Ch. 2.2a Power Rule p. 114-117

3-17 odd, 25-31 odd, 35, 39-49 odd, 53-59 odd, 63, 65

$* \frac{d}{dx} u^n = n \cdot u^{n-1} \cdot u'$

25) $y = \frac{5}{2x^2} = \frac{5}{2} x^{-2}$ | $y' = -\cancel{2} \cdot \frac{5}{\cancel{2}} x^{-3} = \frac{-5}{x^3}$ $\boxed{y' = \frac{-5}{x^3}}$

27) $y = \frac{6}{(5x)^3} = \frac{6}{125x^3} = \frac{6}{125} x^{-3}$ | $y' = -3 \cdot \frac{6}{125} x^{-4} = \frac{-18}{125} x^{-4}$

$\boxed{y' = \frac{-18}{125x^4}}$

29) $y = \frac{\sqrt{x}}{x} = \frac{x^{1/2}}{x^1} = x^{1/2 - 1} = x^{-1/2} = \frac{1}{x^{1/2}}$ or $y = \frac{\cancel{\sqrt{x}}}{\cancel{\sqrt{x}} \cdot \sqrt{x}} = \frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}}$

$y = \frac{1}{x^{1/2}} = x^{-1/2}$ | $y' = -\frac{1}{2} x^{-1/2 - 2/2} = -\frac{1}{2} x^{-3/2}$ | $\boxed{y' = \frac{-1}{2x^{3/2}}}$

Find slope of graph at given point.

31) $f(x) = \frac{8}{x^2}$ at $(2,2)$

$f(x) = 8x^{-2}$

$f'(x) = -2 \cdot 8x^{-3}$

$= \frac{-16}{x^3}$

$f'(2) = \frac{-16}{2^3} = \frac{-16}{8} = -2$

$\boxed{f'(2) = -2}$

35) $y = (4x+1)^2$ at $(0,1)$

$y = (4x+1)(4x+1) = 16x^2 + 8x + 1$

$y' = 32x + 8 + 0 = 32x + 8$

$y'(0) = 32(0) + 8 = 8$

$\boxed{y'(0) = 8}$

2.2a

41) $g(t) = t^2 - \frac{4}{t^3} = t^2 - 4t^{-3}$

$g'(t) = 2t - (-3)\cdot 4t^{-4}$

$g'(t) = 2t + 12t^{-4}$

$\boxed{g'(t) = 2t + \frac{12}{t^4}}$

43) $f(x) = \frac{4x^3+3x^2}{x} = (4x^3+3x^2)x^{-1} = 4x^2+3x$

$f(x) = 4x^2+3x$

$\boxed{f'(x) = 8x+3}$

45) $f(x) = \frac{x^3-3x^2+4}{x^2} = (x^3-3x^2+4)x^{-2} = x - 3 + 4x^{-2}$

$f(x) = x - 3 + 4x^{-2}$

$f'(x) = 1 - 0 + -2\cdot 4x^{-3} = 1 - 8x^{-3}$

$\boxed{f'(x) = 1 - \frac{8}{x^3} \text{ or } \frac{x^3-8}{x^3}}$

47) $y = x(x^2+1) = x^3 + x$

$\boxed{y' = 3x^2+1}$

49) $f(x) = \sqrt{x} - 6\sqrt[3]{x}$

$f(x) = x^{1/2} - 6x^{1/3}$

$f'(x) = \frac{1}{2}x^{-1/2} - \frac{1}{3}\cdot 6x^{1/3-3/3} = \frac{1}{2}x^{-1/2} - 2x^{-2/3}$

$\boxed{f'(x) = \frac{1}{2x^{1/2}} - \frac{2}{x^{2/3}} \text{ or } \frac{1}{2\sqrt{x}} - \frac{2}{\sqrt[3]{x^2}}}$

2.2a (continued)

Find equation of tangent line : $y - y_1 = m(x - x_1)$

steps:
1. find $f'(x)$
2. Find slope $f'(c) = m$
3. Identify point (x, y)
4. Write equation $y - y_1 = m(x - x_1)$

53) $y = x^4 - 3x^2 + 2$

$y' = 4x^3 - 6x$

$y'(1) = 4(1)^3 - 6(1) = -2$

point: $(1, 0)$

slope: $m = -2$

$\boxed{y - 0 = -2(x - 1)}$

55) $y = \frac{2}{\sqrt[4]{x^3}}$ at point $(1, 2)$

$y = \frac{2}{x^{3/4}} = 2x^{-3/4}$

$y' = \frac{-3}{4} \cdot 2x^{-3/4 - 4/4}$

$y' = \frac{-3}{2}x^{-7/4} = \frac{-3}{2x^{7/4}}$

$y'(1) = \frac{-3}{2(1)^{7/4}} = -3/2$

point: $(1, 2)$

slope: $m = -3/2$

$\boxed{y - 2 = -\frac{3}{2}(x - 1)}$

Determine points where function has horizontal tangent line

* set numerator of $f'(x) = 0$, solve for x.

57) $y = x^4 - 2x^2 + 3$

$y' = 4x^3 - 4x$

$0 = 4x^3 - 4x$

$0 = 4x(x^2 - 1)$

$0 = 4x(x-1)(x+1)$

$x = 0, 1, -1$

Horizontal tangents (slope of graph is zero)

$\boxed{(0, 3), (1, 2), (-1, 2)}$

$f(0) = 3 \quad f(-1) = 2$

$f(1) = 2$

59) $y = \frac{1}{x^2} = x^{-2}$

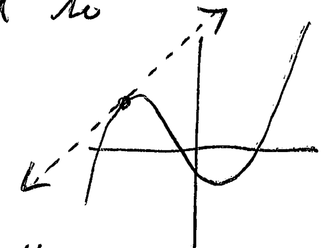
$y' = -2x^{-3}$

$y' = \frac{-2}{x^3}$

$\boxed{-2 \neq 0\text{, so no horizontal tangents.}}$

2.2a

Find K value such that the line is tangent to graph of the function.



* When a line is tangent to a curve:

a) the line and curve share same point (set their equations equal to each other)

b) the line and curve share the same slope (set their derivatives equal to each other)

63) $f(x) = K - x^2$ line: $y = -6x + 1$

set functions equal:

$-6x + 1 = K - x^2$

$-6(3) + 1 = K - 3^2$

$-18 + 1 = K - 9$

$-17 = K - 9$

$\boxed{-8 = K}$

set derivatives equal

$f'(x) = 0 - 2x = -2x$

$y' = -6$

$-2x = -6$

$\underline{x = 3}$

$\boxed{K = -8}$

65) $f(x) = \frac{K}{x}$ line: $y = \frac{-3}{4}x + 3$

set functions equal

$\frac{K}{x} = -\frac{3}{4}x + 3$

$\frac{\frac{3}{4}x^2}{x} = \frac{-3}{4}x + 3$

$\frac{3}{4}x = \frac{-3}{4}x + 3$

$\frac{6}{4}x = 3$

$x = 3 \cdot \frac{4}{6} = 2$

$\boxed{x = 2}$

equate derivatives

$f(x) = \frac{k}{x} = kx^{-1}$

$f'(x) = -1kx^{-2} = \frac{-k}{x^2}$

$y' = \frac{-3}{4}$

$\frac{-k}{x^2} = \frac{-3}{4}$

$-4k = -3x^2$

$k = \frac{3}{4}x^2$

since $x = 2$ and $K = \frac{3}{4}x^2$

$K = \frac{3}{4}(2)^2 = \frac{3}{4}(4) = 3$

$\boxed{K = 3}$