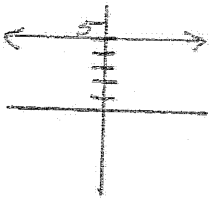
Key

Ch. 2.2a Derivative Rules - Notes

1. Constant Rule: If $f(x)=c$, then $f'(x)=0$

Ex. $f(x)=5 \longrightarrow f'(x)=0$



2. Power Rule: If $f(x)=x^n$, then $f'(x)=n\cdot x^{n-1}$

steps: a) Bring exponent down, in front of variable
b) subtract 1 from original exponent value.

*Important Note: Be sure function is in appropriate form before applying power rule.

→ convert any radicals to rational exponents
→ Move all variables from denominator to numerator (if necessary)

Ex. 1 Find derivatives of the following:

a) $y=x^7 \longrightarrow y'=7x^6$

b) $g(x)=\sqrt[3]{x}$ $\quad g(x)=x^{1/3}$ $\quad g'(x)=\frac{1}{3}x^{1/3-3/3}=\frac{1}{3}x^{-2/3}=\boxed{\frac{1}{3x^{2/3}} \text{ or } \frac{1}{3\sqrt[3]{x^2}}}$

c) $y=\frac{4}{x^5}$ $\quad y=4x^{-5}$ $\quad y'=-20x^{-5-1}=-20x^{-6}=\boxed{\frac{-20}{x^6}}$

d) $y=8x^{2/3}-\sqrt[5]{x}+\frac{2}{\sqrt{x}}+0.875$

$y=8x^{2/3}-x^{1/5}+2x^{-1/2}+0.875$

$y'=\frac{2}{3}\cdot 8x^{2/3-3/3}-\frac{1}{5}x^{1/5-5/5}+\frac{-1}{2}\cdot 2x^{-1/2-2/2}+0$

$y'=\frac{16}{3}x^{-1/3}-\frac{1}{5}x^{-4/5}-1x^{-3/2}+0$

$\boxed{y'=\frac{16}{3x^{1/3}}-\frac{1}{5x^{4/5}}-\frac{1}{x^{3/2}}}$

Ex.2 If $f(x)=x^{-2}$, find $f'(2)$

$f'(x)=-2x^{-3}=\frac{-2}{x^3}$ $\quad f'(2)=\frac{-2}{2^3}=\frac{-2}{8}=\boxed{\frac{-1}{4}}$

Ex.3 If $f(x)=\sqrt[3]{x^2}$, write tangent line equation to $f(x)$ at $x=$

$f(x)=x^{2/3}$ | $f'(x)=\frac{2}{3}x^{-1/3}$ | $f'(8)=\frac{2}{3(8)^{1/3}}=\frac{2}{3\cdot 2}=\frac{1}{3}$

$f'(x)=\frac{2}{3}x^{\frac{2}{3}-\frac{3}{3}}$ | $f'(x)=\frac{2}{3x^{1/3}}$ | $f(8)=\sqrt[3]{8^2}=4$

point: $(8,4)$ | $y-y_1=m(x-x_1)$

slope: $m=1/3$ | $\boxed{y-4=\frac{1}{3}(x-8)}$

Ex.4 $f(x)=\frac{x^4-3x^2+4(\sqrt[3]{x})}{\sqrt{x}}$ find $f'(x)$

$f(x)=\frac{x^4}{x^{1/2}}-\frac{3x^2}{x^{1/2}}+\frac{4x^{1/3}}{x^{1/2}}=x^{7/2}-3x^{3/2}+4x^{-1/6}$

$f'(x)=\frac{7}{2}x^{5/2}-3\cdot\frac{3}{2}x^{1/2}+4\left(-\frac{1}{6}\right)x^{-7/6}$

$=\boxed{\frac{7}{2}x^{5/2}-\frac{9}{2}x^{1/2}-\frac{2}{3x^{7/6}}}$

Ex.5 $f(x)=3x(x+1)^2$ find $f'(x)$

$f(x)=3x(x^2+2x+1)=3x^3+6x^2+3x$

$\boxed{f'(x)=9x^2+12x+3}$

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7) $y = \frac{1}{x^7}$ $\quad y = x^{-7}$ $\quad y' = -7x^{-8}$ $\quad \boxed{y' = \frac{-7}{x^8}}$

9) $f(x) = \sqrt[5]{x}$ $\quad f(x) = x^{1/5}$ $\quad f'(x) = \frac{1}{5}x^{-4/5}$ $\quad \boxed{f'(x) = \frac{1}{5x^{4/5}}}$

27) $y = \frac{3}{(2x)^3} = \frac{3}{8x^3} = \frac{3}{8}x^{-3}$ $\quad y' = -3 \cdot \frac{3}{8}x^{-3-1}$ $\quad y' = \frac{-9}{8}x^{-4}$

$\boxed{y' = \frac{-9}{8x^4}}$

29) $y = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}} = x^{-1/2}$ $\quad y' = -\frac{1}{2}x^{-1/2-\frac{2}{2}}$ $\quad y' = -\frac{1}{2}x^{-3/2} = \boxed{\frac{-1}{2x^{3/2}}}$

35) $y = (2x+1)^2$ at $(0, 1)$

$y = (2x+1)(2x+1)$ $\quad y = 4x^2 + 4x + 1$ $\quad y' = 8x + 4$

$y'(0) = 8(0) + 4 = 4$

$\boxed{y'(0) = 4}$

43) $f(x) = \frac{x^3 - 3x^2 + 4}{x^2} = \frac{x^3}{x^2} - \frac{3x^2}{x^2} + \frac{4}{x^2}$

$f(x) = x - 3 + 4x^{-2}$

$f'(x) = 1 + 0 - 8x^{-3}$ $\quad \boxed{f'(x) = 1 - \frac{8}{x^3}}$

47) $f(x) = \sqrt{x} - 6\sqrt[3]{x} \quad = x^{1/2} - 6x^{1/3}$

$f'(x) = \frac{1}{2}x^{-1/2} - \frac{1}{3}\cdot 6x^{-2/3}$ $\boxed{f'(x) = \frac{1}{2x^{1/2}} - \frac{2}{x^{2/3}}}$

49) $h(s) = 5^{4/5} - 5^{2/3}$

$h'(s) = \frac{4}{5}5^{-1/5} - \frac{2}{3}5^{-1/3}$ $h'(s) = \frac{4}{5s^{1/5}} - \frac{2}{3s^{1/3}}$

55) $f(x) = \frac{2}{\sqrt[4]{x^3}}$ at $(1,2)$ Write equation of tangent line

$f(x) = 2x^{-3/4}$

$f'(x) = \frac{-3}{4}\cdot 2x^{-7/4}$

$f'(x) = \frac{-3}{2x^{7/4}}$

$f'(1) = \frac{-3}{2(1)^{7/4}} = \frac{-3}{2}$

point: $(1,2)$

slope: $m = -3/2$

$y - y_1 = m(x - x_1)$

$\boxed{y - 2 = \frac{-3}{2}(x-1)}$

57) $y = x^4 - 8x^2 + 2$ *Determine point where $f(x)$ has horizontal tangent ↖ slope = 0, $f'(x) = 0$

*set $y'(x) = 0$, solve for x

$y'(x) = 4x^3 - 16x$

$0 = 4x(x^2 - 4)$

$0 = 4x(x+2)(x-2)$

$x = 0, -2, 2$

points are: $y(0) = 2$, $y(2) = -14$, $y(-2) = -14$

$\underline{(0,2), (2,-14), (-2,-14)}$

63) Find k such that line is tangent to graph *set $f'(x)$ = slope of line *set $f(x)$ = equation of line

$f(x) = x^2 - kx$

$f'(x) = 2x - k$

$2x - k = 4$

$k = 2x - 4$

line: $y = 4x - 9$

slope: $m = 4$

$x^2 - kx = 4x - 9$

$x^2 - (2x-4)x = 4x - 9$

$x^2 - 2x^2 + 4x = 4x - 9$

$-x^2 = -9$

$x = \pm 3$

when $x = 3$, $k = 2$

when $x = -3$, $k = -10$