2.2b - PVA p.115-117 #67, 93, 95, 97-100 all, 101, 103, 110, 111, 115

67) $f(x) = Kx^3$ line: $y = x + 1$

set equations equal:

$Kx^3 = x + 1$

$\left(\frac{1}{3x^2}\right)x^3 = x + 1$

$3 \cdot \left( \frac{x}{3} = x + 1 \right)$

$x = 3x + 3$

$-3 = 2x$ $\underline{\underline{x = -3/2}}$

equate derivatives

$f'(x) = 3Kx^2$

$y' = 1$

$3Kx^2 = 1$

$K = \frac{1}{3x^2}$

$K = \frac{1}{3x^2}$

since $x = -3/2$

$K = \frac{1}{3(-3/2)^2} = \frac{1}{3 \cdot 9/4}$

$= \frac{1}{27/4} = 4/27$

$\boxed{K = 4/27}$

Find Rate of Change (R.O.C): Compare avg. rate of change vs. instantaneous Rate of change

93) $f(t) = 4t + 5$ $[1, 2]$

$f(2) = 4(2) + 5 = 13$

$f(1) = 4(1) + 5 = 9$

Avg. ROC $= \frac{f(2) - f(1)}{2 - 1} = \frac{13 - 9}{2 - 1} = 4$

$f'(t) = 4$

$f'(2) = 4$

$f'(1) = 4$

since $f(t)$ is linear, the avg. slope and instantaneous slopes are equal.

95) $f(x) = \frac{-1}{x}$ $[1, 2]$

$f(x) = -x^{-1}$

$f'(x) = 1x^{-2} = \frac{1}{x^2}$

$f'(1) = \frac{1}{1^2} = 1$ $f'(2) = \frac{1}{2^2} = 1/4$

$f(1) = \frac{-1}{1} = -1$

$f(2) = \frac{-1}{2}$

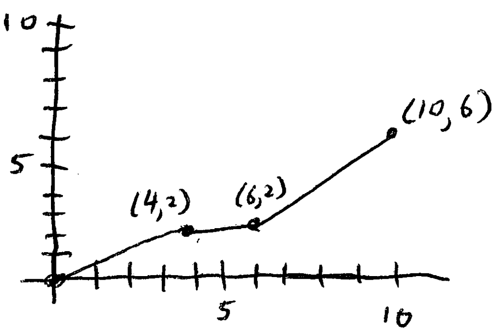
Avg. ROC $= \frac{f(2) - f(1)}{2 - 1}$

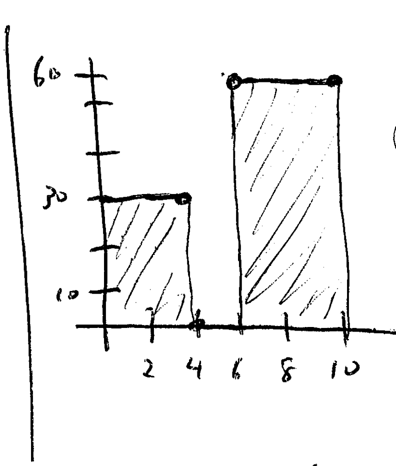
$= \frac{-1/2 - (-1)}{2 - 1}$

Avg. ROC $= -1/2 + 1$

$= \boxed{1/2}$

101) position function $x(t)$



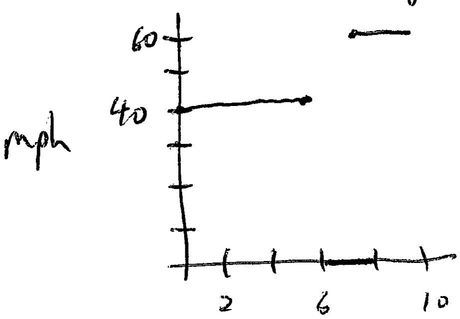



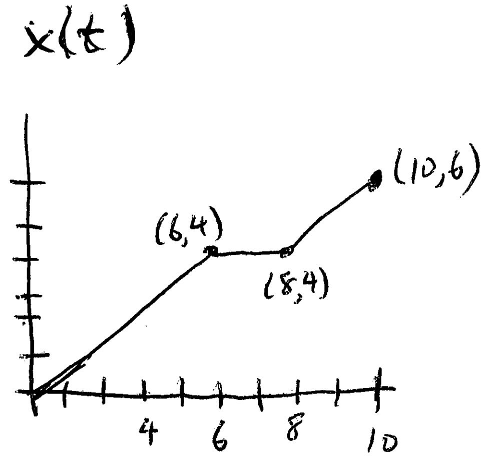

slope : (0,0) to (4,2) $= \frac{2-0}{4-0} = \frac{1}{2}$ $\quad v(t) = \frac{1}{2}$ mi/min. $= \frac{1}{2} \frac{mi}{\cancel{min}} \cdot \frac{60\ \cancel{min}}{hr} = 30$ mph

slope : (4,2) to (6,2) $= 0$ $\quad v(t) = 0$ mi/min $= 0$ mph

slope ; (6,2) to (10,6) $= \frac{6-2}{10-6} = \frac{4}{4}$ $\quad v(t) = 1$ mi/min $= 1 \frac{mi}{\cancel{min}} \cdot \frac{60\ \cancel{min}}{hr} = 60$ mph

103) Given velocity graph, sketch position function




$V_1 = 40 \frac{mi}{hr} \cdot \frac{1\ hr}{60 min} = \frac{2}{3} \cdot 6$ min $= 4$ mi.

(travels 4 miles after 6 mins)

$V_2 = 0$

$V_3 = 60 \frac{mi}{hr} \cdot \frac{1\ hr}{60 min} = 1 \frac{mi}{min} \cdot 2$ min $= 2$ mi.

(travels 2 miles in 2 mins)

# 2.2b

110) $C = \dfrac{1{,}008{,}000}{Q} + 6.3Q$

a) Find Avg. ROC in $[350, 351]$
b) Find $C'(350)$

$C(350) = \dfrac{1{,}008{,}000}{350} + 6.3(350) = 5085$

$C(351) = \dfrac{1{,}008{,}000}{351} + 6.3(351) = 5083.095$

Avg. ROC $= \dfrac{C(351) - C(350)}{351 - 350} = \dfrac{5083.095 - 5085}{1} = \boxed{-\$1.91}$

$C = 1{,}008{,}000Q^{-1} + 6.3Q$

$C'(Q) = -1{,}008{,}000Q^{-2} + 6.3$

$C'(Q) = \dfrac{-1{,}008{,}000}{Q^2} + 6.3$

$C'(350) = \dfrac{-1{,}008{,}000}{350^2} + 6.3$

$\boxed{\dfrac{dC}{dQ} = -1.93}$

115) Find a and b such that f is differentiable everywhere

$f(x) = \begin{cases} ax^3, & x \le 2 \\ x^2 + b, & x > 2 \end{cases}$

* set equations equal
* set derivatives equal

equate equations at $x = 2$

$ax^3 = x^2 + b$ at $x = 2$

$a(2)^3 = (2)^2 + b$

$8a = 4 + b$

$8(1/3) = 4 + b$

$8/3 = 4 + b$

$8/3 - 4 = \frac{8}{3} - \frac{12}{3} = \boxed{-\frac{4}{3} = b}$

equate derivatives at $x = 2$

$f'(x) = \begin{cases} 3ax^2, & x \le 2 \\ 2x + 0, & x > 2 \end{cases}$

$3ax^2 = 2x$ at $x = 2$

$3a(2)^2 = 2(2)$

$12a = 4$

$a = \frac{4}{12} = 1/3$ $\boxed{a = 1/3}$