Key

AP Calculus – 2.2b Notes - Interpreting Limit Definitions of a Derivative

General Limit Definition of the Derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Alternate Limit Definition of a derivative:

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Identify the function we are working with. Then identify the x-value for the instantaneous rate of change (slope of the tangent line at a point).

1. $\lim_{h \to 0} \frac{5\ln\left(\frac{2}{4+h}\right) - 5\ln\left(\frac{1}{2}\right)}{h}$

Function: $f(x) =$ $5\ln\left(\frac{2}{x}\right)$

Instantaneous rate at $x =$ 4

$$f'(x) = \lim_{h \to 0} \frac{5\ln\left(\frac{2}{x+h}\right) - 5\ln\left(\frac{2}{x}\right)}{h}$$

$$\boxed{f'(4)} = \lim_{h \to 0} \frac{5\ln\left(\frac{2}{4+h}\right) - 5\ln\left(\frac{2}{4}\right)}{h}$$

* This notation represents $f'(4)$ which is the slope or rate of change of $f(x)$ at $x = 4$

2. $\lim_{x \to \frac{\pi}{2}} \frac{\sin x - 1}{x - \frac{\pi}{2}}$

$$f'(c) = \lim_{x \to \pi/2} \frac{\sin x - \sin(\pi/2)}{x - \pi/2}$$

Function: $f(x) =$ $\sin x$

Instantaneous rate at $x =$ $\pi/2$

$$\boxed{f'(\pi/2)} = \lim_{x \to \pi/2} \frac{\sin x - 1}{x - \pi/2}$$

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24. $\lim_{h \to 0} \frac{3(1+h)^2 - 7(1+h) + 1 + (3)}{h}$

Function: $f(x) =$ $3x^2 - 7x + 1$

Instantaneous rate at $x =$ 1

$$f'(x) = \lim_{h \to 0} \frac{3(x+h)^2 - 7(x+h) + 1 - (3x^2 - 7x + 1)}{h}$$

$$f'(1) = \lim_{h \to 0} \frac{3(1+h)^2 - 7(1+h) + 1 - (-3)}{h}$$

This problem represents $f'(1)$

25. $\lim_{x \to \frac{\pi}{2}} \frac{6x^2 \sin x - \frac{3\pi^2}{2}}{x - \frac{\pi}{2}}$

Function: $f(x) =$ $6x^2 \sin x$

Instantaneous rate at $x =$ $\pi/2$

$$\boxed{f'\left(\tfrac{\pi}{2}\right)} = \lim_{x \to \pi/2} \frac{f(x) - f(\pi/2)}{x - \pi/2}$$

26. $\lim_{h \to 0} \frac{\log(2 - 4(h-5)) - \log(22)}{h}$

Function: $f(x) =$ $\log(2 - 4x)$

Instantaneous rate at $x =$ -5

$$f'(x) = \lim_{h \to 0} \frac{\log[2 - 4(x+h)] - \log(2-4x)}{h}$$

$$\boxed{f'(-5)} = \lim_{h \to 0} \frac{\log[2 - 4(-5+h)] - \log 22}{h}$$

27. $\lim_{x\to 5} \frac{\frac{1}{\sqrt{3x}}-\frac{1}{\sqrt{15}}}{x-5}$

Function: $f(x) = \frac{1}{\sqrt{3x}}$

Instantaneous rate at $x = 5$

$\boxed{f'(5)} = \lim_{x\to 5} \frac{f(x)-f(5)}{x-5}$

28. $\lim_{h\to 0} \frac{e^{6(3+h)+1}-e^{19}}{h}$

Function: $f(x) = e^{6x+1}$

Instantaneous rate at $x = 3$

$f'(x) = \lim_{h\to 0} \frac{e^{6(x+h)+1} - e^{6x+1}}{h}$

$f'(3) = \lim_{h\to 0} \frac{e^{6(3+h)+1} - e^{19}}{h}$

29. Let f be the function defined by $f(x) = \ln 7x$. The average rate of change of f over the interval $[2, a]$ is 41, where $a > 2$. Which of the following is an equation that could be used to find the value of a?

(A) $f(a) = 41$

(B) $f(a) - f(2) = 41$

(C) $\frac{f(a)-f(2)}{a-2} = 41$

(D) $\frac{f(a)+f(2)}{2} = 41$

Avg. ROC is slope $\rightarrow \frac{y_2-y_1}{x_2-x_1}$

Avg. ROC on interval $[2,a] \rightarrow \boxed{\frac{f(a)-f(2)}{a-2} = 41}$

30. Find the average rate of change of $f(x) = \sin x \ln x$ on the interval $1 \le x \le a$.

$\frac{f(a)-f(1)}{a-1} \rightarrow \frac{\sin a \ln a - \sin 1 \ln 1}{a-1} \rightarrow \frac{\sin a \ln a - 0}{a-1}$

$\rightarrow \boxed{\frac{\sin a \ln a}{a-1}}$

31) Let $f'(x) = \lim_{h\to 0} \frac{(x+h)^2-x^2}{h}$. For what value of x does $f(x) = 4$?

(A) -4 (B) -1 (C) 1 $\boxed{\text{(D) } 2}$ (E) 4

The function being referenced is $f(x) = x^2$

$x^2 = 4 \rightarrow x = \pm 2 \quad x = 2, x = -2$

General Limit Definition of the Derivative: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f(x) = 5x^2 - x \quad \rightarrow$ Find $f'(1)$

$f(\ ) = 5(\ )^2 - (\ )$

$f(x+h) = 5(x+h)^2 - (x+h)$

$$f'(x) = \lim_{h \to 0} \frac{5(x+h)^2 - (x+h) - (5x^2 - x)}{h} \rightarrow f'(1) = \lim_{h \to 0} \frac{5(1+h)^2 - (1+h) - 4}{h}$$

Given the above expression, What does this represent?

The function that is being referenced is $f(x) = 5x^2 - x$
This represents $f'(1)$, the rate of change of $f(x)$ at $x = 1$

$$f'(x) = \lim_{h \to 0} \frac{5(x^2 + 2xh + h^2) - x - h - 5x^2 + x}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{5x^2} + 10xh + 5h^2 \cancel{- x} - h \cancel{- 5x^2} \cancel{+ x}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{h}(10x + 5h - 1)}{\cancel{h}}$$

$$f'(x) = 10x + 5(0) - 1$$

$$\boxed{f'(x) = 10x - 1}$$

$$f'(1) = 10(1) - 1 = 9$$

$$\boxed{f'(1) = 9}$$

Alternative Limit Definition of a Derivative: $f'(c) = \lim_{x\to c} \frac{f(x)-f(c)}{x-c}$

$f(x) = 5x^2 - x$ Find $f'(1)$

$$f'(c) = \lim_{x\to c} \frac{f(x)-f(c)}{x-c}$$

$f(1) = 5(1)^2 - 1 = 4$

$$f'(1) = \lim_{x\to 1} \frac{f(x)-f(1)}{x-1}$$

$$\boxed{f'(1) = \lim_{x\to 1} \frac{5x^2-x-4}{x-1}}$$

$$f'(1) = \lim_{x\to 1} \frac{(5x+4)\cancel{(x-1)}}{\cancel{x-1}}$$

$$f'(1) = 5(1)+4 = 9$$

$$\boxed{f'(1) = 9}$$

Given: $\lim_{x\to 1} \frac{5x^2-x-4}{x-1}$

What does the above represent?

The function being referenced is $f(x) = 5x^2 - x$.
This represents $f'(1)$, the rate of change of $f(x)$ at $x=1$