97. $f(x) = \begin{cases} x\sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$

Using the Squeeze Theorem, you have $-|x| \le x\sin(1/x) \le |x|$, $x \neq 0$. So, $\lim_{x\to 0} x\sin(1/x) = 0 = f(0)$ and f is continuous at $x = 0$. Using the alternative form of the derivative, you have

$$\lim_{x\to 0}\frac{f(x) - f(0)}{x - 0} = \lim_{x\to 0}\frac{x\sin(1/x) - 0}{x - 0} = \lim_{x\to 0}\left(\sin\frac{1}{x}\right).$$

Because this limit does not exist ($\sin(1/x)$ oscillates between -1 and 1), the function is not differentiable at $x = 0$.

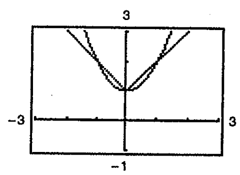
$g(x) = \begin{cases} x^2\sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$

Using the Squeeze Theorem again, you have $-x^2 \le x^2\sin(1/x) \le x^2$, $x \neq 0$. So, $\lim_{x\to 0} x^2\sin(1/x) = 0 = g(0)$ and g is continuous at $x = 0$. Using the alternative form of the derivative again, you have

$$\lim_{x\to 0}\frac{g(x) - g(0)}{x - 0} = \lim_{x\to 0}\frac{x^2\sin(1/x) - 0}{x - 0} = \lim_{x\to 0} x\sin\frac{1}{x} = 0.$$

Therefore, g is differentiable at $x = 0$, $g'(0) = 0$.

98.



As you zoom in, the graph of $y_1 = x^2 + 1$ appears to be locally the graph of a horizontal line, whereas the graph of $y_2 = |x| + 1$ always has a sharp corner at $(0, 1)$. y_2 is not differentiable at $(0, 1)$.

Section 2.2 Basic Differentiation Rules and Rates of Change

1. (a) $y = x^{1/2}$

$y' = \frac{1}{2}x^{-1/2}$

$y'(1) = \frac{1}{2}$

(b) $y = x^3$

$y' = 3x^2$

$y'(1) = 3$

2. (a) $y = x^{-1/2}$

$y' = -\frac{1}{2}x^{-3/2}$

$y'(1) = -\frac{1}{2}$

(b) $y = x^{-1}$

$y' = -x^{-2}$

$y'(1) = -1$

3. $y = 12$

$y' = 0$

4. $f(x) = -9$

$f'(x) = 0$

5. $y = x^7$

$y' = 7x^6$

6. $y = x^{12}$

$y' = 12x^{11}$

7. $y = \frac{1}{x^5} = x^{-5}$

$y' = -5x^{-6} = -\frac{5}{x^6}$

8. $y = \frac{3}{x^7} = 3x^{-7}$

$y' = 3(-7x^{-8}) = -\frac{21}{x^8}$

9. $y = \sqrt[5]{x} = x^{1/5}$
$y' = \frac{1}{5}x^{-4/5} = \frac{1}{5x^{4/5}}$

10. $y = \sqrt[4]{x} = x^{1/4}$
$y' = \frac{1}{4}x^{-3/4} = \frac{1}{4x^{3/4}}$

11. $f(x) = x + 11$
$f'(x) = 1$

12. $g(x) = 6x + 3$
$g'(x) = 6$

13. $f(t) = -2t^2 + 3t - 6$
$f'(t) = -4t + 3$

14. $y = t^2 - 3t + 1$
$y' = 2t - 3$

15. $g(x) = x^2 + 4x^3$
$g'(x) = 2x + 12x^2$

16. $y = 4x - 3x^3$
$y' = 4 - 9x^2$

17. $s(t) = t^3 + 5t^2 - 3t + 8$
$s'(t) = 3t^2 + 10t - 3$

18. $y = 2x^3 + 6x^2 - 1$
$y' = 6x^2 + 12x$

19. $y = \frac{\pi}{2}\sin\theta - \cos\theta$
$y' = \frac{\pi}{2}\cos\theta + \sin\theta$

20. $g(t) = \pi\cos t$
$g'(t) = -\pi\sin t$

21. $y = x^2 - \frac{1}{2}\cos x$
$y' = 2x + \frac{1}{2}\sin x$

22. $y = 7 + \sin x$
$y' = \cos x$

23. $y = \frac{1}{x} - 3\sin x$
$y' = -\frac{1}{x^2} - 3\cos x$

24. $y = \frac{5}{(2x)^3} + 2\cos x = \frac{5}{8}x^{-3} + 2\cos x$
$y' = \frac{5}{8}(-3)x^{-4} - 2\sin x = -\frac{15}{8x^4} - 2\sin x$

	Function	Rewrite	Differentiate	Simplify
25.	$y = \frac{5}{2x^2}$	$y = \frac{5}{2}x^{-2}$	$y' = -5x^{-3}$	$y' = -\frac{5}{x^3}$
26.	$y = \frac{3}{2x^4}$	$y = \frac{3}{2}x^{-4}$	$y' = -6x^{-5}$	$y' = -\frac{6}{x^5}$
27.	$y = \frac{6}{(5x)^3}$	$y = \frac{6}{125}x^{-3}$	$y' = -\frac{18}{125}x^{-4}$	$y' = -\frac{18}{125x^4}$
28.	$y = \frac{\pi}{(3x)^2}$	$y = \frac{\pi}{9}x^{-2}$	$y' = -\frac{2\pi}{9}x^{-3}$	$y' = -\frac{2\pi}{9x^3}$
29.	$y = \frac{\sqrt{x}}{x}$	$y = x^{-1/2}$	$y' = -\frac{1}{2}x^{-3/2}$	$y' = -\frac{1}{2x^{3/2}}$
30.	$y = \frac{4}{x^{-3}}$	$y = 4x^3$	$y' = 12x^2$	$y' = 12x^2$

31. $f(x) = \frac{8}{x^2} = 8x^{-2}, (2, 2)$
$f'(x) = -16x^{-3} = -\frac{16}{x^3}$
$f'(2) = -2$

32. $f(t) = 2 - \frac{4}{t} = 2 - 4t^{-1}, (4, 1)$
$f'(t) = 4t^{-2} = \frac{4}{t^2}$
$f'(4) = \frac{1}{4}$

33. $f(x) = -\frac{1}{2} + \frac{7}{5}x^3, \left(0, -\frac{1}{2}\right)$

$f'(x) = \frac{21}{5}x^2$

$f'(0) = 0$

34. $y = 2x^4 - 3, (1, -1)$

$y' = 8x^3$

$y'(1) = 8$

35. $y = (4x + 1)^2, (0, 1) = 16x^2 + 8x + 1$

$y' = 32x + 8$

$y'(0) = 32(0) + 8 = 8$

36. $f(x) = 2(x - 4)^2, (2, 8)$

$= 2x^2 - 16x + 32$

$f'(x) = 4x - 16$

$f'(2) = 8 - 16 = -8$

37. $f(\theta) = 4\sin\theta - \theta, (0, 0)$

$f'(\theta) = 4\cos\theta - 1$

$f'(0) = 4(1) - 1 = 3$

38. $g(t) = -2\cos t + 5, (\pi, 7)$

$g'(t) = 2\sin t$

$g'(\pi) = 0$

39. $f(x) = x^2 + 5 - 3x^{-2}$

$f'(x) = 2x + 6x^{-3} = 2x + \frac{6}{x^3}$

40. $f(x) = x^3 - 2x + 3x^{-3}$

$f'(x) = 3x^2 - 2 - 9x^{-4} = 3x^2 - 2 - \frac{9}{x^4}$

41. $g(t) = t^2 - \frac{4}{t^3} = t^2 - 4t^{-3}$

$g'(t) = 2t + 12t^{-4} = 2t + \frac{12}{t^4}$

42. $f(x) = 8x + \frac{3}{x^2} = 8x + 3x^{-2}$

$f'(x) = 8 - 6x^{-3} = 8 - \frac{6}{x^3}$

43. $f(x) = \frac{4x^3 + 3x^2}{x} = 4x^2 + 3x$

$f'(x) = 8x + 3$

44. $f(x) = \frac{2x^4 - x}{x^3} = 2x - x^{-2}$

$f'(x) = 2 + 2x^{-3} = 2 + \frac{2}{x^3}$

45. $f(x) = \frac{x^3 - 3x^2 + 4}{x^2} = x - 3 + 4x^{-2}$

$f'(x) = 1 - \frac{8}{x^3} = \frac{x^3 - 8}{x^3}$

46. $h(x) = \frac{4x^3 + 2x + 5}{x} = 4x^2 + 2 + 5x^{-1}$

$h'(x) = 8x - 5x^{-2} = 8x - \frac{5}{x^2}$

47. $y = x(x^2 + 1) = x^3 + x$

$y' = 3x^2 + 1$

48. $y = x^2(2x^2 - 3x) = 2x^4 - 3x^3$

$y' = 8x^3 - 9x^2 = x^2(8x - 9)$

49. $f(x) = \sqrt{x} - 6\sqrt[3]{x} = x^{1/2} - 6x^{1/3}$

$f'(x) = \frac{1}{2}x^{-1/2} - 2x^{-2/3} = \frac{1}{2\sqrt{x}} - \frac{2}{x^{2/3}}$

50. $f(t) = t^{2/3} - t^{1/3} + 4$

$f'(t) = \frac{2}{3}t^{-1/3} - \frac{1}{3}t^{-2/3} = \frac{2}{3t^{1/3}} - \frac{1}{3t^{2/3}}$

51. $f(x) = 6\sqrt{x} + 5\cos x = 6x^{1/2} + 5\cos x$

$f'(x) = 3x^{-1/2} - 5\sin x = \frac{3}{\sqrt{x}} - 5\sin x$

52. $f(x) = \frac{2}{\sqrt[3]{x}} + 3\cos x = 2x^{-1/3} + 3\cos x$

$f'(x) = -\frac{2}{3}x^{-4/3} - 3\sin x = -\frac{2}{3x^{4/3}} - 3\sin x$

53. (a) $y = x^4 - 3x^2 + 2$

$y' = 4x^3 - 6x$

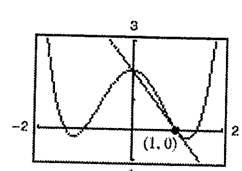
At $(1, 0)$: $y' = 4(1)^3 - 6(1) = -2$

Tangent line: $y - 0 = -2(x - 1)$

$y = -2x + 2$

$2x + y - 2 = 0$

(b)



54. (a) $y = x^3 - 3x$

$y' = 3x^2 - 3$

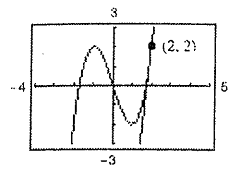
At $(2, 2)$: $y' = 3(2)^2 - 3 = 9$

Tangent line: $y - 2 = 9(x - 2)$

$y = 9x - 16$

$9x - y - 16 = 0$

(b)



55. (a) $f(x) = \dfrac{2}{\sqrt[4]{x^3}} = 2x^{-3/4}$

$f'(x) = -\dfrac{3}{2}x^{-7/4} = -\dfrac{3}{2x^{7/4}}$

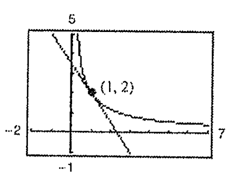
At $(1, 2)$: $f'(1) = -\dfrac{3}{2}$

Tangent line: $y - 2 = -\dfrac{3}{2}(x - 1)$

$y = -\dfrac{3}{2}x + \dfrac{7}{2}$

$3x + 2y - 7 = 0$

(b)



56. (a) $y = (x - 2)(x^2 + 3x) = x^3 + x^2 - 6x$

$y' = 3x^2 + 2x - 6$

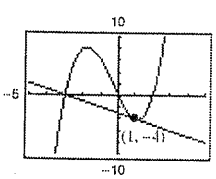
At $(1, -4)$: $y' = 3(1)^2 + 2(1) - 6 = -1$

Tangent line: $y - (-4) = -1(x - 1)$

$y = -x - 3$

$x + y + 3 = 0$

(b)



57. $y = x^4 - 2x^2 + 3$

$y' = 4x^3 - 4x$

$= 4x(x^2 - 1)$

$= 4x(x - 1)(x + 1)$

$y' = 0 \Rightarrow x = 0, \pm 1$

Horizontal tangents: $(0, 3), (1, 2), (-1, 2)$

58. $y = x^3 + x$

$y' = 3x^2 + 1 > 0$ for all x.

Therefore, there are no horizontal tangents.

59. $y = \dfrac{1}{x^2} = x^{-2}$

$y' = -2x^{-3} = -\dfrac{2}{x^3}$ cannot equal zero.

Therefore, there are no horizontal tangents.

60. $y = x^2 + 9$

$y' = 2x = 0 \Rightarrow x = 0$

At $x = 0, y = 1$.

Horizontal tangent: $(0, 9)$

61. $y = x + \sin x, 0 \le x < 2\pi$

$y' = 1 + \cos x = 0$

$\cos x = -1 \Rightarrow x = \pi$

At $x = \pi$: $y = \pi$

Horizontal tangent: (π, π)

62. $y = \sqrt{3}x + 2\cos x, 0 \le x < 2\pi$

$y' = \sqrt{3} - 2\sin x = 0$

$\sin x = \dfrac{\sqrt{3}}{2} \Rightarrow x = \dfrac{\pi}{3}$ or $\dfrac{2\pi}{3}$

At $x = \dfrac{\pi}{3}$: $y = \dfrac{\sqrt{3}\pi + 3}{3}$

At $x = \dfrac{2\pi}{3}$: $y = \dfrac{2\sqrt{3}\pi - 3}{3}$

Horizontal tangents: $\left(\dfrac{\pi}{3}, \dfrac{\sqrt{3}\pi + 3}{3}\right), \left(\dfrac{2\pi}{3}, \dfrac{2\sqrt{3}\pi - 3}{3}\right)$

63. $k - x^2 = -6x + 1$ Equate functions.

$-2x = -6$ Equate derivatives.

So, $x = 3$ and $k - 9 = -18 + 1 \Rightarrow k = -8$.

64. $kx^2 = -2x + 3$ Equate functions.

$2kx = -2$ Equate derivatives.

So, $k = -\frac{2}{2x} = -\frac{1}{x}$, and $\left(-\frac{1}{x}\right)x^2 = -2x + 3 \Rightarrow -x = -2x + 3 \Rightarrow x = 3 \Rightarrow k = -\frac{1}{3}$.

65. $\frac{k}{x} = -\frac{3}{4}x + 3$ Equate functions.

$-\frac{k}{x^2} = -\frac{3}{4}$ Equate derivatives.

So, $k = \frac{3}{4}x^2$ and

$$\frac{\frac{3}{4}x^2}{x} = -\frac{3}{4}x + 3 \Rightarrow \frac{3}{4}x = -\frac{3}{4}x + 3$$

$$\Rightarrow \frac{3}{2}x = 3 \Rightarrow x = 2 \Rightarrow k = 3.$$

66. $k\sqrt{x} = x + 4$ Equate functions.

$\frac{k}{2\sqrt{x}} = 1$ Equate derivatives.

So, $k = 2\sqrt{x}$ and

$$\left(2\sqrt{x}\right)\sqrt{x} = x + 4 \Rightarrow 2x = x + 4 \Rightarrow x = 4 \Rightarrow k = 4.$$

67. $kx^3 = x + 1$ Equate equations.

$3kx^2 = 1$ Equate derivatives.

So, $k = \frac{1}{3x^2}$ and

$$\left(\frac{1}{3x^2}\right)x^3 = x + 1$$

$$\frac{1}{3}x = x + 1$$

$$x = -\frac{3}{2}, k = \frac{4}{27}.$$

68. $kx^4 = 4x - 1$ Equate equations.

$4kx^3 = 4$ Equate derivatives.

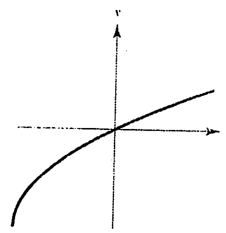
So, $k = \frac{1}{x^3}$ and

$$\left(\frac{1}{x^3}\right)x^4 = 4x - 1$$

$$x = 4x - 1$$

$$x = \frac{1}{3} \text{ and } k = 27.$$

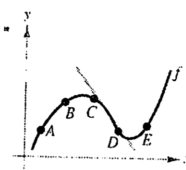
69. The graph of a function f such that $f' > 0$ for all x and the rate of change of the function is decreasing (i.e., $f'' < 0$) would, in general, look like the graph below.



70. (a) The slope appears to be steepest between A and B.

(b) The average rate of change between A and B is **greater** than the instantaneous rate of change at B.

(c)



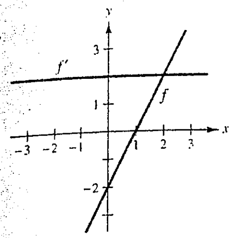
71. $g(x) = f(x) + 6 \Rightarrow g'(x) = f'(x)$

72. $g(x) = 2f(x) \Rightarrow g'(x) = 2f'(x)$

73. $g(x) = -5f(x) \Rightarrow g'(x) = -5f'(x)$

74. $g(x) = 3f(x) - 1 \Rightarrow g'(x) = 3f'(x)$

75.

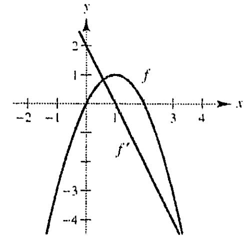


If f is linear then its derivative is a constant function.

$f(x) = ax + b$

$f'(x) = a$

76.



If f is quadratic, then its derivative is a linear function.

$f(x) = ax^2 + bx + c$

$f'(x) = 2ax + b$

77. Let (x_1, y_1) and (x_2, y_2) be the points of tangency on $y = x^2$ and $y = -x^2 + 6x - 5$, respectively.

The derivatives of these functions are:

$y' = 2x \Rightarrow m = 2x_1$ and $y' = -2x + 6 \Rightarrow m = -2x_2 + 6$

$m = 2x_1 = -2x_2 + 6$

$x_1 = -x_2 + 3$

Because $y_1 = x_1^2$ and $y_2 = -x_2^2 + 6x_2 - 5$:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\left(-x_2^2 + 6x_2 - 5\right) - \left(x_1^2\right)}{x_2 - x_1} = -2x_2 + 6$$

$$\frac{\left(-x_2^2 + 6x_2 - 5\right) - \left(-x_2 + 3\right)^2}{x_2 - \left(-x_2 + 3\right)} = -2x_2 + 6$$

$$\left(-x_2^2 + 6x_2 - 5\right) - \left(x_2^2 - 6x_2 + 9\right) = \left(-2x_2 + 6\right)\left(2x_2 - 3\right)$$

$$-2x_2^2 + 12x_2 - 14 = -4x_2^2 + 18x_2 - 18$$

$$2x_2^2 - 6x_2 + 4 = 0$$

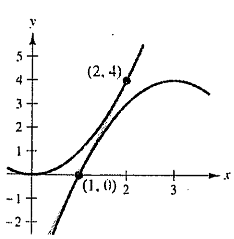
$$2(x_2 - 2)(x_2 - 1) = 0$$

$$x_2 = 1 \text{ or } 2$$

$x_2 = 1 \Rightarrow y_2 = 0, x_1 = 2$ and $y_1 = 4$

So, the tangent line through $(1, 0)$ and $(2, 4)$ is

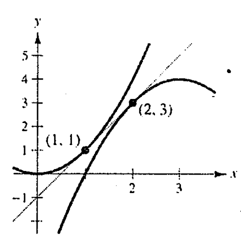
$$y - 0 = \left(\frac{4 - 0}{2 - 1}\right)(x - 1) \Rightarrow y = 4x - 4.$$



$x_2 = 2 \Rightarrow y_2 = 3, x_1 = 1$ and $y_1 = 1$

So, the tangent line through $(2, 3)$ and $(1, 1)$ is

$$y - 1 = \left(\frac{3 - 1}{2 - 1}\right)(x - 1) \Rightarrow y = 2x - 1.$$



78. m_1 is the slope of the line tangent to $y = x$. m_2 is the slope of the line tangent to $y = 1/x$. Because

$$y = x \Rightarrow y' = 1 \Rightarrow m_1 = 1 \text{ and } y = \frac{1}{x} \Rightarrow y' = -\frac{1}{x^2} \Rightarrow m_2 = -\frac{1}{x^2}.$$

The points of intersection of $y = x$ and $y = 1/x$ are

$$x = \frac{1}{x} \Rightarrow x^2 = 1 \Rightarrow x = \pm 1.$$

At $x = \pm 1$, $m_2 = -1$. Because $m_2 = -1/m_1$, these tangent lines are perpendicular at the points of intersection.

79. $f(x) = 3x + \sin x + 2$

$f'(x) = 3 + \cos x$

Because $|\cos x| \le 1$, $f'(x) \ne 0$ for all x and f does not have a horizontal tangent line.

80. $f(x) = x^5 + 3x^3 + 5x$

$f'(x) = 5x^4 + 9x^2 + 5$

Because $5x^4 + 9x^2 \ge 0$, $f'(x) \ge 5$. So, f does not have a tangent line with a slope of 3.

81. $f(x) = \sqrt{x}, (-4, 0)$

$$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$\frac{1}{2\sqrt{x}} = \frac{0 - y}{-4 - x}$$

$$4 + x = 2\sqrt{x}y$$

$$4 + x = 2\sqrt{x}\sqrt{x}$$

$$4 + x = 2x$$

$$x = 4, y = 2$$

The point $(4, 2)$ is on the graph of f.

Tangent line: $y - 2 = \frac{0 - 2}{-4 - 4}(x - 4)$

$$4y - 8 = x - 4$$

$$0 = x - 4y + 4$$

82. $f(x) = \frac{2}{x}, (5, 0)$

$$f'(x) = -\frac{2}{x^2}$$

$$-\frac{2}{x^2} = \frac{0 - y}{5 - x}$$

$$-10 + 2x = -x^2y$$

$$-10 + 2x = -x^2\left(\frac{2}{x}\right)$$

$$-10 + 2x = -2x$$

$$4x = 10$$

$$x = \frac{5}{2}, y = \frac{4}{5}$$

The point $\left(\frac{5}{2}, \frac{4}{5}\right)$ is on the graph of f. The slope of the tangent line is $f'\left(\frac{5}{2}\right) = -\frac{8}{25}$.

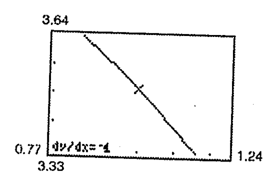
Tangent line: $y - \frac{4}{5} = -\frac{8}{25}\left(x - \frac{5}{2}\right)$

$$25y - 20 = -8x + 20$$

$$8x + 25y - 40 = 0$$

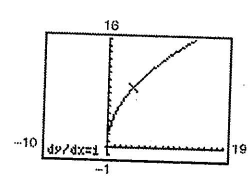
83. $f'(1)$ appears to be close to -1.

$f'(1) = -1$



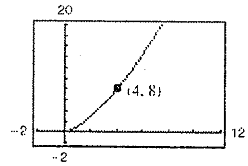
84. $f'(4)$ appears to be close to 1.

$f'(4) = 1$



85. (a) One possible secant is between $(3.9, 7.7019)$ and $(4, 8)$:

$$y - 8 = \frac{8 - 7.7019}{4 - 3.9}(x - 4)$$
$$y - 8 = 2.981(x - 4)$$
$$y = S(x) = 2.981x - 3.924$$

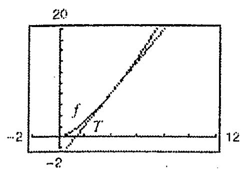


(b) $f'(x) = \frac{3}{2}x^{1/2} \Rightarrow f'(4) = \frac{3}{2}(2) = 3$

$T(x) = 3(x - 4) + 8 = 3x - 4$

The slope (and equation) of the secant line approaches that of the tangent line at $(4, 8)$ as you choose points closer and closer to $(4, 8)$.

(c) As you move further away from $(4, 8)$, the accuracy of the approximation T gets worse.



(d)

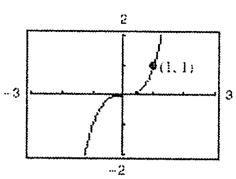
Δx	−3	−2	−1	−0.5	−0.1	0	0.1	0.5	1	2	3
$f(4 + \Delta x)$	1	2.828	5.196	6.548	7.702	8	8.302	9.546	11.180	14.697	18.520
$T(4 + \Delta x)$	−1	2	5	6.5	7.7	8	8.3	9.5	11	14	17

86. (a) Nearby point: $(1.0073138, 1.0221024)$

Secant line: $y - 1 = \frac{1.0221024 - 1}{1.0073138 - 1}(x - 1)$

$y = 3.022(x - 1) + 1$

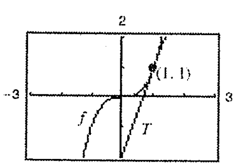
(Answers will vary.)



(b) $f'(x) = 3x^2$

$T(x) = 3(x - 1) + 1 = 3x - 2$

(c) The accuracy worsens as you move away from $(1, 1)$.



(d)

Δx	−3	−2	−1	−0.5	−0.1	0	0.1	0.5	1	2	3
$f(x)$	−8	−1	0	0.125	0.729	1	1.331	3.375	8	27	64
$T(x)$	−8	−5	−2	−0.5	0.7	1	1.3	2.5	4	7	10

The accuracy decreases more rapidly than in Exercise 85 because $y = x^3$ is less "linear" than $y = x^{3/2}$.

87. False. Let $f(x) = x$ and $g(x) = x + 1$. Then $f'(x) = g'(x) = x$, but $f(x) \neq g(x)$.

88. True. If $f(x) = g(x) + c$, then $f'(x) = g'(x) + 0 = g'(x)$.

89. False. If $y = \pi^2$, then $dy/dx = 0$. (π^2 is a constant.)

90. True. If $y = x/\pi = (1/\pi) \cdot x$, then
$dy/dx = (1/\pi)(1) = 1/\pi$.

91. True. If $g(x) = 3f(x)$, then $g'(x) = 3f'(x)$.

92. False. If $f(x) = \dfrac{1}{x_n} = x^{-n}$, then

$$f'(x) = -nx^{-n-1} = \frac{-n}{x^{n+1}}.$$

93. $f(t) = 4t + 5, \quad [1, 2]$

$f'(t) = 4$. So, $f'(1) = f'(2) = 4$.

Instantaneous rate of change is the constant 4. Average rate of change:

$$\frac{f(2) - f(1)}{2 - 1} = \frac{13 - 9}{1} = 4$$

(These are the same because f is a line of slope 4.)

94. $f(t) = t^2 - 7, \quad [3, 3.1]$

$f'(t) = 2t$

Instantaneous rate of change:

At $(3, 2)$: $f'(3) = 6$

At $(3.1, 2.61)$: $f'(3.1) = 6.2$

Average rate of change:

$$\frac{f(3.1) - f(3)}{3.1 - 3} = \frac{2.61 - 2}{0.1} = 6.1$$

95. $f(x) = -\dfrac{1}{x}, \quad [1, 2]$

$$f'(x) = \frac{1}{x^2}$$

Instantaneous rate of change:

$(1, -1) \Rightarrow f'(1) = 1$

$\left(2, -\dfrac{1}{2}\right) \Rightarrow f'(2) = \dfrac{1}{4}$

Average rate of change:

$$\frac{f(2) - f(1)}{2 - 1} = \frac{(-1/2) - (-1)}{2 - 1} = \frac{1}{2}$$

96. $f(x) = \sin x, \quad \left[0, \dfrac{\pi}{6}\right]$

$f'(x) = \cos x$

Instantaneous rate of change:

$(0, 0) \Rightarrow f'(0) = 1$

$\left(\dfrac{\pi}{6}, \dfrac{1}{2}\right) \Rightarrow f'\left(\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2} \approx 0.866$

Average rate of change:

$$\frac{f(\pi/6) - f(0)}{(\pi/6) - 0} = \frac{(1/2) - 0}{(\pi/6) - 0} = \frac{3}{\pi} \approx 0.955$$

97. (a) $s(t) = -16t^2 + 1362$

$v(t) = -32t$

(b) $\dfrac{s(2) - s(1)}{2 - 1} = 1298 - 1346 = -48$ ft/sec

(c) $v(t) = s'(t) = -32t$

When $t = 1$: $v(1) = -32$ ft/sec

When $t = 2$: $v(2) = -64$ ft/sec

(d) $-16t^2 + 1362 = 0$

$$t^2 = \frac{1362}{16} \Rightarrow t = \frac{\sqrt{1362}}{4} \approx 9.226 \text{ sec}$$

(e) $v\left(\dfrac{\sqrt{1362}}{4}\right) = -32\left(\dfrac{\sqrt{1362}}{4}\right)$

$= -8\sqrt{1362} \approx -295.242$ ft/sec

98.
$$\begin{aligned}
s(t) &= -16t^2 - 22t + 220 \\
v(t) &= -32t - 22 \\
v(3) &= -118 \text{ ft/sec} \\
s(t) &= -16t^2 - 22t + 220 \\
&= 112 \text{ (height after falling 108 ft)} \\
-16t^2 - 22t + 108 &= 0 \\
-2(t - 2)(8t + 27) &= 0 \\
t &= 2 \\
v(2) &= -32(2) - 22 \\
&= -86 \text{ ft/sec}
\end{aligned}$$

99.
$$\begin{aligned}
s(t) &= -4.9t^2 + v_0 t + s_0 \\
&= -4.9t^2 + 120t \\
v(t) &= -9.8t + 120 \\
v(5) &= -9.8(5) + 120 = 71 \text{ m/sec} \\
v(10) &= -9.8(10) + 120 = 22 \text{ m/sec}
\end{aligned}$$

100. $s(t) = -4.9t^2 + v_0t + s_0$

$= -4.9t^2 + s_0 = 0$ when $t = 5.6$.

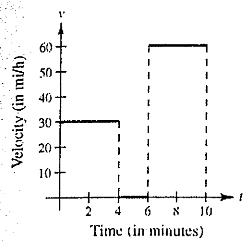
$s_0 = 4.9t^2 = 4.9(5.6)^2 \approx 153.7$ m

101. From $(0, 0)$ to $(4, 2)$, $s(t) = \frac{1}{2}t \Rightarrow v(t) = \frac{1}{2}$ mi/min.

$v(t) = \frac{1}{2}(60) = 30$ mi/h for $0 < t < 4$

Similarly, $v(t) = 0$ for $4 < t < 6$. Finally, from $(6, 2)$ to $(10, 6)$,

$s(t) = t - 4 \Rightarrow v(t) = 1$ mi/min. $= 60$ mi/h.



(The velocity has been converted to miles per hour.)

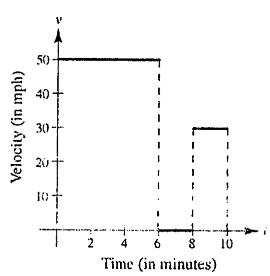
102. From $(0, 0)$ to $(6, 5)$, $s(t) = \frac{5}{6}t \Rightarrow v(t) = \frac{5}{6}$ mi/min.

$v(t) = \frac{5}{6}(60) = 50$ mi/h for $0 < t < 6$

Similarly, $v(t) = 0$ for $6 < t < 8$.

Finally, from $(8, 5)$ to $(10, 6)$,

$s(t) = \frac{1}{2}t + 1 \Rightarrow v(t) = \frac{1}{2}$ mi/min $= 30$ mi/h.



(The velocity has been converted to miles per hour.)

103. $v = 40$ mi/h $= \frac{2}{3}$ mi/min

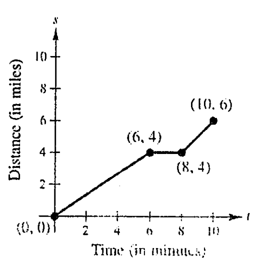
$\left(\frac{2}{3}\text{ mi/min}\right)(6\text{ min}) = 4$ mi

$v = 0$ mi/h $= 0$ mi/min

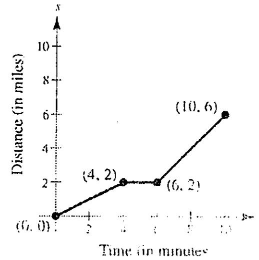
$(0\text{ mi/min})(2\text{ min}) = 0$ mi

$v = 60$ mi/h $= 1$ mi/min

$(1\text{ mi/min})(2\text{ min}) = 2$ mi



104. This graph corresponds with Exercise 101.



105. $V = s^3, \dfrac{dV}{ds} = 3s^2$

When $s = 6$ cm, $\dfrac{dV}{ds} = 108\text{ cm}^3$ per cm change in s.

106. $A = s^2, \dfrac{dA}{ds} = 2s$

When $s = 6$ m, $\dfrac{dA}{ds} = 12\text{ m}^2$ per m change in s.

107. (a) Using a graphing utility,

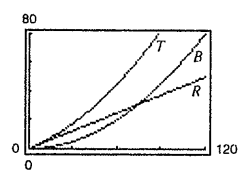
$R(v) = 0.417v - 0.02.$

(b) Using a graphing utility,

$B(v) = 0.0056v^2 + 0.001v + 0.04.$

(c) $T(v) = R(v) + B(v) = 0.0056v^2 + 0.418v + 0.02$

(d)



(e) $\dfrac{dT}{dv} = 0.0112v + 0.418$

For $v = 40$, $T'(40) \approx 0.866$

For $v = 80$, $T'(80) \approx 1.314$

For $v = 100$, $T'(100) \approx 1.538$

(f) For increasing speeds, the total stopping distance increases.

108. $C = (\text{gallons of fuel used})(\text{cost per gallon})$

$$= \left(\frac{15{,}000}{x}\right)(3.48) = \frac{52{,}200}{x}$$

$$\frac{dC}{dx} = -\frac{52{,}200}{x^2}$$

x	10	15	20	25	30	35	40
C	5220	3480	2610	2088	1740	1491.4	1305
dC/dx	–522	–232	–130.5	–83.52	–58	–42.61	–32.63

The driver who gets 15 miles per gallon would benefit more. The rate of change at $x = 15$ is larger in absolute value than that at $x = 35$.

109. $s(t) = -\frac{1}{2}at^2 + c$ and $s'(t) = -at$

Average velocity: $$\frac{s(t_0 + \Delta t) - s(t_0 - \Delta t)}{(t_0 + \Delta t) - (t_0 - \Delta t)} = \frac{\left[-(1/2)a(t_0 + \Delta t)^2 + c\right] - \left[-(1/2)a(t_0 - \Delta t)^2 + c\right]}{2\Delta t}$$

$$= \frac{-(1/2)a\left(t_0^2 + 2t_0\Delta t + (\Delta t)^2\right) + (1/2)a\left(t_0^2 - 2t_0\Delta t + (\Delta t)^2\right)}{2\,\Delta t}$$

$$= \frac{-2at_0\,\Delta t}{2\,\Delta t} = -at_0 = s'(t_0) \quad \text{instantaneous velocity at } t = t_0$$

110. $$C = \frac{1{,}008{,}000}{Q} + 6.3Q$$

$$\frac{dC}{dQ} = -\frac{1{,}008{,}000}{Q} + 6.3$$

$C(351) - C(350) \approx 5083.095 - 5085 \approx -\$1.91$

When $Q = 350$, $\frac{dC}{dQ} \approx -\$1.93$.

111. $y = ax^2 + bx + c$

Because the parabola passes through $(0, 1)$ and $(1, 0)$, you have:

$(0, 1)$: $1 = a(0)^2 + b(0) + c \Rightarrow c = 1$

$(1, 0)$: $0 = a(1)^2 + b(1) + 1 \Rightarrow b = -a - 1$

So, $y = ax^2 + (-a - 1)x + 1$.

From the tangent line $y = x - 1$, you know that the derivative is 1 at the point $(1, 0)$.

$y' = 2ax + (-a - 1)$

$1 = 2a(1) + (-a - 1)$

$1 = a - 1$

$a = 2$

$b = -a - 1 = -3$

Therefore, $y = 2x^2 - 3x + 1$.

112. $y = \frac{1}{x}, x > 0$

$$y' = -\frac{1}{x^2}$$

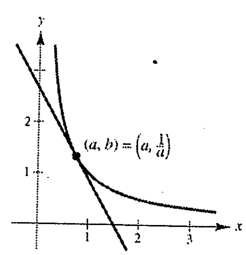
At (a, b), the equation of the tangent line is

$$y - \frac{1}{a} = -\frac{1}{a^2}(x - a) \quad \text{or} \quad y = -\frac{x}{a^2} + \frac{2}{a}.$$

The x-intercept is $(2a, 0)$.

The y-intercept is $\left(0, \frac{2}{a}\right)$.

The area of the triangle is $A = \frac{1}{2}bh = \frac{1}{2}(2a)\left(\frac{2}{a}\right) = 2$.



113. $y = x^3 - 9x$

$y' = 3x^2 - 9$

Tangent lines through $(1, -9)$:

$$y + 9 = (3x^2 - 9)(x - 1)$$
$$(x^3 - 9x) + 9 = 3x^3 - 3x^2 - 9x + 9$$
$$0 = 2x^3 - 3x^2 = x^2(2x - 3)$$
$$x = 0 \text{ or } x = \tfrac{3}{2}$$

The points of tangency are $(0, 0)$ and $\left(\tfrac{3}{2}, -\tfrac{81}{8}\right)$.

At $(0, 0)$, the slope is $y'(0) = -9$. At $\left(\tfrac{3}{2}, -\tfrac{81}{8}\right)$, the slope is $y'\left(\tfrac{3}{2}\right) = -\tfrac{9}{4}$.

Tangent Lines:

$$y - 0 = -9(x - 0) \quad \text{and} \quad y + \tfrac{81}{8} = -\tfrac{9}{4}\left(x - \tfrac{3}{2}\right)$$
$$y = -9x \qquad\qquad y = -\tfrac{9}{4}x - \tfrac{27}{4}$$
$$9x + y = 0 \qquad\qquad 9x + 4y + 27 = 0$$

114. $y = x^2$

$y' = 2x$

(a) Tangent lines through $(0, a)$:

$$y - a = 2x(x - 0)$$
$$x^2 - a = 2x^2$$
$$-a = x^2$$
$$\pm\sqrt{-a} = x$$

The points of tangency are $\left(\pm\sqrt{-a}, -a\right)$. At $\left(\sqrt{-a}, -a\right)$, the slope is $y'\left(\sqrt{-a}\right) = 2\sqrt{-a}$.

At $\left(-\sqrt{-a}, -a\right)$, the slope is $y'\left(-\sqrt{-a}\right) = -2\sqrt{-a}$.

Tangent lines: $y + a = 2\sqrt{-a}\left(x - \sqrt{-a}\right)$ and $y + a = -2\sqrt{-a}\left(x + \sqrt{-a}\right)$

$$y = 2\sqrt{-a}x + a \qquad\qquad y = -2\sqrt{-a}x + a$$

Restriction: a must be negative.

(b) Tangent lines through $(a, 0)$:

$$y - 0 = 2x(x - a)$$
$$x^2 = 2x^2 - 2ax$$
$$0 = x^2 - 2ax = x(x - 2a)$$

The points of tangency are $(0, 0)$ and $\left(2a, 4a^2\right)$. At $(0, 0)$, the slope is $y'(0) = 0$. At $\left(2a, 4a^2\right)$, the slope is $y'(2a) = 4a$,

Tangent lines: $y - 0 = 0(x - 0)$ and $y - 4a^2 = 4a(x - 2a)$

$$y = 0 \qquad\qquad y = 4ax - 4a^2$$

Restriction: None, a can be any real number.

115. $f(x) = \begin{cases} ax^3, & x \le 2 \\ x^2 + b, & x > 2 \end{cases}$

f must be continuous at $x = 2$ to be differentiable at $x = 2$.

$$\left.\begin{aligned} \lim_{x\to 2^-} f(x) &= \lim_{x\to 2^-} ax^3 = 8a \\ \lim_{x\to 2^+} f(x) &= \lim_{x\to 2^+} \left(x^2 + b\right) = 4 + b \end{aligned}\right\} \begin{aligned} 8a &= 4 + b \\ 8a - 4 &= b \end{aligned}$$

$$f'(x) = \begin{cases} 3ax^2, & x < 2 \\ 2x, & x > 2 \end{cases}$$

For f to be differentiable at $x = 2$, the left derivative must equal the right derivative.

$$\begin{aligned} 3a(2)^2 &= 2(2) \\ 12a &= 4 \\ a &= \tfrac{1}{3} \\ b &= 8a - 4 = -\tfrac{4}{3} \end{aligned}$$

116. $f(x) = \begin{cases} \cos x, & x < 0 \\ ax + b, & x \ge 0 \end{cases}$

$f(0) = b = \cos(0) = 1 \Rightarrow b = 1$

$$f'(x) = \begin{cases} -\sin x, & x < 0 \\ a, & x > 0 \end{cases}$$

So, $a = 0$.

Answer: $a = 0$; $b = 1$

117. $f_1(x) = |\sin x|$ is differentiable for all $x \ne n\pi$, n an integer.

$f_2(x) = \sin|x|$ is differentiable for all $x \ne 0$.

You can verify this by graphing f_1 and f_2 and observing the locations of the sharp turns.

118. Let $f(x) = \cos x$.

$$\begin{aligned} f'(x) &= \lim_{\Delta x\to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x\to 0} \frac{\cos x \cos \Delta x - \sin x \sin \Delta x - \cos x}{\Delta x} \\ &= \lim_{\Delta x\to 0} \frac{\cos x(\cos \Delta x - 1)}{\Delta x} - \lim_{\Delta x\to 0} \sin x\left(\frac{\sin \Delta x}{\Delta x}\right) \\ &= 0 - \sin x(1) = -\sin x \end{aligned}$$

119. You are given $f : R \to R$ satisfying

$(*) f'(x) = \dfrac{f(x + n) - f(x)}{n}$ for all real numbers x and all positive integers n. You claim that $f(x) = mx + b$, $m, b \in R$.

For this case,

$$f'(x) = m = \frac{[m(x + n) + b] - [mx + b]}{n} = m.$$

Furthermore, these are the only solutions:

Note first that $f'(x + 1) = \dfrac{f(x + 2) - f(x + 1)}{1}$, and $f'(x) = f(x + 1) - f(x)$. From $(*)$ you have

$$\begin{aligned} 2f'(x) &= f(x + 2) - f(x) \\ &= [f(x + 2) - f(x + 1)] + [f(x + 1) - f(x)] \\ &= f'(x + 1) + f'(x). \end{aligned}$$

Thus, $f'(x) = f'(x + 1)$.

Let $g(x) = f(x + 1) - f(x)$.

Let $m = g(0) = f(1) - f(0)$.

Let $b = f(0)$. Then

$$\begin{aligned} g'(x) &= f'(x + 1) - f'(x) = 0 \\ g(x) &= \text{constant} = g(0) = m \\ f'(x) &= f(x + 1) - f(x) = g(x) = m \\ &\Rightarrow f(x) = mx + b. \end{aligned}$$