Section 2.3 Product and Quotient Rules and Higher-Order Derivatives

1. $g(x) = (x^2 + 3)(x^2 - 4x)$

$g'(x) = (x^2 + 3)(2x - 4) + (x^2 - 4x)(2x)$

$= 2x^3 - 4x^2 + 6x - 12 + 2x^3 - 8x^2$

$= 4x^3 - 12x^2 + 6x - 12$

$= 2(2x^3 - 6x^2 + 3x - 6)$

2. $y = (3x - 4)(x^3 + 5)$

$y' = (3x - 4)(3x^2) + (x^3 + 5)(3)$

$= 9x^3 - 12x^2 + 3x^3 + 15$

$= 12x^3 - 12x^2 + 15$

3. $h(t) = \sqrt{t}(1 - t^2) = t^{1/2}(1 - t^2)$

$h'(t) = t^{1/2}(-2t) + (1 - t^2)\frac{1}{2}t^{-1/2}$

$= -2t^{3/2} + \frac{1}{2t^{1/2}} - \frac{1}{2}t^{3/2}$

$= -\frac{5}{2}t^{3/2} + \frac{1}{2t^{1/2}}$

$= \frac{1 - 5t^2}{2t^{1/2}} = \frac{1 - 5t^2}{2\sqrt{t}}$

4. $g(s) = \sqrt{s}(s^2 + 8) = s^{1/2}(s^2 + 8)$

$g'(s) = s^{1/2}(2s) + (s^2 + 8)\frac{1}{2}s^{-1/2}$

$= 2s^{3/2} + \frac{1}{2}s^{3/2} + 4s^{-1/2}$

$= \frac{5}{2}s^{3/2} + \frac{4}{s^{1/2}}$

$= \frac{5s^2 + 8}{2\sqrt{s}}$

5. $f(x) = x^3 \cos x$

$f'(x) = x^3(-\sin x) + \cos x(3x^2)$

$= 3x^2 \cos x - x^3 \sin x$

$= x^2(3 \cos x - x \sin x)$

6. $g(x) = \sqrt{x} \sin x$

$g'(x) = \sqrt{x} \cos x + \sin x\left(\frac{1}{2\sqrt{x}}\right)$

$= \sqrt{x} \cos x + \frac{1}{2\sqrt{x}} \sin x$

7. $f(x) = \frac{x}{x^2 + 1}$

$f'(x) = \frac{(x^2 + 1)(1) - x(2x)}{(x^2 + 1)^2} = \frac{1 - x^2}{(x^2 + 1)^2}$

8. $g(t) = \frac{3t^2 - 1}{2t + 5}$

$g'(t) = \frac{(2t + 5)(6t) - (3t^2 - 1)(2)}{(2t + 5)^2}$

$= \frac{12t^2 + 30t - 6t^2 + 2}{(2t + 5)^2}$

$= \frac{6t^2 + 30t + 2}{(2t + 5)^2}$

9. $h(x) = \frac{\sqrt{x}}{x^3 + 1} = \frac{x^{1/2}}{x^3 + 1}$

$h'(x) = \frac{(x^3 + 1)\frac{1}{2}x^{-1/2} - x^{1/2}(3x^2)}{(x^3 + 1)^2}$

$= \frac{x^3 + 1 - 6x^3}{2x^{1/2}(x^3 + 1)^2}$

$= \frac{1 - 5x^3}{2\sqrt{x}(x^3 + 1)^2}$

10. $f(x) = \frac{x^2}{2\sqrt{x} + 1}$

$f'(x) = \frac{(2\sqrt{x} + 1)(2x) - x^2(x^{-1/2})}{(2\sqrt{x} + 1)^2}$

$= \frac{4x^{3/2} + 2x - x^{3/2}}{(2\sqrt{x} + 1)^2}$

$= \frac{3x^{3/2} + 2x}{(2\sqrt{x} + 1)^2}$

$= \frac{x(3\sqrt{x} + 2)}{(2\sqrt{x} + 1)^2}$

11. $g(x) = \frac{\sin x}{x^2}$

$g'(x) = \frac{x^2(\cos x) - \sin x(2x)}{(x^2)^2} = \frac{x \cos x - 2 \sin x}{x^3}$

12. $f(t) = \frac{\cos t}{t^3}$

$f'(t) = \frac{t^3(-\sin t) - \cos t(3t^2)}{(t^3)^2} = -\frac{t \sin t + 3\cos t}{t^4}$

13. $f(x) = (x^3 + 4x)(3x^2 + 2x - 5)$

$f'(x) = (x^3 + 4x)(6x + 2) + (3x^2 + 2x - 5)(3x^2 + 4)$

$= 6x^4 + 24x^2 + 2x^3 + 8x + 9x^4 + 6x^3 - 15x^2 + 12x^2 + 8x - 20$

$= 15x^4 + 8x^3 + 21x^2 + 16x - 20$

$f'(0) = -20$

14. $y = (x^2 - 3x + 2)(x^3 + 1)$

$y' = (x^2 - 3x + 2)(3x^2) + (x^3 + 1)(2x - 3)$

$= 3x^4 - 9x^3 + 6x^2 + 2x^4 - 3x^3 + 2x - 3$

$= 5x^4 - 12x^3 + 6x^2 + 2x - 3$

$y'(2) = 5(2^4) - 12(2^3) + 6(2^2) + 2(2) - 3 = 9$

15. $f(x) = \frac{x^2 - 4}{x - 3}$

$f'(x) = \frac{(x - 3)(2x) - (x^2 - 4)(1)}{(x - 3)^2}$

$= \frac{2x^2 - 6x - x^2 + 4}{(x - 3)^2}$

$= \frac{x^2 - 6x + 4}{(x - 3)^2}$

$f'(1) = \frac{1 - 6 + 4}{(1 - 3)^2} = -\frac{1}{4}$

16. $f(x) = \frac{x - 4}{x + 4}$

$f'(x) = \frac{(x + 4)(1) - (x - 4)(1)}{(x + 4)^2}$

$= \frac{x + 4 - x + 4}{(x + 4)^2}$

$= \frac{8}{(x + 4)^2}$

$f'(3) = \frac{8}{(3 + 4)^2} = \frac{8}{49}$

17. $f(x) = x \cos x$

$f'(x) = (x)(-\sin x) + (\cos x)(1) = \cos x - x \sin x$

$f'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} - \frac{\pi}{4}\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{8}(4 - \pi)$

18. $f(x) = \frac{\sin x}{x}$

$f'(x) = \frac{(x)(\cos x) - (\sin x)(1)}{x^2}$

$= \frac{x \cos x - \sin x}{x^2}$

$f'\left(\frac{\pi}{6}\right) = \frac{(\pi/6)(\sqrt{3}/2) - (1/2)}{\pi^2/36}$

$= \frac{3\sqrt{3}\pi - 18}{\pi^2}$

$= \frac{3(\sqrt{3}\pi - 6)}{\pi^2}$

	Function	*Rewrite*	*Differentiate*	*Simplify*
19.	$y = \frac{x^2 + 3x}{7}$	$y = \frac{1}{7}x^2 + \frac{3}{7}x$	$y' = \frac{2}{7}x + \frac{3}{7}$	$y' = \frac{2x + 3}{7}$
20.	$y = \frac{5x^2 - 3}{4}$	$y = \frac{5}{4}x^2 - \frac{3}{4}$	$y' = \frac{10}{4}x$	$y' = \frac{5x}{2}$

Function	Rewrite	Differentiate	Simplify
21. $y = \dfrac{6}{7x^2}$	$y = \dfrac{6}{7}x^{-2}$	$y' = -\dfrac{12}{7}x^{-3}$	$y' = -\dfrac{12}{7x^3}$
22. $y = \dfrac{10}{3x^3}$	$y = \dfrac{10}{3}x^{-3}$	$y' = -\dfrac{30}{3}x^{-4}$	$y' = -\dfrac{10}{x^4}$
23. $y = \dfrac{4x^{3/2}}{x}$	$y = 4x^{1/2}, x > 0$	$y' = 2x^{-1/2}$	$y' = \dfrac{2}{\sqrt{x}}, x > 0$
24. $y = \dfrac{2x}{x^{1/3}}$	$y = 2x^{2/3}$	$y' = \dfrac{4}{3}x^{-1/3}$	$y' = \dfrac{4}{3x^{1/3}}$

25. $f(x) = \dfrac{4 - 3x - x^2}{x^2 - 1}$

$$f'(x) = \frac{(x^2 - 1)(-3 - 2x) - (4 - 3x - x^2)(2x)}{(x^2 - 1)^2}$$
$$= \frac{-3x^2 + 3 - 2x^3 + 2x - 8x + 6x^2 + 2x^3}{(x^2 - 1)^2}$$
$$= \frac{3x^2 - 6x + 3}{(x^2 - 1)^2}$$
$$= \frac{3(x^2 - 2x + 1)}{(x^2 - 1)^2}$$
$$= \frac{3(x - 1)^2}{(x - 1)^2(x + 1)^2} = \frac{3}{(x + 1)^2}, x \neq 1$$

26. $f(x) = \dfrac{x^2 + 5x + 6}{x^2 - 4}$

$$f'(x) = \frac{(x^2 - 4)(2x + 5) - (x^2 + 5x + 6)(2x)}{(x^2 - 4)^2}$$
$$= \frac{2x^3 + 5x^2 - 8x - 20 - 2x^3 - 10x^2 - 12x}{(x^2 - 4)^2}$$
$$= \frac{-5x^2 - 20x - 20}{(x^2 - 4)^2}$$
$$= \frac{-5(x^2 + 4x + 4)}{(x - 2)^2(x + 2)^2}$$
$$= \frac{-5(x + 2)^2}{(x - 2)^2(x + 2)^2}$$
$$= -\frac{5}{(x - 2)^2}, x \neq 2, -2$$

Alternate solution:

$$f(x) = \frac{x^2 + 5x + 6}{x^2 - 4}$$
$$= \frac{(x + 3)(x + 2)}{(x + 2)(x - 2)}$$
$$= \frac{x + 3}{x - 2}, \ x \neq -2$$
$$f'(x) = \frac{(x - 2)(1) - (x + 3)(1)}{(x - 2)^2}$$
$$= -\frac{5}{(x - 2)^2}$$

27. $f(x) = x\left(1 - \frac{4}{x+3}\right) = x - \frac{4x}{x+3}$

$$f'(x) = 1 - \frac{(x+3)4 - 4x(1)}{(x+3)^2}$$
$$= \frac{(x^2 + 6x + 9) - 12}{(x+3)^2}$$
$$= \frac{x^2 + 6x - 3}{(x+3)^2}$$

28. $f(x) = x^4\left[1 - \frac{2}{x+1}\right] = x^4\left[\frac{x-1}{x+1}\right]$

$$f'(x) = x^4\left[\frac{(x+1) - (x-1)}{(x+1)^2}\right] + \left[\frac{x-1}{x+1}\right](4x^3)$$
$$= x^4\left[\frac{2}{(x+1)^2}\right] + \left[\frac{x^2-1}{(x+1)^2}\right](4x^3)$$
$$= 2x^3\left[\frac{2x^2 + x - 2}{(x+1)^2}\right]$$

29. $f(x) = \frac{3x-1}{\sqrt{x}} = 3x^{1/2} - x^{-1/2}$

$$f'(x) = \frac{3}{2}x^{-1/2} + \frac{1}{2}x^{-3/2} = \frac{3x+1}{2x^{3/2}}$$

Alternate solution:

$$f(x) = \frac{3x-1}{\sqrt{x}} = \frac{3x-1}{x^{1/2}}$$
$$f'(x) = \frac{x^{1/2}(3) - (3x-1)\left(\frac{1}{2}\right)\left(x^{-1/2}\right)}{x}$$
$$= \frac{\frac{1}{2}x^{-1/2}(3x+1)}{x}$$
$$= \frac{3x+1}{2x^{3/2}}$$

30. $f(x) = \sqrt[3]{x}\left(\sqrt{x} + 3\right) = x^{1/3}\left(x^{1/2} + 3\right)$

$$f'(x) = x^{1/3}\left(\frac{1}{2}x^{-1/2}\right) + \left(x^{1/2} + 3\right)\left(\frac{1}{3}x^{-2/3}\right)$$
$$= \frac{5}{6}x^{-1/6} + x^{-2/3}$$
$$= \frac{5}{6x^{1/6}} + \frac{1}{x^{2/3}}$$

Alternate solution:

$$f(x) = \sqrt[3]{x}\left(\sqrt{x} + 3\right)$$
$$= x^{5/6} + 3x^{1/3}$$
$$f'(x) = \frac{5}{6}x^{-1/6} + x^{-2/3}$$
$$= \frac{5}{6x^{1/6}} + \frac{1}{x^{2/3}}$$

31. $h(s) = \left(s^3 - 2\right)^2 = s^6 - 4s^3 + 4$

$$h'(s) = 6s^5 - 12s^2 = 6s^2\left(s^3 - 2\right)$$

32. $h(x) = \left(x^2 + 3\right)^3 = x^6 + 9x^4 + 27x^2 + 27$

$$h'(x) = 6x^5 + 36x^3 + 54x$$
$$= 6x\left(x^4 + 6x^2 + 9\right)$$
$$= 6x\left(x^2 + 3\right)^2$$

33. $f(x) = \frac{2 - (1/x)}{x - 3} = \frac{2x - 1}{x(x-3)} = \frac{2x-1}{x^2 - 3x}$

$$f'(x) = \frac{\left(x^2 - 3x\right)2 - (2x-1)(2x-3)}{\left(x^2 - 3x\right)^2}$$
$$= \frac{2x^2 - 6x - 4x^2 + 8x - 3}{\left(x^2 - 3x\right)^2}$$
$$= \frac{-2x^2 + 2x - 3}{\left(x^2 - 3x\right)^2} = -\frac{2x^2 - 2x + 3}{x^2(x-3)^2}$$

34. $g(x) = x^2\left(\frac{2}{x} - \frac{1}{x+1}\right) = 2x - \frac{x^2}{x+1}$

$$g'(x) = 2 - \frac{(x+1)2x - x^2(1)}{(x+1)^2} = \frac{2\left(x^2 + 2x + 1\right) - x^2 - 2x}{(x+1)^2} = \frac{x^2 + 2x + 2}{(x+1)^2}$$

35. $f(x) = \left(2x^3 + 5x\right)(x-3)(x+2)$

$$f'(x) = \left(6x^2 + 5\right)(x-3)(x+2) + \left(2x^3 + 5x\right)(1)(x+2) + \left(2x^3 + 5x\right)(x-3)(1)$$
$$= \left(6x^2 + 5\right)\left(x^2 - x - 6\right) + \left(2x^3 + 5x\right)(x+2) + \left(2x^3 + 5x\right)(x-3)$$
$$= \left(6x^4 + 5x^2 - 6x^3 - 5x - 36x^2 - 30\right) + \left(2x^4 + 4x^3 + 5x^2 + 10x\right) + \left(2x^4 + 5x^2 - 6x^3 - 15x\right)$$
$$= 10x^4 - 8x^3 - 21x^2 - 10x - 30$$

Note: You could simplify first: $f(x) = \left(2x^3 + 5x\right)\left(x^2 - x - 6\right)$

36. $f(x) = (x^3 - x)(x^2 + 2)(x^2 + x - 1)$

$$\begin{aligned} f'(x) &= (3x^2 - 1)(x^2 + 2)(x^2 + x - 1) + (x^3 - x)(2x)(x^2 + x - 1) + (x^3 - x)(x^2 + 2)(2x + 1) \\ &= (3x^4 + 5x^2 - 2)(x^2 + x - 1) + (2x^4 - 2x^2)(x^2 + x - 1) + (x^5 + x^3 - 2x)(2x + 1) \\ &= (3x^6 + 5x^4 - 2x^2 + 3x^5 + 5x^3 - 2x - 3x^4 - 5x^2 + 2) \\ &\quad + (2x^6 - 2x^4 + 2x^5 - 2x^3 - 2x^4 + 2x^2) \\ &\quad + (2x^6 + 2x^4 - 4x^2 + x^5 + x^3 - 2x) \\ &= 7x^6 + 6x^5 + 4x^3 - 9x^2 - 4x + 2 \end{aligned}$$

37. $f(x) = \dfrac{x^2 + c^2}{x^2 - c^2}$

$$\begin{aligned} f'(x) &= \frac{(x^2 - c^2)(2x) - (x^2 + c^2)(2x)}{(x^2 - c^2)^2} \\ &= -\frac{4xc^2}{(x^2 - c^2)^2} \end{aligned}$$

38. $f(x) = \dfrac{c^2 - x^2}{c^2 + x^2}$

$$\begin{aligned} f'(x) &= \frac{(c^2 + x^2)(-2x) - (c^2 - x^2)(2x)}{(c^2 + x^2)^2} \\ &= -\frac{4xc^2}{(c^2 + x^2)^2} \end{aligned}$$

39. $f(t) = t^2 \sin t$

$f'(t) = t^2 \cos t + 2t \sin t = t(t \cos t + 2 \sin t)$

40. $f(\theta) = (\theta + 1)\cos\theta$

$$\begin{aligned} f'(\theta) &= (\theta + 1)(-\sin\theta) + (\cos\theta)(1) \\ &= \cos\theta - (\theta + 1)\sin\theta \end{aligned}$$

41. $f(t) = \dfrac{\cos t}{t}$

$f'(t) = \dfrac{-t \sin t - \cos t}{t^2} = -\dfrac{t \sin t + \cos t}{t^2}$

42. $f(x) = \dfrac{\sin x}{x^3}$

$f'(x) = \dfrac{x^3 \cos x - \sin x(3x^2)}{(x^3)^2} = \dfrac{x \cos x - 3 \sin x}{x^4}$

43. $f(x) = -x + \tan x$

$f'(x) = -1 + \sec^2 x = \tan^2 x$

44. $y = x + \cot x$

$y' = 1 - \csc^2 x = -\cot^2 x$

45. $g(t) = \sqrt[4]{t} + 6 \csc t = t^{1/4} + 6 \csc t$

$$\begin{aligned} g'(t) &= \frac{1}{4}t^{-3/4} - 6 \csc t \cot t \\ &= \frac{1}{4t^{3/4}} - 6 \csc t \cot t \end{aligned}$$

46. $h(x) = \dfrac{1}{x} - 12 \sec x = x^{-1} - 12 \sec x$

$$\begin{aligned} h'(x) &= -x^{-2} - 12 \sec x \tan x \\ &= \frac{-1}{x^2} - 12 \sec x \tan x \end{aligned}$$

47. $y = \dfrac{3(1 - \sin x)}{2 \cos x} = \dfrac{3 - 3 \sin x}{2 \cos x}$

$$\begin{aligned} y' &= \frac{(-3 \cos x)(2 \cos x) - (3 - 3 \sin x)(-2 \sin x)}{(2 \cos x)^2} \\ &= \frac{-6 \cos^2 x + 6 \sin x - 6 \sin^2 x}{4 \cos^2 x} \\ &= \frac{3}{2}(-1 + \tan x \sec x - \tan^2 x) \\ &= \frac{3}{2} \sec x(\tan x - \sec x) \end{aligned}$$

48. $y = \dfrac{\sec x}{x}$

$$\begin{aligned} y' &= \frac{x \sec x \tan x - \sec x}{x^2} \\ &= \frac{\sec x(x \tan x - 1)}{x^2} \end{aligned}$$

49. $y = -\csc x - \sin x$

$$\begin{aligned} y' &= \csc x \cot x - \cos x \\ &= \frac{\cos x}{\sin^2 x} - \cos x \\ &= \cos x(\csc^2 x - 1) \\ &= \cos x \cot^2 x \end{aligned}$$

50. $y = x \sin x + \cos x$

$y' = x \cos x + \sin x - \sin x = x \cos x$

51. $f(x) = x^2 \tan x$

$f'(x) = x^2 \sec^2 x + 2x \tan x = x(x \sec^2 x + 2 \tan x)$

52. $f(x) = \sin x \cos x$

$f'(x) = \sin x(-\sin x) + \cos x(\cos x) = \cos 2x$

53. $y = 2x \sin x + x^2 \cos x$

$y' = 2x \cos x + 2 \sin x + x^2(-\sin x) + 2x \cos x$

$= 4x \cos x + (2 - x^2) \sin x$

54. $h(\theta) = 5\theta \sec \theta + \theta \tan \theta$

$h'(\theta) = 5\theta \sec \theta \tan \theta + 5 \sec \theta + \theta \sec^2 \theta + \tan \theta$

55. $g(x) = \left(\dfrac{x+1}{x+2}\right)(2x - 5)$

$g'(x) = \left(\dfrac{x+1}{x+2}\right)(2) + (2x - 5)\left[\dfrac{(x+2)(1) - (x+1)(1)}{(x+2)^2}\right]$

$= \dfrac{2x^2 + 8x - 1}{(x+2)^2}$

(Form of answer may vary.)

56. $f(x) = \left(\dfrac{x^2 - x - 3}{x^2 + 1}\right)(x^2 + x + 1)$

$f'(x) = 2\dfrac{x^5 + 2x^3 + 2x^2 - 2}{(x^2 + 1)^2}$

(Form of answer may vary.)

57. $g(\theta) = \dfrac{\theta}{1 - \sin \theta}$

$g'(\theta) = \dfrac{1 - \sin \theta + \theta \cos \theta}{(1 - \sin \theta)^2}$

(Form of answer may vary.)

58. $f(\theta) = \dfrac{\sin \theta}{1 - \cos \theta}$

$f'(\theta) = \dfrac{1}{\cos \theta - 1} = \dfrac{\cos \theta - 1}{(1 - \cos \theta)^2}$

(Form of answer may vary.)

59. $y = \dfrac{1 + \csc x}{1 - \csc x}$

$y' = \dfrac{(1 - \csc x)(-\csc x \cot x) - (1 + \csc x)(\csc x \cot x)}{(1 - \csc x)^2} = \dfrac{-2 \csc x \cot x}{(1 - \csc x)^2}$

$y'\left(\dfrac{\pi}{6}\right) = \dfrac{-2(2)(\sqrt{3})}{(1 - 2)^2} = -4\sqrt{3}$

60. $f(x) = \tan x \cot x = 1$

$f'(x) = 0$

$f'(1) = 0$

61. $h(t) = \dfrac{\sec t}{t}$

$h'(t) = \dfrac{t(\sec t \tan t) - (\sec t)(1)}{t^2} = \dfrac{\sec t(t \tan t - 1)}{t^2}$

$h'(\pi) = \dfrac{\sec \pi(\pi \tan \pi - 1)}{\pi^2} = \dfrac{1}{\pi^2}$

62. $f(x) = \sin x(\sin x + \cos x)$

$f'(x) = \sin x(\cos x - \sin x) + (\sin x + \cos x)\cos x$

$= \sin x \cos x - \sin^2 x + \sin x \cos x + \cos^2 x$

$= \sin 2x + \cos 2x$

$f'\left(\dfrac{\pi}{4}\right) = \sin \dfrac{\pi}{2} + \cos \dfrac{\pi}{2} = 1$

63. (a) $f(x) = (x^3 + 4x - 1)(x - 2), \quad (1, -4)$

$f'(x) = (x^3 + 4x - 1)(1) + (x - 2)(3x^2 + 4)$

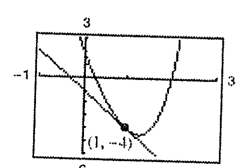
$= x^3 + 4x - 1 + 3x^3 - 6x^2 + 4x - 8$

$= 4x^3 - 6x^2 + 8x - 9$

$f'(1) = -3$; Slope at $(1, -4)$

Tangent line: $y + 4 = -3(x - 1) \Rightarrow y = -3x - 1$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = -3$ at $(1, -4)$.

64. (a) $f(x) = (x-2)(x^2+4),\ (1, -5)$

$f'(x) = (x-2)(2x) + (x^2+4)(1)$

$= 2x^2 - 4x + x^2 + 4$

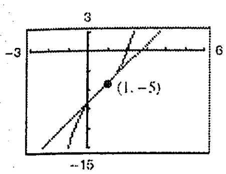
$= 3x^2 - 4x + 4$

$f'(1) = -3$; Slope at $(1, -5)$

Tangent line:

$y - (-5) = 3(x-1) \Rightarrow y = 3x - 8$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = 3$ at $(1, -5)$.

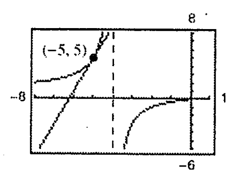
65. (a) $f(x) = \dfrac{x}{x+4},\quad (-5, 5)$

$f'(x) = \dfrac{(x+4)(1) - x(1)}{(x+4)^2} = \dfrac{4}{(x+4)^2}$

$f'(-5) = \dfrac{4}{(-5+4)^2} = 4$; Slope at $(-5, 5)$

Tangent line: $y - 5 = 4(x+5) \Rightarrow y = 4x + 25$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = 4$ at $(-5, 5)$.

66. (a) $f(x) = \dfrac{x+3}{x-3},\quad (4, 7)$

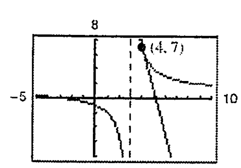
$f'(x) = \dfrac{(x-3)(1) - (x+3)(1)}{(x-3)^2} = -\dfrac{6}{(x-3)^2}$

$f'(4) = \dfrac{-6}{1} = -6$; Slope at $(4, 7)$

Tangent line:

$y - 7 = -6(x-4) \Rightarrow y = -6x + 31$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = -6$ at $(4, 7)$.

67. (a) $f(x) = \tan x,\quad \left(\dfrac{\pi}{4}, 1\right)$

$f'(x) = \sec^2 x$

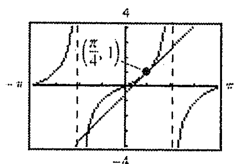
$f'\left(\dfrac{\pi}{4}\right) = 2$; Slope at $\left(\dfrac{\pi}{4}, 1\right)$

Tangent line: $y - 1 = 2\left(x - \dfrac{\pi}{4}\right)$

$y - 1 = 2x - \dfrac{\pi}{2}$

$4x - 2y - \pi + 2 = 0$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = 2$ at $\left(\dfrac{\pi}{4}, 1\right)$.

68. (a) $f(x) = \sec x,\quad \left(\dfrac{\pi}{3}, 2\right)$

$f'(x) = \sec x \tan x$

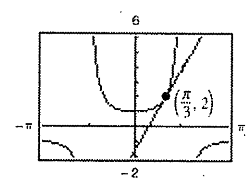
$f'\left(\dfrac{\pi}{3}\right) = 2\sqrt{3}$; Slope at $\left(\dfrac{\pi}{3}, 2\right)$

Tangent line:

$y - 2 = 2\sqrt{3}\left(x - \dfrac{\pi}{3}\right)$

$6\sqrt{3}x - 3y + 6 - 2\sqrt{3}\pi = 0$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = 2\sqrt{3}$ at $\left(\dfrac{\pi}{3}, 2\right)$.

69. $f(x) = \dfrac{8}{x^2+4};\quad (2, 1)$

$f'(x) = \dfrac{(x^2+4)(0) - 8(2x)}{(x^2+4)^2} = \dfrac{-16x}{(x^2+4)^2}$

$f'(2) = \dfrac{-16(2)}{(4+4)^2} = -\dfrac{1}{2}$

$y - 1 = -\dfrac{1}{2}(x-2)$

$y = -\dfrac{1}{2}x + 2$

$2y + x - 4 = 0$

70. $f(x) = \dfrac{27}{x^2 + 9};\ \left(-3, \dfrac{3}{2}\right)$

$$f'(x) = \frac{(x^2 + 9)(0) - 27(2x)}{(x^2 + 9)^2} = \frac{-54x}{(x^2 + 9)^2}$$

$$f'(-3) = \frac{-54(-3)}{(9 + 9)^2} = \frac{1}{2}$$

$$y - \frac{3}{2} = \frac{1}{2}(x + 3)$$

$$y = \frac{1}{2}x + 3$$

$$2y - x - 6 = 0$$

71. $f(x) = \dfrac{16x}{x^2 + 16};\ \left(-2, -\dfrac{8}{5}\right)$

$$f'(x) = \frac{(x^2 + 16)(16) - 16x(2x)}{(x^2 + 16)^2} = \frac{256 - 16x^2}{(x^2 + 16)^2}$$

$$f'(-2) = \frac{256 - 16(4)}{20^2} = \frac{12}{25}$$

$$y + \frac{8}{5} = \frac{12}{25}(x + 2)$$

$$y = \frac{12}{25}x - \frac{16}{25}$$

$$25y - 12x + 16 = 0$$

72. $f(x) = \dfrac{4x}{x^2 + 6};\ \left(2, \dfrac{4}{5}\right)$

$$f'(x) = \frac{(x^2 + 6)(4) - 4x(2x)}{(x^2 + 6)^2} = \frac{24 - 4x^2}{(x^2 + 6)^2}$$

$$f'(2) = \frac{24 - 16}{10^2} = \frac{2}{25}$$

$$y - \frac{4}{5} = \frac{2}{25}(x - 2)$$

$$y = \frac{2}{25}x + \frac{16}{25}$$

$$25y - 2x - 16 = 0$$

73. $f(x) = \dfrac{2x - 1}{x^2} = 2x^{-1} - x^{-2}$

$$f'(x) = -2x^{-2} + 2x^{-3} = \frac{2(-x + 1)}{x^3}$$

$f'(x) = 0$ when $x = 1$, and $f(1) = 1$.

Horizontal tangent at $(1, 1)$.

74. $f(x) = \dfrac{x^2}{x^2 + 1}$

$$f'(x) = \frac{(x^2 + 1)(2x) - (x^2)(2x)}{(x^2 + 1)^2}$$

$$= \frac{2x}{(x^2 + 1)^2}$$

$f'(x) = 0$ when $x = 0$.

Horizontal tangent is at $(0, 0)$.

75. $f(x) = \dfrac{x^2}{x - 1}$

$$f'(x) = \frac{(x - 1)(2x) - x^2(1)}{(x - 1)^2}$$

$$= \frac{x^2 - 2x}{(x - 1)^2} = \frac{x(x - 2)}{(x - 1)^2}$$

$f'(x) = 0$ when $x = 0$ or $x = 2$.

Horizontal tangents are at $(0, 0)$ and $(2, 4)$.

76. $f(x) = \dfrac{x - 4}{x^2 - 7}$

$$f'(x) = \frac{(x^2 - 7)(1) - (x - 4)(2x)}{(x^2 - 7)^2}$$

$$= \frac{x^2 - 7 - 2x^2 + 8x}{(x^2 - 7)^2}$$

$$= -\frac{x^2 - 8x + 7}{(x^2 - 7)^2} = -\frac{(x - 7)(x - 1)}{(x^2 - 7)^2}$$

$f'(x) = 0$ for $x = 1, 7$; $f(1) = \dfrac{1}{2}$, $f(7) = \dfrac{1}{14}$

f has horizontal tangents at $\left(1, \dfrac{1}{2}\right)$ and $\left(7, \dfrac{1}{14}\right)$.

77. $f(x) = \dfrac{x+1}{x-1}$

$f'(x) = \dfrac{(x-1)-(x+1)}{(x-1)^2} = \dfrac{-2}{(x-1)^2}$

$2y + x = 6 \Rightarrow y = -\frac{1}{2}x + 3$; Slope: $-\frac{1}{2}$

$\dfrac{-2}{(x-1)^2} = -\dfrac{1}{2}$

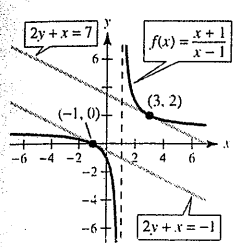
$(x-1)^2 = 4$

$x - 1 = \pm 2$

$x = -1, 3;\ f(-1) = 0,\ f(3) = 2$

$y - 0 = -\frac{1}{2}(x+1) \Rightarrow y = -\frac{1}{2}x - \frac{1}{2}$

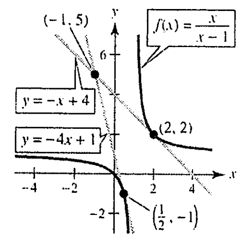
$y - 2 = -\frac{1}{2}(x-3) \Rightarrow y = -\frac{1}{2}x + \frac{7}{2}$



78. $f(x) = \dfrac{x}{x-1}$

$f'(x) = \dfrac{(x-1)-x}{(x-1)^2} = \dfrac{-1}{(x-1)^2}$

Let $(x, y) = (x, x/(x-1))$ be a point of tangency on the graph of f.



$\dfrac{5 - (x/(x-1))}{-1-x} = \dfrac{-1}{(x-1)^2}$

$\dfrac{4x-5}{(x-1)(x+1)} = \dfrac{1}{(x-1)^2}$

$(4x-5)(x-1) = x + 1$

$4x^2 - 10x + 4 = 0$

$(x-2)(2x-1) = 0 \Rightarrow x = \frac{1}{2}, 2$

$f\left(\frac{1}{2}\right) = -1,\ f(2) = 2;\ f'\left(\frac{1}{2}\right) = -4,\ f'(2) = -1$

Two tangent lines:

$y + 1 = -4\left(x - \frac{1}{2}\right) \Rightarrow y = -4x + 1$

$y - 2 = -1(x-2) \Rightarrow y = -x + 4$

79. $f'(x) = \dfrac{(x+2)3 - 3x(1)}{(x+2)^2} = \dfrac{6}{(x+2)^2}$

$g'(x) = \dfrac{(x+2)5 - (5x+4)(1)}{(x+2)^2} = \dfrac{6}{(x+2)^2}$

$g(x) = \dfrac{5x+4}{(x+2)} = \dfrac{3x}{(x+2)} + \dfrac{2x+4}{(x+2)} = f(x) + 2$

f and g differ by a constant.

80. $f'(x) = \dfrac{x(\cos x - 3) - (\sin x - 3x)(1)}{x^2} = \dfrac{x\cos x - \sin x}{x^2}$

$g'(x) = \dfrac{x(\cos x + 2) - (\sin x + 2x)(1)}{x^2} = \dfrac{x\cos x - \sin x}{x^2}$

$g(x) = \dfrac{\sin x + 2x}{x} = \dfrac{\sin x - 3x + 5x}{x} = f(x) + 5$

f and g differ by a constant.

81. (a) $p'(x) = f'(x)g(x) + f(x)g'(x)$

$$p'(1) = f'(1)g(1) + f(1)g'(1) = 1(4) + 6\left(-\frac{1}{2}\right) = 1$$

(b) $$q'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

$$q'(4) = \frac{3(-1) - 7(0)}{3^2} = -\frac{1}{3}$$

82. (a) $p'(x) = f'(x)g(x) + f(x)g'(x)$

$$p'(4) = \frac{1}{2}(8) + 1(0) = 4$$

(b) $$q'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

$$q'(7) = \frac{4(2) - 4(-1)}{4^2} = \frac{12}{16} = \frac{3}{4}$$

83. Area $= A(t) = (6t + 5)\sqrt{t} = 6t^{3/2} + 5t^{1/2}$

$$A'(t) = 9t^{1/2} + \frac{5}{2}t^{-1/2} = \frac{18t + 5}{2\sqrt{t}} \text{ cm}^2/\text{sec}$$

84. $V = \pi r^2 h = \pi(t + 2)\left(\frac{1}{2}\sqrt{t}\right) = \frac{1}{2}\left(t^{3/2} + 2t^{1/2}\right)\pi$

$$V'(t) = \frac{1}{2}\left(\frac{3}{2}t^{1/2} + t^{-1/2}\right)\pi = \frac{3t + 2}{4t^{1/2}}\pi \text{ in.}^3/\text{sec}$$

85. $C = 100\left(\frac{200}{x^2} + \frac{x}{x + 30}\right),\ 1 \le x$

$$\frac{dC}{dx} = 100\left(-\frac{400}{x^3} + \frac{30}{(x + 30)^2}\right)$$

(a) When $x = 10$: $\frac{dC}{dx} = -\$38.13$ thousand/100 components

(b) When $x = 15$: $\frac{dC}{dx} = -\$10.37$ thousand/100 components

(c) When $x = 20$: $\frac{dC}{dx} = -\$3.80$ thousand/100 components

As the order size increases, the cost per item decreases.

86. $P(t) = 500\left[1 + \frac{4t}{50 + t^2}\right]$

$$P'(t) = 500\left[\frac{(50 + t^2)(4) - (4t)(2t)}{(50 + t^2)^2}\right] = 500\left[\frac{200 - 4t^2}{(50 + t^2)^2}\right] = 2000\left[\frac{50 - t^2}{(50 + t^2)^2}\right]$$

$P'(2) \approx 31.55$ bacteria/h

87. (a) $\sec x = \frac{1}{\cos x}$

$$\frac{d}{dx}[\sec x] = \frac{d}{dx}\left[\frac{1}{\cos x}\right] = \frac{(\cos x)(0) - (1)(-\sin x)}{(\cos x)^2} = \frac{\sin x}{\cos x \cos x} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = \sec x \tan x$$

(b) $\csc x = \frac{1}{\sin x}$

$$\frac{d}{dx}[\csc x] = \frac{d}{dx}\left[\frac{1}{\sin x}\right] = \frac{(\sin x)(0) - (1)(\cos x)}{(\sin x)^2} = -\frac{\cos x}{\sin x \sin x} = -\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} = -\csc x \cot x$$

(c) $\cot x = \frac{\cos x}{\sin x}$

$$\frac{d}{dx}[\cot x] = \frac{d}{dx}\left[\frac{\cos x}{\sin x}\right] = \frac{\sin x(-\sin x) - (\cos x)(\cos x)}{(\sin x)^2} = -\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x} = -\csc^2 x$$

88. $f(x) = \sec x$

$g(x) = \csc x, \ [0, 2\pi)$

$f'(x) = g'(x)$

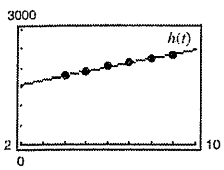
$$\sec x \tan x = -\csc x \cot x \Rightarrow \frac{\sec x \tan x}{\csc x \cot x} = -1 \Rightarrow \frac{\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}}{\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}} = -1 \Rightarrow \frac{\sin^3 x}{\cos^3 x} = -1 \Rightarrow \tan^3 x = -1 \Rightarrow \tan x = -1$$

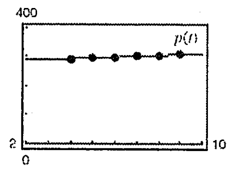
$$x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

89. (a) $h(t) = 112.4t + 1332$

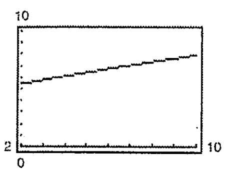
$p(t) = 2.9t + 282$

(b)





(c) $A = \dfrac{112.4t + 1332}{2.9t + 282}$



A represents the average health care expenses per person (in thousands of dollars).

(d) $A'(t) \approx \dfrac{3407.5}{(t + 98.53)^2} \approx \dfrac{27{,}834}{8.41t^2 + 1635.6t + 79{,}524}$

$A'(t)$ represents the rate of change of the average health care expenses per person per year t.

90. (a) $\sin \theta = \dfrac{r}{r + h}$

$r + h = r \csc \theta$

$h = r \csc \theta - r = r(\csc \theta - 1)$

(b) $h'(\theta) = r(-\csc \theta \cdot \cot \theta)$

$h'(30^\circ) = h'\left(\dfrac{\pi}{6}\right)$

$= -3960\left(2 \cdot \sqrt{3}\right) = -7920\sqrt{3}$ mi/rad

91. $f(x) = x^4 + 2x^3 - 3x^2 - x$

$f'(x) = 4x^3 + 6x^2 - 6x - 1$

$f''(x) = 12x^2 + 12x - 6$

92. $f(x) = 4x^5 - 2x^3 + 5x^2$

$f'(x) = 20x^4 - 6x^2 + 10x$

$f''(x) = 80x^3 - 12x + 10$

93. $f(x) = 4x^{3/2}$

$f'(x) = 6x^{1/2}$

$f''(x) = 3x^{-1/2} = \dfrac{3}{\sqrt{x}}$

94. $f(x) = x^2 + 3x^{-3}$

$f'(x) = 2x - 9x^{-4}$

$f''(x) = 2 + 36x^{-5} = 2 + \dfrac{36}{x^5}$

95. $f(x) = \dfrac{x}{x - 1}$

$f'(x) = \dfrac{(x - 1)(1) - x(1)}{(x - 1)^2} = \dfrac{-1}{(x - 1)^2}$

$f''(x) = \dfrac{2}{(x - 1)^3}$

96. $f(x) = \dfrac{x^2 + 3x}{x - 4}$

$$f'(x) = \frac{(x-4)(2x+3) - (x^2+3x)(1)}{(x-4)^2}$$

$$= \frac{2x^2 - 5x - 12 - x^2 - 3x}{(x-4)^2} = \frac{x^2 - 8x - 12}{x^2 - 8x + 16}$$

$$f''(x) = \frac{(x-4)^2(2x-8) - (x^2 - 8x - 12)(2x-8)}{(x-4)^4}$$

$$= \frac{(x-4)\left[(x-4)(2x-8) - 2(x^2 - 8x - 12)\right]}{(x-4)^4}$$

$$= \frac{(x-4)(2x-8) - 2(x^2 - 8x - 12)}{(x-4)^3}$$

$$= \frac{2x^2 - 16x + 32 - 2x^2 + 16x + 24}{(x-4)^3}$$

$$= \frac{56}{(x-4)^3}$$

97. $f(x) = x \sin x$

$f'(x) = x \cos x + \sin x$

$f''(x) = x(-\sin x) + \cos x + \cos x$

$= -x \sin x + 2 \cos x$

98. $f(x) = \sec x$

$f'(x) = \sec x \tan x$

$f''(x) = \sec x(\sec^2 x) + \tan x(\sec x \tan x)$

$= \sec x(\sec^2 x + \tan^2 x)$

99. $f'(x) = x^2$

$f''(x) = 2x$

100. $f''(x) = 2 - 2x^{-1}$

$f'''(x) = 2x^{-2} = \dfrac{2}{x^2}$

101. $f'''(x) = 2\sqrt{x}$

$f^{(4)}(x) = \dfrac{1}{2}(2)x^{-1/2} = \dfrac{1}{\sqrt{x}}$

102. $f^{(4)}(x) = 2x + 1$

$f^{(5)}(x) = 2$

$f^{(6)}(x) = 0$

103. $f(x) = 2g(x) + h(x)$

$f'(x) = 2g'(x) + h'(x)$

$f'(2) = 2g'(2) + h'(2)$

$= 2(-2) + 4$

$= 0$

104. $f(x) = 4 - h(x)$

$f'(x) = -h'(x)$

$f'(2) = -h'(2) = -4$

105. $f(x) = \dfrac{g(x)}{h(x)}$

$$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{\left[h(x)\right]^2}$$

$$f'(2) = \frac{h(2)g'(2) - g(2)h'(2)}{\left[h(2)\right]^2}$$

$$= \frac{(-1)(-2) - (3)(4)}{(-1)^2}$$

$= -10$

106. $f(x) = g(x)h(x)$

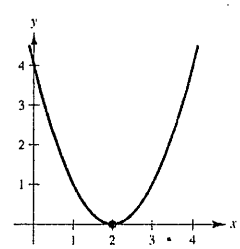
$f'(x) = g(x)h'(x) + h(x)g'(x)$

$f'(2) = g(2)h'(2) + h(2)g'(2)$

$= (3)(4) + (-1)(-2)$

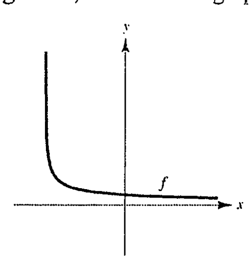
$= 14$

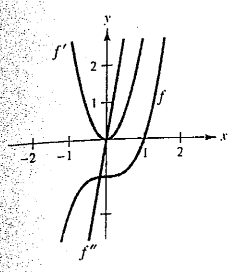
107. The graph of a differentiable function f such that $f(2) = 0$, $f' < 0$ for $-\infty < x < 2$, and $f' > 0$ for $2 < x < \infty$ would, in general, look like the graph below.



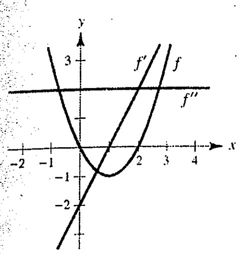
One such function is $f(x) = (x - 2)^2$.

108. The graph of a differentiable function f such that $f > 0$ and $f' < 0$ for all real numbers x would, in general, look like the graph below.

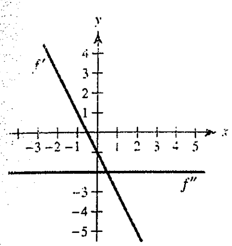


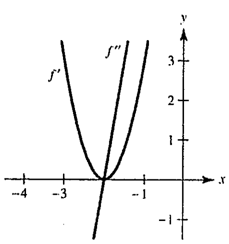
109. 

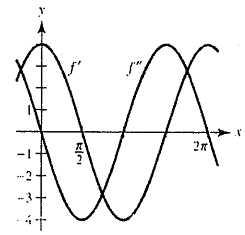
It appears that f is cubic, so f' would be quadratic and f'' would be linear.

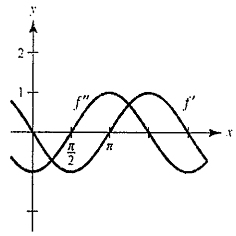
110. 

It appears that f is quadratic so f' would be linear and f'' would be constant.

111. 

112. 

113. 

114. 

115. $v(t) = 36 - t^2, 0 \le t \le 6$

$a(t) = v'(t) = -2t$

$v(3) = 27$ m/sec

$a(3) = -6$ m/sec^2

The speed of the object is decreasing.

116. $v(t) = \dfrac{100t}{2t + 15}$

$$a(t) = v'(t) = \frac{(2t + 15)(100) - (100t)(2)}{(2t + 15)^2} = \frac{1500}{(2t + 15)^2}$$

(a) $a(5) = \dfrac{1500}{[2(5) + 15]^2} = 2.4$ ft/sec^2

(b) $a(10) = \dfrac{1500}{[2(10) + 15]^2} \approx 1.2$ ft/sec^2

(c) $a(20) = \dfrac{1500}{[2(20) + 15]^2} \approx 0.5$ ft/sec^2

117. $s(t) = -8.25t^2 + 66t$

$v(t) = s'(t) = 16.50t + 66$

$a(t) = v'(t) = -16.50$

t(sec)	0	1	2	3	4
$s(t)$ (ft)	0	57.75	99	123.75	132
$v(t) = s'(t)$ (ft/sec)	66	49.5	33	16.5	0
$a(t) = v'(t)$ (ft/sec^2)	−16.5	−16.5	−16.5	−16:5	−16.5

Average velocity on:

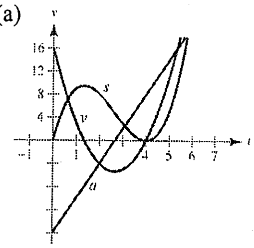
$[0, 1]$ is $\dfrac{57.75 - 0}{1 - 0} = 57.75$

$[1, 2]$ is $\dfrac{99 - 57.75}{2 - 1} = 41.25$

$[2, 3]$ is $\dfrac{123.75 - 99}{3 - 2} = 24.75$

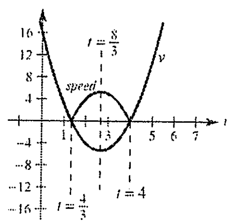
$[3, 4]$ is $\dfrac{132 - 123.75}{4 - 3} = 8.25$

118. (a)



s position function
v velocity function
a acceleration function

(b) The speed of the particle is the absolute value of its velocity. So, the particle's speed is slowing down on the intervals $(0, 4/3)$ and $(8/3, 4)$ and it speeds up on the intervals $(4/3, 8/3)$ and $(4, 6)$.



119. $f(x) = x^n$

$f^{(n)}(x) = n(n-1)(n-2)\cdots(2)(1) = n!$

Note: $n! = n(n-1)\cdots 3\cdot 2\cdot 1$ (read "n factorial")

120. $f(x) = \dfrac{1}{x}$

$$f^{(n)}(x) = \frac{(-1)^n(n)(n-1)(n-2)\cdots(2)(1)}{x^{n+1}} = \frac{(-1)^n n!}{x^{n+1}}$$

121. $f(x) = g(x)h(x)$

(a)
$$\begin{aligned}
f'(x) &= g(x)h'(x) + h(x)g'(x)\\
f''(x) &= g(x)h''(x) + g'(x)h'(x) + h(x)g''(x) + h'(x)g'(x)\\
&= g(x)h''(x) + 2g'(x)h'(x) + h(x)g''(x)\\
f'''(x) &= g(x)h'''(x) + g'(x)h''(x) + 2g'(x)h''(x) + 2g''(x)h'(x) + h(x)g'''(x) + h'(x)g''(x)\\
&= g(x)h'''(x) + 3g'(x)h''(x) + 3g''(x)h'(x) + g'''(x)h(x)\\
f^{(4)}(x) &= g(x)h^{(4)}(x) + g'(x)h'''(x) + 3g'(x)h'''(x) + 3g''(x)h''(x) + 3g''(x)h''(x) + 3g'''(x)h'(x)\\
&\quad + g'''(x)h'(x) + g^{(4)}(x)h(x)\\
&= g(x)h^{(4)}(x) + 4g'(x)h'''(x) + 6g''(x)h''(x) + 4g'''(x)h'(x) + g^{(4)}(x)h(x)
\end{aligned}$$

(b)
$$\begin{aligned}
f^{(n)}(x) &= g(x)h^{(n)}(x) + \frac{n(n-1)(n-2)\cdots(2)(1)}{1[(n-1)(n-2)\cdots(2)(1)]}g'(x)h^{(n-1)}(x) + \frac{n(n-1)(n-2)\cdots(2)(1)}{(2)(1)[(n-2)(n-3)\cdots(2)(1)]}g''(x)h^{(n-2)}(x)\\
&\quad + \frac{n(n-1)(n-2)\cdots(2)(1)}{(3)(2)(1)[(n-3)(n-4)\cdots(2)(1)]}g'''(x)h^{(n-3)}(x) + \cdots\\
&\quad + \frac{n(n-1)(n-2)\cdots(2)(1)}{[(n-1)(n-2)\cdots(2)(1)](1)}g^{(n-1)}(x)h'(x) + g^{(n)}(x)h(x)\\
&= g(x)h^{(n)}(x) + \frac{n!}{1!(n-1)!}g'(x)h^{(n-1)}(x) + \frac{n!}{2!(n-2)!}g''(x)h^{(n-2)}(x) + \cdots\\
&\quad + \frac{n!}{(n-1)!1!}g^{(n-1)}(x)h'(x) + g^{(n)}(x)h(x)
\end{aligned}$$

Note: $n! = n(n-1)\cdots 3\cdot 2\cdot 1$ (read "n factorial")

122. $[xf(x)]' = xf'(x) + f(x)$

$[xf(x)]'' = xf''(x) + f'(x) + f'(x) = xf''(x) + 2f'(x)$

$[xf(x)]''' = xf'''(x) + f''(x) + 2f''(x) = xf'''(x) + 3f''(x)$

In general, $[xf(x)]^{(n)} = xf^{(n)}(x) + nf^{(n-1)}(x)$.

123. $f(x) = x^n \sin x$

$f'(x) = x^n \cos x + nx^{n-1} \sin x$

When $n = 1$: $f'(x) = x \cos x + \sin x$

When $n = 2$: $f'(x) = x^2 \cos x + 2 \sin x$

When $n = 3$: $f'(x) = x^3 \cos x + 3x^2 \sin x$

When $n = 4$: $f'(x) = x^4 \cos x + 4x^3 \sin x$

For general n, $f'(x) = x^n \cos x + nx^{n-1} \sin x$.

124. $f(x) = \dfrac{\cos x}{x^n} = x^{-n} \cos x$

$$\begin{aligned} f'(x) &= -x^{-n} \sin x - nx^{-n-1} \cos x \\ &= -x^{-n-1}(x \sin x + n \cos x) \\ &= -\frac{x \sin x + n \cos x}{x^{n+1}} \end{aligned}$$

When $n = 1$: $f'(x) = -\dfrac{x \sin x + \cos x}{x^2}$

When $n = 2$: $f'(x) = -\dfrac{x \sin x + 2 \cos x}{x^3}$

When $n = 3$: $f'(x) = -\dfrac{x \sin x + 3 \cos x}{x^4}$

When $n = 4$: $f'(x) = -\dfrac{x \sin x + 4 \cos x}{x^5}$

For general n, $f'(x) = -\dfrac{x \sin x + n \cos x}{x^{n+1}}$.

125. $y = \dfrac{1}{x},\ y' = -\dfrac{1}{x^2},\ y'' = \dfrac{2}{x^3}$

$$x^3y'' + 2x^2y' = x^3\left[\frac{2}{x^3}\right] + 2x^2\left[-\frac{1}{x^2}\right] = 2 - 2 = 0$$

126.
$$\begin{aligned} y &= 2x^3 - 6x + 10 \\ y' &= 6x^2 - 6 \\ y'' &= 12x \\ y''' &= 12 \end{aligned}$$
$$-y''' - xy'' - 2y' = -12 - x(12x) - 2(6x^2 - 6) = -24x^2$$

127.
$$\begin{aligned} y &= 2 \sin x + 3 \\ y' &= 2 \cos x \\ y'' &= -2 \sin x \\ y'' + y &= -2 \sin x + (2 \sin x + 3) = 3 \end{aligned}$$

128.
$$\begin{aligned} y &= 3 \cos x + \sin x \\ y' &= -3 \sin x + \cos x \\ y'' &= -3 \cos x - \sin x \\ y'' + y &= (-3 \cos x - \sin x) + (3 \cos x + \sin x) = 0 \end{aligned}$$

129. False. If $y = f(x)g(x)$, then

$$\frac{dy}{dx} = f(x)g'(x) + g(x)f'(x).$$

130. True. y is a fourth-degree polynomial.

$$\frac{d^n y}{dx^n} = 0 \text{ when } n > 4.$$

131. True

$$\begin{aligned} h'(c) &= f(c)g'(c) + g(c)f'(c) \\ &= f(c)(0) + g(c)(0) \\ &= 0 \end{aligned}$$

132. True

133. True

134. True. If $v(t) = c$ then $a(t) = v'(t) = 0$.

135. $f(x) = x|x| = \begin{cases} x^2, & x \ge 0 \\ -x^2, & x < 0 \end{cases}$

$$f'(x) = \begin{cases} 2x, & x > 0 \\ -2x, & x < 0 \end{cases} = 2|x|$$

$$f''(x) = \begin{cases} 2, & x > 0 \\ -2, & x < 0 \end{cases}$$

$f''(0)$ does not exist because the left and right derivatives do not agree at $x = 0$.