2017

2.3 Product, Quotient Rules

p.125-128
#13, 15, 21-31 odd, 69, 75, 77, 81, 91, 93, 95, 99, 101, 103, 105

*product rule: $\frac{d}{dx} f(x)g(x) = f'(x)g(x) + f(x)g'(x)$

13) $f(x) = (x^3+4x)(3x^2+2x-5)$ $\quad c=0$

$$f'(x) = (3x^2+4)(3x^2+2x-5) + (x^3+4x)(6x+2)$$

$$f'(0) = (0+4)(0+0-5) + (0+0)(0+2) = \boxed{-20}$$

Quotient Rule:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

15) $f(x) = \frac{x^2-4}{x-3}$ $\quad c=1$

$$f'(x) = \frac{(2x)(x-3)-(x^2-4)(1)}{(x-3)^2} = \frac{2x^2-6x-x^2+4}{(x-3)^2} = \frac{x^2-6x+4}{(x-3)^2}$$

$$f'(1) = \frac{1-6+4}{(1-3)^2} = \boxed{\frac{-1}{4}}$$

21) $y = \frac{6}{7x^2} = \frac{6}{7}x^{-2}$ $\quad y' = -2\cdot\frac{6}{7}x^{-3} = -\frac{12}{7}x^{-3} = \boxed{\frac{-12}{7x^3}}$

23) $y = \frac{4x^{3/2}}{x} = 4x^{3/2}x^{-1} = 4x^{1/2}$ $\quad y' = \frac{1}{2}\cdot 4x^{-1/2} = \boxed{\frac{2}{x^{1/2}} = \frac{2}{\sqrt{x}}}$

25) $f(x) = \frac{4-3x-x^2}{x^2-1}$ $\quad f'(x) = \frac{(-3-2x)(x^2-1)-(4-3x-x^2)(2x)}{(x^2-1)^2}$

$$f'(x) = \frac{-3x^2+3-2x^3+2x-8x+6x^2+2x^3}{(x^2-1)^2} = \frac{3x^2-6x+3}{(x^2-1)^2} = \frac{3(x^2-2x+1)}{(x^2-1)^2} = \frac{3(x-1)^2}{(x-1)^2(x+1)^2}$$

$$= \boxed{\frac{3}{(x+1)^2}}$$

27) $f(x) = x\left(1 - \frac{4}{x+3}\right) = x - \frac{4x}{x+3} = \frac{x(x+3)}{x+3} - \frac{4x}{x+3}$

$f(x) = \frac{x^2+3x-4x}{x+3} = \frac{x^2-x}{x+3}$ ← *Apply quotient rule

$f'(x) = \frac{(2x-1)(x+3)-(x^2-x)(1)}{(x+3)^2} = \frac{2x^2+6x-x-3-x^2+x}{(x+3)^2} = \boxed{\frac{x^2+6x-3}{(x+3)^2}}$

29) $f(x) = \frac{3x-1}{\sqrt{x}} = \frac{3x-1}{x^{1/2}} = (3x-1)x^{-1/2}$

$f(x) = 3x^{1/2} - x^{-1/2}$ | $f'(x) = \frac{1}{2}\cdot 3x^{-1/2} - \frac{-1}{2}x^{-3/2} = \frac{3}{2\sqrt{x}} + \frac{1}{2x^{3/2}}$

$= \boxed{\frac{3}{2\sqrt{x}} + \frac{1}{2\sqrt{x^3}}}$

31) $h(s) = (s^3-2)^2 = (s^3-2)(s^3-2)$

$h(s) = s^6 - 4s^3 + 4$ | $\boxed{h'(s) = 6s^5 - 12s^2}$

*Apply quotient rule

65) Find equation of tangent line $f(x) = \frac{x}{x+4}$ $(-5, 5)$

$f'(x) = \frac{1(x+4) - x(1)}{(x+4)^2} = \frac{x+4-x}{(x+4)^2} = \frac{4}{(x+4)^2}$ | $f'(-5) = \frac{4}{(-5+4)^2} = \frac{4}{(-1)^2} = 4$

point: $(-5, 5)$

slope: $m = 4$

$y - y_1 = m(x - x_1)$

$\boxed{y - 5 = 4(x+5)}$

69) Find equation of tangent line $f(x)=\frac{8}{x^2+4}$ point: $(2,1)$

$$f'(x)=\frac{0(x^2+4)-8(2x)}{(x^2+4)^2}=\frac{0-16x}{(x^2+4)^2}=\frac{-16x}{(x^2+4)^2}$$

$$f'(2)=\frac{-16(2)}{(2^2+4)^2}=\frac{-32}{8^2}=\frac{-32}{64}=-\tfrac{1}{2}$$

point: $(2,1)$
slope: $m=-\tfrac{1}{2}$

$$\boxed{y-1=-\tfrac{1}{2}(x-2)}$$

75) Horizontal tangent line: * set numerator of $f'(x)=0$, solve for x

$$f(x)=\frac{x^2}{x-1}$$

$$f'(x)=\frac{2x(x-1)-(x^2)(1)}{(x-1)^2}=\frac{2x^2-2x-x^2}{(x-1)^2}=\frac{x^2-2x}{(x-1)^2}$$

$$f'(x)=\frac{x^2-2x}{(x-1)^2}$$

set $x^2-2x=0$
$x(x-2)=0$
$x=0,2$

$f(0)=0$
$f(2)=4$

Horizontal tangents are at $(0,0)$ and $(2,4)$

77) Find equation of tangent line to $f(x)=\frac{x+1}{x-1}$ and parallel to $2y+x=6$

* set $f'(x)$ equal to slope of line

$2y+x=6$
$2y=-x+6$
$y=\frac{-1}{2}x+3$
slope: $m=-\tfrac{1}{2}$

$$f'(x)=\frac{(1)(x-1)-(x+1)(1)}{(x-1)^2}$$

$$f'(x)=\frac{\cancel{x}-1-\cancel{x}-1}{(x-1)^2}$$

$$f'(x)=\frac{-2}{(x-1)^2}$$

$$\frac{-2}{(x-1)^2}=\frac{-1}{2}$$

$-(x-1)^2=-4$
$(x-1)^2=4$
$\sqrt{(x-1)^2}=\pm\sqrt{4}$
$x-1=\pm 2$
$x=1\pm 2$
$x=-1,3$

$f(-1)=0$
$f(3)=2$

point: $(-1,0)$
slope: $m=-\tfrac{1}{2}$

$$\boxed{y-0=-\tfrac{1}{2}(x+1)}$$

point: $(3,2)$
slope: $m=-\tfrac{1}{2}$

$$\boxed{y-2=-\tfrac{1}{2}(x-3)}$$

Find 2^{nd} Derivative

95) $f(x) = \dfrac{x}{x-1}$

$$f'(x) = \frac{1(x-1) - x(1)}{(x-1)^2}$$

$$f'(x) = \frac{x-1-x}{(x-1)^2} = \boxed{\frac{-1}{(x-1)^2}}$$

$$f'(x) = \frac{-1}{x^2-2x+1}$$

$$f''(x) = \frac{\cancel{0(x^2-2x+1)} - (-1)(2x-2)}{(x^2-2x+1)^2}$$

$$f''(x) = \frac{2x-2}{(x-1)^4} = \frac{2(x-1)}{(x-1)^4} = \boxed{\frac{2}{(x-1)^3}}$$