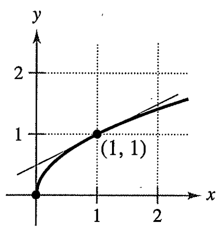
2.2 Exercises

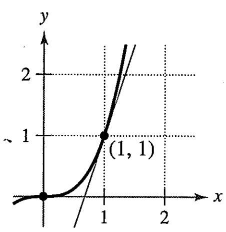
See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Estimating Slope **In Exercises 1 and 2, use the graph to estimate the slope of the tangent line to $y = x^n$ at the point (1, 1). Verify your answer analytically. To print an enlarged copy of the graph, go to *MathGraphs.com*.**

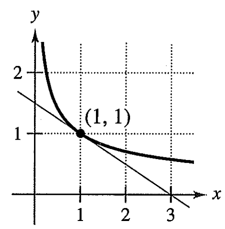
1. (a) $y = x^{1/2}$



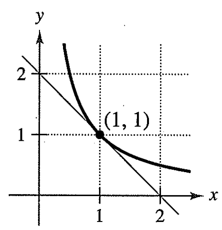
(b) $y = x^3$



2. (a) $y = x^{-1/2}$



(b) $y = x^{-1}$



Finding a Derivative **In Exercises 3–24, use the rules of differentiation to find the derivative of the function.**

3. $y = 12$
4. $f(x) = -9$
5. $y = x^7$
6. $y = x^{12}$
7. $y = \dfrac{1}{x^5}$
8. $y = \dfrac{3}{x^7}$
9. $f(x) = \sqrt[5]{x}$
10. $g(x) = \sqrt[4]{x}$
11. $f(x) = x + 11$
12. $g(x) = 6x + 3$
13. $f(t) = -2t^2 + 3t - 6$
14. $y = t^2 - 3t + 1$
15. $g(x) = x^2 + 4x^3$
16. $y = 4x - 3x^3$
17. $s(t) = t^3 + 5t^2 - 3t + 8$
18. $y = 2x^3 + 6x^2 - 1$
19. $y = \dfrac{\pi}{2}\sin\theta - \cos\theta$
20. $g(t) = \pi\cos t$
21. $y = x^2 - \frac{1}{2}\cos x$
22. $y = 7 + \sin x$
23. $y = \dfrac{1}{x} - 3\sin x$
24. $y = \dfrac{5}{(2x)^3} + 2\cos x$

Rewriting a Function Before Differentiating **In Exercises 25–30, complete the table to find the derivative of the function.**

Original Function	Rewrite	Differentiate	Simplify
25. $y = \dfrac{5}{2x^2}$			
26. $y = \dfrac{3}{2x^4}$			
27. $y = \dfrac{6}{(5x)^3}$			
28. $y = \dfrac{\pi}{(3x)^2}$			
29. $y = \dfrac{\sqrt{x}}{x}$			
30. $y = \dfrac{4}{x^{-3}}$			

Finding the Slope of a Graph **In Exercises 31–38, find the slope of the graph of the function at the given point. Use the *derivative* feature of a graphing utility to confirm your results.**

Function	Point
31. $f(x) = \dfrac{8}{x^2}$	$(2, 2)$
32. $f(t) = 2 - \dfrac{4}{t}$	$(4, 1)$
33. $f(x) = -\frac{1}{2} + \frac{7}{5}x^3$	$\left(0, -\frac{1}{2}\right)$
34. $y = 2x^4 - 3$	$(1, -1)$
35. $y = (4x + 1)^2$	$(0, 1)$
36. $f(x) = 2(x - 4)^2$	$(2, 8)$
37. $f(\theta) = 4\sin\theta - \theta$	$(0, 0)$
38. $g(t) = -2\cos t + 5$	$(\pi, 7)$

Finding a Derivative **In Exercises 39–52, find the derivative of the function.**

39. $f(x) = x^2 + 5 - 3x^{-2}$
40. $f(x) = x^3 - 2x + 3x^{-3}$
41. $g(t) = t^2 - \dfrac{4}{t^3}$
42. $f(x) = 8x + \dfrac{3}{x^2}$
43. $f(x) = \dfrac{4x^3 + 3x^2}{x}$
44. $f(x) = \dfrac{2x^4 - x}{x^3}$
45. $f(x) = \dfrac{x^3 - 3x^2 + 4}{x^2}$
46. $h(x) = \dfrac{4x^3 + 2x + 5}{x}$
47. $y = x(x^2 + 1)$
48. $y = x^2(2x^2 - 3x)$
49. $f(x) = \sqrt{x} - 6\sqrt[3]{x}$
50. $f(t) = t^{2/3} - t^{1/3} + 4$
51. $f(x) = 6\sqrt{x} + 5\cos x$
52. $f(x) = \dfrac{2}{\sqrt[3]{x}} + 3\cos x$

Finding an Equation of a Tangent Line **In Exercises 53–56, (a) find an equation of the tangent line to the graph of f at the given point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the *derivative* feature of a graphing utility to confirm your results.**

Function	Point
53. $y = x^4 - 3x^2 + 2$	$(1, 0)$
54. $y = x^3 - 3x$	$(2, 2)$
55. $f(x) = \dfrac{2}{\sqrt[4]{x^3}}$	$(1, 2)$
56. $y = (x - 2)(x^2 + 3x)$	$(1, -4)$

Horizontal Tangent Line **In Exercises 57–62, determine the point(s) (if any) at which the graph of the function has a horizontal tangent line.**

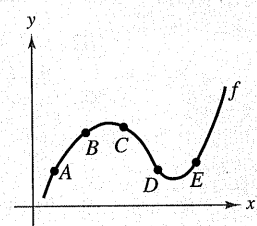
57. $y = x^4 - 2x^2 + 3$
58. $y = x^3 + x$
59. $y = \dfrac{1}{x^2}$
60. $y = x^2 + 9$
61. $y = x + \sin x, \quad 0 \le x < 2\pi$
62. $y = \sqrt{3}x + 2\cos x, \quad 0 \le x < 2\pi$

Finding a Value **In Exercises 63–68, find k such that the line is tangent to the graph of the function.**

Function	Line
63. $f(x) = k - x^2$	$y = -6x + 1$
64. $f(x) = kx^2$	$y = -2x + 3$
65. $f(x) = \dfrac{k}{x}$	$y = -\dfrac{3}{4}x + 3$
66. $f(x) = k\sqrt{x}$	$y = x + 4$
67. $f(x) = kx^3$	$y = x + 1$
68. $f(x) = kx^4$	$y = 4x - 1$

69. **Sketching a Graph** Sketch the graph of a function f such that $f' > 0$ for all x and the rate of change of the function is decreasing.

70. **HOW DO YOU SEE IT?** Use the graph of f to answer each question. To print an enlarged copy of the graph, go to *MathGraphs.com*.



(a) Between which two consecutive points is the average rate of change of the function greatest?

(b) Is the average rate of change of the function between A and B greater than or less than the instantaneous rate of change at B?

(c) Sketch a tangent line to the graph between C and D such that the slope of the tangent line is the same as the average rate of change of the function between C and D.

WRITING ABOUT CONCEPTS

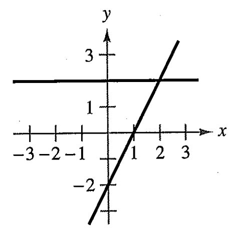
Exploring a Relationship **In Exercises 71–74, the relationship between f and g is given. Explain the relationship between f' and g'.**

71. $g(x) = f(x) + 6$
72. $g(x) = 2f(x)$
73. $g(x) = -5f(x)$
74. $g(x) = 3f(x) - 1$

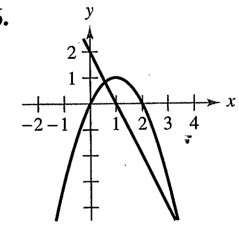
WRITING ABOUT CONCEPTS (continued)

A Function and Its Derivative **In Exercises 75 and 76, the graphs of a function f and its derivative f' are shown in the same set of coordinate axes. Label the graphs as f or f' and write a short paragraph stating the criteria you used in making your selection. To print an enlarged copy of the graph, go to *MathGraphs.com*.**

75.



76.



77. **Finding Equations of Tangent Lines** Sketch the graphs of $y = x^2$ and $y = -x^2 + 6x - 5$, and sketch the two lines that are tangent to both graphs. Find equations of these lines.

78. **Tangent Lines** Show that the graphs of the two equations

$$y = x \quad \text{and} \quad y = \frac{1}{x}$$

have tangent lines that are perpendicular to each other at their point of intersection.

79. **Tangent Line** Show that the graph of the function

$$f(x) = 3x + \sin x + 2$$

does not have a horizontal tangent line.

80. **Tangent Line** Show that the graph of the function

$$f(x) = x^5 + 3x^3 + 5x$$

does not have a tangent line with a slope of 3.

Finding an Equation of a Tangent Line **In Exercises 81 and 82, find an equation of the tangent line to the graph of the function f through the point (x_0, y_0) not on the graph. To find the point of tangency (x, y) on the graph of f, solve the equation**

$$f'(x) = \frac{y_0 - y}{x_0 - x}.$$

81. $f(x) = \sqrt{x}$
$(x_0, y_0) = (-4, 0)$

82. $f(x) = \dfrac{2}{x}$
$(x_0, y_0) = (5, 0)$

83. **Linear Approximation** Use a graphing utility with a square window setting to zoom in on the graph of

$$f(x) = 4 - \tfrac{1}{2}x^2$$

to approximate $f'(1)$. Use the derivative to find $f'(1)$.

84. **Linear Approximation** Use a graphing utility with a square window setting to zoom in on the graph of

$$f(x) = 4\sqrt{x} + 1$$

to approximate $f'(4)$. Use the derivative to find $f'(4)$.

85. Linear Approximation Consider the function $f(x) = x^{3/2}$ with the solution point $(4, 8)$.

(a) Use a graphing utility to graph f. Use the *zoom* feature to obtain successive magnifications of the graph in the neighborhood of the point $(4, 8)$. After zooming in a few times, the graph should appear nearly linear. Use the *trace* feature to determine the coordinates of a point near $(4, 8)$. Find an equation of the secant line $S(x)$ through the two points.

(b) Find the equation of the line

$$T(x) = f'(4)(x - 4) + f(4)$$

tangent to the graph of f passing through the given point. Why are the linear functions S and T nearly the same?

(c) Use a graphing utility to graph f and T in the same set of coordinate axes. Note that T is a good approximation of f when x is close to 4. What happens to the accuracy of the approximation as you move farther away from the point of tangency?

(d) Demonstrate the conclusion in part (c) by completing the table.

Δx	-3	-2	-1	-0.5	-0.1	0
$f(4 + \Delta x)$						
$T(4 + \Delta x)$						

Δx	0.1	0.5	1	2	3
$f(4 + \Delta x)$					
$T(4 + \Delta x)$					

86. Linear Approximation Repeat Exercise 85 for the function $f(x) = x^3$, where $T(x)$ is the line tangent to the graph at the point $(1, 1)$. Explain why the accuracy of the linear approximation decreases more rapidly than in Exercise 85.

True or False? **In Exercises 87–92, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

87. If $f'(x) = g'(x)$, then $f(x) = g(x)$.

88. If $f(x) = g(x) + c$, then $f'(x) = g'(x)$.

89. If $y = \pi^2$, then $dy/dx = 2\pi$.

90. If $y = x/\pi$, then $dy/dx = 1/\pi$.

91. If $g(x) = 3f(x)$, then $g'(x) = 3f'(x)$.

92. If $f(x) = \dfrac{1}{x^n}$, then $f'(x) = \dfrac{1}{nx^{n-1}}$.

Finding Rates of Change **In Exercises 93–96, find the average rate of change of the function over the given interval. Compare this average rate of change with the instantaneous rates of change at the endpoints of the interval.**

93. $f(t) = 4t + 5, \quad [1, 2]$

94. $f(t) = t^2 - 7, \quad [3, 3.1]$

95. $f(x) = \dfrac{-1}{x}, \quad [1, 2]$

96. $f(x) = \sin x, \quad \left[0, \dfrac{\pi}{6}\right]$

Vertical Motion **In Exercises 97 and 98, use the position function $s(t) = -16t^2 + v_0 t + s_0$ for free-falling objects.**

97. A silver dollar is dropped from the top of a building that is 1362 feet tall.

(a) Determine the position and velocity functions for the coin.

(b) Determine the average velocity on the interval $[1, 2]$.

(c) Find the instantaneous velocities when $t = 1$ and $t = 2$.

(d) Find the time required for the coin to reach ground level.

(e) Find the velocity of the coin at impact.

98. A ball is thrown straight down from the top of a 220-foot building with an initial velocity of -22 feet per second. What is its velocity after 3 seconds? What is its velocity after falling 108 feet?

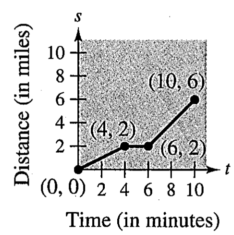
Vertical Motion **In Exercises 99 and 100, use the position function $s(t) = -4.9t^2 + v_0 t + s_0$ for free-falling objects.**

99. A projectile is shot upward from the surface of Earth with an initial velocity of 120 meters per second. What is its velocity after 5 seconds? After 10 seconds?

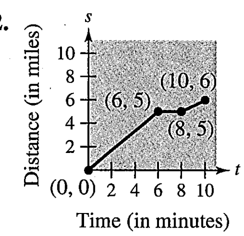
100. To estimate the height of a building, a stone is dropped from the top of the building into a pool of water at ground level. The splash is seen 5.6 seconds after the stone is dropped. What is the height of the building?

Think About It **In Exercises 101 and 102, the graph of a position function is shown. It represents the distance in miles that a person drives during a 10-minute trip to work. Make a sketch of the corresponding velocity function.**

101.

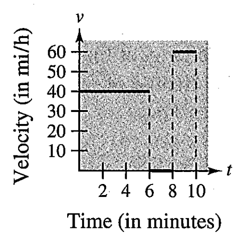


102.

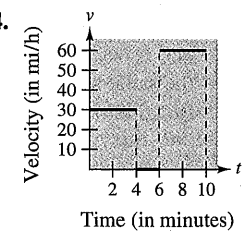


Think About It **In Exercises 103 and 104, the graph of a velocity function is shown. It represents the velocity in miles per hour during a 10-minute trip to work. Make a sketch of the corresponding position function.**

103.



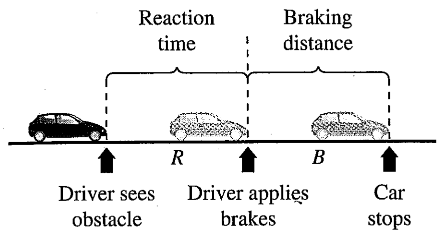
104.



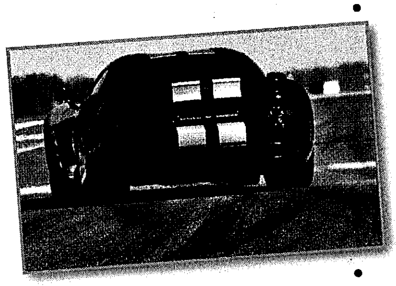
105. Volume The volume of a cube with sides of length s is given by $V = s^3$. Find the rate of change of the volume with respect to s when $s = 6$ centimeters.

106. Area The area of a square with sides of length s is given by $A = s^2$. Find the rate of change of the area with respect to s when $s = 6$ meters.

107. Modeling Data The stopping distance of an automobile, on dry, level pavement, traveling at a speed v (in kilometers per hour) is the distance R (in meters) the car travels during the reaction time of the driver plus the distance B (in meters) the car travels after the brakes are applied (see figure). The table shows the results of an experiment.



Speed, v	20	40	60	80	100
Reaction Time Distance, R	8.3	16.7	25.0	33.3	41.7
Braking Time Distance, B	2.3	9.0	20.2	35.8	55.9



(a) Use the regression capabilities of a graphing utility to find a linear model for reaction time distance R.

(b) Use the regression capabilities of a graphing utility to find a quadratic model for braking time distance B.

(c) Determine the polynomial giving the total stopping distance T.

(d) Use a graphing utility to graph the functions R, B, and T in the same viewing window.

(e) Find the derivative of T and the rates of change of the total stopping distance for $v = 40$, $v = 80$, and $v = 100$.

(f) Use the results of this exercise to draw conclusions about the total stopping distance as speed increases.

108. Fuel Cost A car is driven 15,000 miles a year and gets x miles per gallon. Assume that the average fuel cost is \$3.48 per gallon. Find the annual cost of fuel C as a function of x and use this function to complete the table.

x	10	15	20	25	30	35	40
C							
dC/dx							

Who would benefit more from a one-mile-per-gallon increase in fuel efficiency—the driver of a car that gets 15 miles per gallon, or the driver of a car that gets 35 miles per gallon? Explain.

109. Velocity Verify that the average velocity over the time interval $[t_0 - \Delta t, t_0 + \Delta t]$ is the same as the instantaneous velocity at $t = t_0$ for the position function

$$s(t) = -\frac{1}{2}at^2 + c.$$

110. Inventory Management The annual inventory cost C for a manufacturer is

$$C = \frac{1{,}008{,}000}{Q} + 6.3Q$$

where Q is the order size when the inventory is replenished. Find the change in annual cost when Q is increased from 350 to 351, and compare this with the instantaneous rate of change when $Q = 350$.

111. Finding an Equation of a Parabola Find an equation of the parabola $y = ax^2 + bx + c$ that passes through $(0, 1)$ and is tangent to the line $y = x - 1$ at $(1, 0)$.

112. Proof Let (a, b) be an arbitrary point on the graph of $y = 1/x$, $x > 0$. Prove that the area of the triangle formed by the tangent line through (a, b) and the coordinate axes is 2.

113. Finding Equation(s) of Tangent Line(s) Find the equation(s) of the tangent line(s) to the graph of the curve $y = x^3 - 9x$ through the point $(1, -9)$ not on the graph.

114. Finding Equation(s) of Tangent Line(s) Find the equation(s) of the tangent line(s) to the graph of the parabola $y = x^2$ through the given point not on the graph.

(a) $(0, a)$ (b) $(a, 0)$

Are there any restrictions on the constant a?

Making a Function Differentiable **In Exercises 115 and 116, find a and b such that f is differentiable everywhere.**

115. $f(x) = \begin{cases} ax^3, & x \le 2 \\ x^2 + b, & x > 2 \end{cases}$

116. $f(x) = \begin{cases} \cos x, & x < 0 \\ ax + b, & x \ge 0 \end{cases}$

117. Determining Differentiability Where are the functions $f_1(x) = |\sin x|$ and $f_2(x) = \sin |x|$ differentiable?

118. Proof Prove that $\dfrac{d}{dx}[\cos x] = -\sin x$.

FOR FURTHER INFORMATION For a geometric interpretation of the derivatives of trigonometric functions, see the article "Sines and Cosines of the Times" by Victor J. Katz in *Math Horizons*. To view this article, go to *MathArticles.com*.

PUTNAM EXAM CHALLENGE

119. Find all differentiable functions $f: \mathbb{R} \to \mathbb{R}$ such that

$$f'(x) = \frac{f(x + n) - f(x)}{n}$$

for all real numbers x and all positive integers n.

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