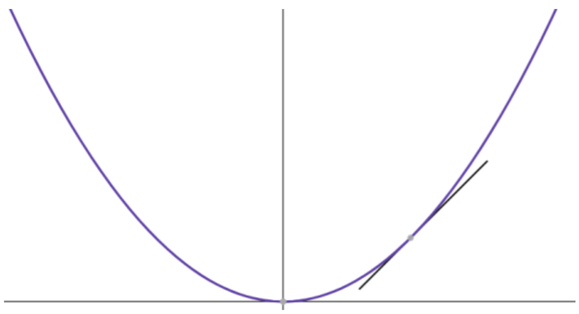
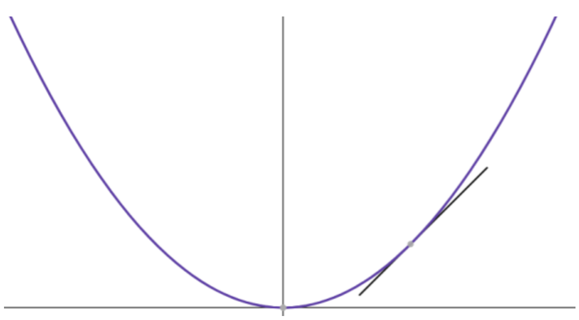
Example 1: (a) Find the general derivative of $f(x) = x^2$

(b) Write the equation of the tangent line to $f(x)$ at $x = 1$ (point-slope form: $\boldsymbol{y - y_1 = m(x - x_1)}$)



(c) Write the equation of the tangent line to $f(x)$ at $x = -5$



To Recap:

$*$ $\boldsymbol{f(x)}$ is the **height-finding formula** (finds the y-value of graph at that point)

*Since $f(1) = 1$, this tells us that when x = 1, the height of the graph has a y-value of 1

*$\boldsymbol{f'(x)}$ is the **slope-finding formula** for the $f(x)$ graph.

*Since $f'(1) = 2$, this tells us that when x = 1, the slope of the tangent line to $f(x)$ has a slope (steepness) of 2.

General Limit Definition of the Derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Alternate Limit Definition of a derivative:

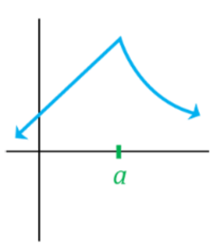
$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Example 2: (a) Find the general derivative of $f(x) = \sqrt{x}$

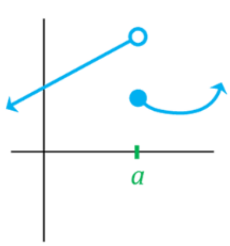
(b) Write the equation of the tangent line to $f(x)$ at $x = 2$ (point-slope form: $\boldsymbol{y - y_1 = m(x - x_1)}$)

Example 3: Use the alternative derivative definition to find slope of $f(x) = \sqrt{x}$ at x = 2.

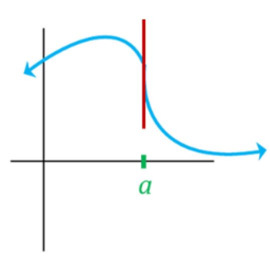
Differentiability: In order for a function to be **differentiable** (smooth curve) at a point a, then the graph must be continuous at that point, cannot contain a sharp turn & cannot have a vertical tangent at the point.



Cusp / Corner



Discontinuous



Vertical Tangent

General Limit Definition of the Derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

Classwork Examples:

Find the derivative using limits

1. $f(x) = 7 - 6x$

2. $y = 5x^2 - x$

3. $y = \sqrt{5x+2}$

4. $f(x) = \frac{1}{x-2}$