Key

2.3-2.4 Review WS #3 – Power Rule, Particle Motion, Product & Quotient Rule

No negative exponents in answer.

1. Find $\frac{dy}{dx}$ if $y = 7x^3(x-1)^2 - \frac{3x^2}{\sqrt{11}} + 2\pi x - 5e^4 + 2\sqrt[5]{x^4} + \frac{2}{3\sqrt{x^7}}$

$y = 7x^3(x^2-2x+1) - \frac{3}{\sqrt{11}}x^2 + 2\pi x - 5e^4 + 2x^{4/5} + \frac{2}{3}x^{-7/2}$

$y = 7x^5 - 14x^4 + 7x^3 - \frac{3}{\sqrt{11}}x^2 + 2\pi x - 5e^4 + 2x^{4/5} + \frac{2}{3}x^{-7/2}$

$\frac{dy}{dx} = 35x^4 - 56x^3 + 21x^2 - \frac{6}{\sqrt{11}}x + 2\pi - 0 + \frac{8}{5}x^{-1/5} - \frac{7}{3}x^{-9/2}$

$$\boxed{\frac{dy}{dx} = 35x^4 - 56x^3 + 21x^2 - \frac{6}{\sqrt{11}}x + 2\pi + \frac{8}{5x^{1/5}} - \frac{7}{3x^{9/2}}}$$

2. Find $\frac{dy}{dx}$ if $y = \frac{4x(\sqrt{x}+3x-1)}{5(\sqrt{x^9})}$

$y = \frac{4x^{3/2}+12x^2+4x}{5x^{9/2}} \rightarrow \frac{4x^{3/2}}{5x^{9/2}} + \frac{12x^2}{5x^{9/2}} + \frac{4x}{5x^{9/2}}$

$y = \frac{4}{5}x^{-6/2} + \frac{12}{5}x^{-5/2} + \frac{4}{5}x^{-7/2}$

$\frac{dy}{dx} = \frac{-6}{2}\cdot\frac{4}{5}x^{-8/2} + \frac{-5}{2}\cdot\frac{12}{5}x^{-7/2} - \frac{7}{2}\cdot\frac{4}{5}x^{-9/2}$

$$\boxed{\frac{dy}{dx} = \frac{-12}{5x^4} - \frac{6}{x^{7/2}} - \frac{14}{5x^{9/2}}}$$

3. If $f(x) = \frac{x}{x^2-2}$ find $f'(x)$ (simplify fully). Then write the equation of the line tangent to $f(x)$ at $x = 1$ in point-slope form.

$f(1) = \frac{1}{1^2-2} = \frac{1}{-1} = -1$

$f'(x) = \frac{(1)(x^2-2)-(x)(2x)}{(x^2-2)^2}$ (f', g, f, g'; g^2)

$f'(x) = \frac{x^2-2-2x^2}{(x^2-2)^2}$

$f'(x) = \frac{-1x^2-2}{(x^2-2)^2}$

$f'(1) = \frac{-1-2}{(1-2)^2} = \frac{-3}{1} = -3$

point: $(1,-1)$

slope: $m = -3$

$y - y_1 = m(x-x_1)$

$$\boxed{y+1 = -3(x-1)}$$

4. Particle moves along the x-axis so that its position at time t is given $x(t) = t^3 - 6t^2 - 15t + 1$ where $x(t)$ is in meters per minute and $t \geq 0$. Use this to answer the questions below. **Include units with your answers**

a) Find the velocity and acceleration function

$v(t) = 3t^2 - 12t - 15$

$a(t) = 6t - 12$

b) What is its velocity at $t = 1$ seconds?

$v(1) = 3(1)^2 - 12(1) - 15 = -24$ meters/min

c) What is its acceleration at t = 2 seconds?

$a(2) = 6(2) - 12 = 0$ meters/min^2

d) Find the average velocity of particle in [1, 2]

avg. velocity = change in position / change in time $\rightarrow \frac{x(2)-x(1)}{2-1}$

$x(2) = -45$

$x(1) = -19$

$= \frac{-45-(-19)}{2-1} =$ -26 m/min

e) When is the particle at rest?

*set $v(t) = 0$

$0 = 3(t^2 - 4t - 5)$

$0 = 3(t-5)(t+1)$

$t = 5, t = -1$

$t = 5$ secs

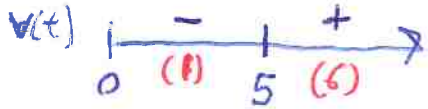
f) Find the average acceleration of particle in [1, 3]

avg. acceleration = change in velocity / change in time $\rightarrow \frac{v(3)-v(1)}{3-1} \rightarrow \frac{-24-(-24)}{2} =$ 0 m/min^2

$v(1) = -24$

$v(3) = -24$

g) When is the particle moving right? When does particle change directions? (Create Sign Line) Give justification.



particle moving right $(5, \infty)$ b/c $v(t) > 0$

particle changes direction at $t = 5$ since $v(t)$ change signs.

h) What is displacement of particle from t = 1 to t = 4? Show work.

$x(1) = -19$

$x(4) = -91$

$x(4) - x(1)$

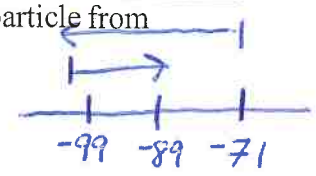
$-91 - (-19) =$ -72 meters

i) What is the total distance of particle from t =3 to t = 6? Show work.

$x(3) = -71$

$x(5) = -99$ (28)

$x(6) = -89$ (10)



$28 + 10 =$ 38 feet

j) Is the speed increasing or decreasing at t = 1? Justify.

$v(1) = -24$

$a(1) = -6$

speed is increasing at $t = 1$ since $v(t)$ and $a(t)$ have same signs.

k) Is velocity increasing or decreasing at t = 3? Justify.

$a(3) = 6$

Velocity is increasing since $a(3) > 0$