Ch. 2.3a Derivative of Polynomial (Power Rule)

p. 190–193 #7–21 odd, 33, 37, 47, 57, 61, 63

$\boxed{\text{Find derivative } f'(x)}$

7) $f(x) = 3x + \sqrt{2}$ $\boxed{f'(x) = 3}$

9) $f(x) = x^2 + 3x + 4$ $\boxed{f'(x) = 2x + 3}$

11) $f(u) = 8u^5 - 5u + 1$ $\boxed{f'(u) = 40u^4 - 5}$

13) $f(s) = as^3 + \frac{3}{2}s^2$ $f'(s) = 3as^2 + \frac{3}{2} \cdot 2s \rightarrow \boxed{f'(s) = 3as^2 + 3s}$

15) $f(t) = \frac{1}{3}(t^5 - 8)$ $f(t) = \frac{1}{3}t^5 - \frac{8}{3}$ $f'(t) = \frac{1}{3} \cdot 5t^4 - 0$ $\boxed{f'(t) = \frac{5}{3}t^4}$

17) $f(t) = \frac{t^3 + 2}{5}$ $f(t) = \frac{t^3}{5} + \frac{2}{5}$ $f(t) = \frac{1}{5}t^3 + \frac{2}{5}$ $f'(t) = \frac{1}{5} \cdot 3t^2 + 0$

$\boxed{f'(t) = \frac{3}{5}t^2}$

19) $f(x) = \frac{x^3 + 2x + 1}{7} \rightarrow f(x) = \frac{x^3}{7} + \frac{2x}{7} + \frac{1}{7} \rightarrow f(x) = \frac{1}{7}x^3 + \frac{2}{7}x + \frac{1}{7}$

$\boxed{f'(x) = \frac{3}{7}x^2 + \frac{2}{7}}$

21) $f(x) = ax^2 + bx + c$ $\boxed{f'(x) = 2ax + b}$

33) a) Find slope of tangent line at point
b) Find tangent line equation
c) Find equation of normal line
d) Graph f, tangent, and normal line

$f(x) = x^3 + 3x - 1$ at $(0, -1)$

$f'(x) = 3x^2 + 3$

a) $f'(0) = 3(0)^2 + 3 = 3$

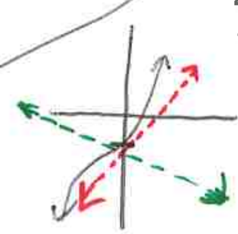
b) tangent line: $f'(0) = 3$
$f(0) = -1$

point: $(0, -1)$ slope: $m = 3$

$y + 1 = 3(x - 0) \rightarrow \boxed{y = 3x - 1}$

c) normal line $m = -\frac{1}{3}$ point: $(0, -1)$

$y + 1 = -\frac{1}{3}(x - 0)$ $y = -\frac{1}{3}x - 1$



37) a) Find points where horizontal tangent line occurs (set $f'(x)=0$)

b) Find equation for each horizontal line

c) Solve inequality $f'(x)>0$

d) Solve inequality $f'(x)<0$

e) Graph f and horizontal tangent lines

f) Describe graph of f in relation to results found in c and d.

$f(x) = 3x^2 - 12x + 4$

a) $f'(x) = 6x - 12$

$0 = 6x - 12$

$x = 2$

$\boxed{\text{(horizontal tangent occurs at } x=2)}$

b) $f(2) = 3(2)^2 - 12(2) + 4$

$f(2) = -8$

$\boxed{\text{horizontal tangent line is } y=-8}$

c) $f'(x) = 6x - 12$

$6x - 12 > 0$

$6x > 12$

$x > 2$

$\boxed{f'(x) > 0 \text{ when } x > 2}$

d) $f'(x) = 6x - 12$

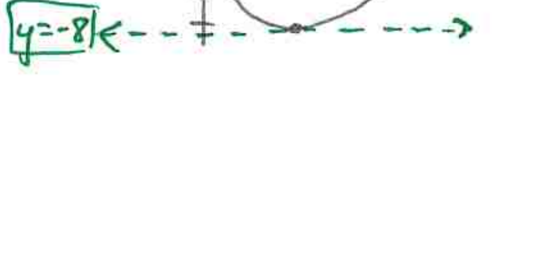
$6x - 12 < 0$

$6x < 12$

$x < 2$

$\boxed{f'(x) < 0 \text{ when } x < 2}$

e)



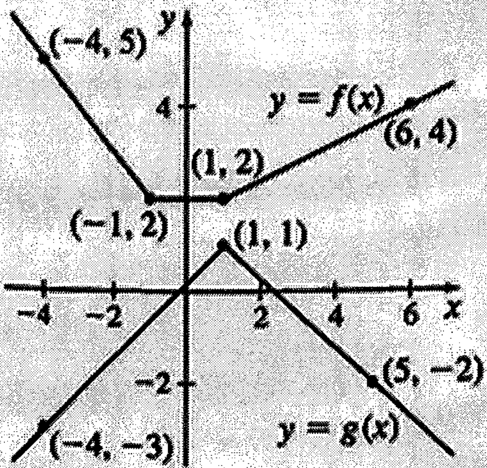
f) when $x>2$, $f'(x)>0$, meaning that to the right of $x=2$, slope of graph is positive

when $x<2$, $f'(x)<0$, meaning that to the left of $x=2$, slope of graph is negative.

2.3 Exercise Problems

In Problems 47 and 48, use the graphs to find each derivative.

47. Let $u(x) = f(x) + g(x)$ and $v(x) = f(x) - g(x)$.



(a) $u'(0)$ (b) $u'(4)$
(c) $v'(-2)$ (d) $v'(6)$
(e) $3u'(5)$ (f) $-2v'(3)$

a) $u(x) = f(x) + g(x)$
$u'(x) = f'(x) + g'(x)$

$u'(0) = f'(0) + g'(0)$
$u'(0) = 0 + \frac{4}{5} = \boxed{\frac{4}{5}}$

$g'(0) = \frac{1-(-3)}{1-(-4)} \to \frac{4}{5}$

b) $u'(4) = f'(4) + g'(4) \to u'(4) = \frac{2}{5} + \frac{-3}{4} = \boxed{\frac{-7}{20}}$

$g'(4) = \frac{-2-1}{5-1} = \frac{-3}{4}$

c) $v'(-2) = f'(-2) - g'(-2) \to -1 - \frac{4}{5} = \boxed{\frac{-9}{5}}$

$f'(-2) = \frac{2-5}{-1-(-4)} = \frac{-3}{3}$

d) $v'(6) = f'(6) - g'(6) \to \frac{2}{5} - \frac{-3}{4} = \boxed{\frac{23}{20}}$

$f'(6) = \frac{4-2}{6-1} = \frac{2}{5}$

e) $3u'(5) = 3\left[f'(5) + g'(5)\right] \to 3\left[\frac{2}{5} + \frac{-3}{4}\right] = 3\left(\frac{-7}{20}\right) = \boxed{\frac{-21}{20}}$

f) $-2v'(3) = -2\left[f'(3) - g'(3)\right] \to -2\left[\frac{2}{5} - \left(\frac{-3}{4}\right)\right] \to -2\left(\frac{-23}{20}\right) \to \boxed{\frac{-23}{10}}$

Find equation of tangent line that is parallel of line L.

57) $f(x) = 3x^2 - x$ $\quad L: y = 5x$

i) find slope of line: $m = 5$

ii) find $f'(x) = 6x - 1$

iii) set slope of line equal to derivative to find x

$5 = 6x - 1$ | $x = 1$

$6 = 6x$

iv) Find point: $f(1) = 3(1)^2 - 1 = 2$

v) Gather info to find tangent line equation:

point: $(1, 2)$ | $y - y_1 = m(x - x_1)$ | OR

slope: $m = 5$ | $\boxed{y - 2 = 5(x - 1)}$ | $\boxed{y = 5x - 3}$

61) $f(x) = \frac{1}{3}x^3 - x^2$ $\quad L: y = 3x - 2$

i) slope of line: $m = 3$

ii) $f'(x) = \frac{1}{3} \cdot 3x^2 - 2x \rightarrow f'(x) = x^2 - 2x$

iii) $3 = x^2 - 2x$ | $0 = (x-3)(x+1)$ | 2 different points

$0 = x^2 - 2x - 3$ | $x = 3, x = -1$

iv) $f(3) = \frac{1}{3}(3)^3 - 3^2 = 0$ $\qquad f(-1) = \frac{1}{3}(-1)^3 - (-1)^2 = \frac{-1}{3} - 1 = -4/3$

v) point: $(3, 0)$ | point: $(-1, -4/3)$

slope: $m = 3$ | slope: $m = 3$

$y - y_1 = m(x - x_1)$

$\boxed{y - 0 = 3(x - 3)}$ and $\boxed{y + 4/3 = 3(x + 1)}$

63) Tangent Line: Let $f(x) = 4x^3 - 3x - 1$

a) Find equation of tangent line at $x = 2$

$f(2) = 4(2)^3 - 3(2) - 1$ | $f'(x) = 12x^2 - 3$

$f(2) = 25$ | $f'(2) = 12(2)^2 - 3 = 45$

point: $(2, 25)$ $\rightarrow$ $y - y_1 = m(x - x_1)$

slope: $m = 45$ $\rightarrow$ $\boxed{y - 25 = 45(x - 2)}$

b) Find points where $f(x)$ is parallel to $y = x + 12$

slope: $m = 1$

$f'(x) = 12x^2 - 3$

$1 = 12x^2 - 3$ | $\frac{4}{12} = x^2$ | $x = \pm\sqrt{1/3} \rightarrow x = \pm\sqrt{3}/3$

$4 = 12x^2$ | $x^2 = 1/3$ |

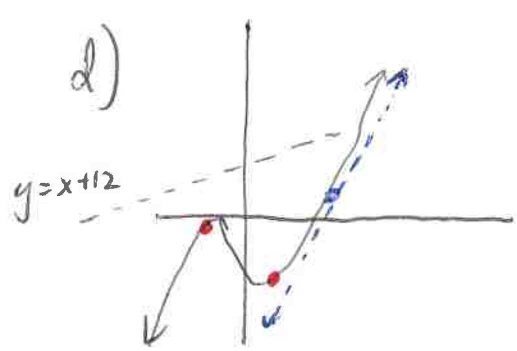
$$\boxed{\begin{aligned} f(\sqrt{3}/3) &= \frac{-5\sqrt{3}}{9} - 1 \\ f(-\sqrt{3}/3) &= \frac{5\sqrt{3}}{9} - 1 \end{aligned}}$$

c) Find equation of tangent line of points from (b)

$$y - \left(\frac{-5\sqrt{3}}{9} - 1\right) = 1\left(x - \frac{\sqrt{3}}{3}\right)$$

$$y - \left(\frac{5\sqrt{3}}{9} - 1\right) = 1\left(x + \frac{\sqrt{3}}{3}\right)$$

d)



Ch 2.2 Derivative as a Function and Differentiability

p. 182-183 AP Practice