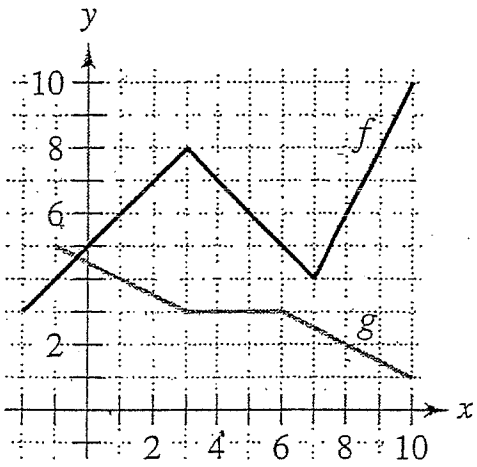
Ch.2.4 p.139

In Exercises 99 and 100, the graphs of f and g are shown. Let $h(x) = f(g(x))$ and $s(x) = g(f(x))$. Find each derivative, if it exists. If the derivative does not exist, explain why.

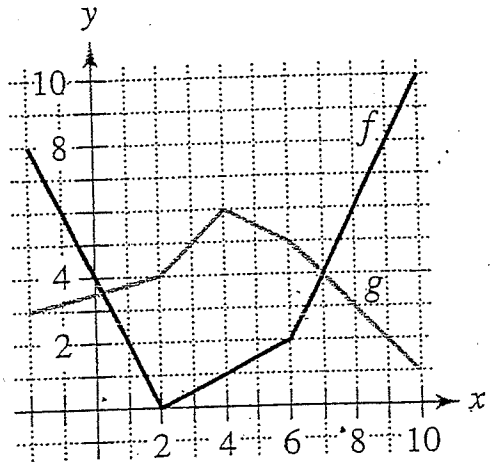
99. (a) Find $h'(1)$.

(b) Find $s'(5)$.



100. (a) Find $h'(3)$.

(b) Find $s'(9)$.



Ch.2.4 p.139

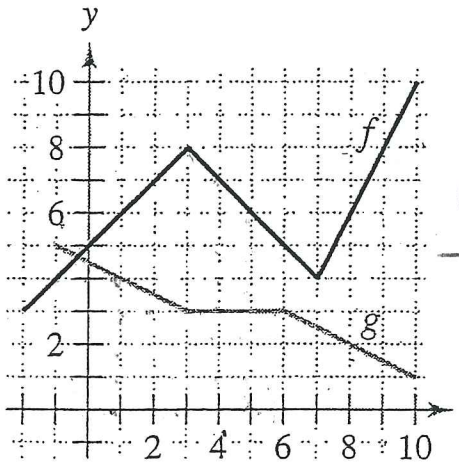
In Exercises 99 the graphs of f and g are shown. Let $h(x) = f(g(x))$ and $s(x) = g(f(x))$. Find each derivative, if it exists. If the derivative does not exist, explain why.

$h(x) = f(g(x))$

$s(x) = g(f(x))$

99. (a) Find $h'(1)$.

(b) Find $s'(5)$.



@ $h'(x) = f'[g(x)] \cdot g'(x)$

$h'(1) = f'[g(1)] \cdot g'(1)$

$h'(1) = f'[4] \cdot (-\frac{1}{2})$

$= (-1)(-\frac{1}{2})$

$\boxed{h'(1) = \frac{1}{2}}$

$g(1) = 4$

$g'(1) = -\frac{1}{2}$

$f'(4) = -1$

$s'(x) = g'(f(x)) \cdot f'(x)$

$s'(5) = g'(f(5)) \cdot f'(5)$

$s'(5) = g'(6) \cdot (-1)$

$\boxed{s'(5) = \text{DNE}}$

$f(5) = 6$

$f'(5) = -1$

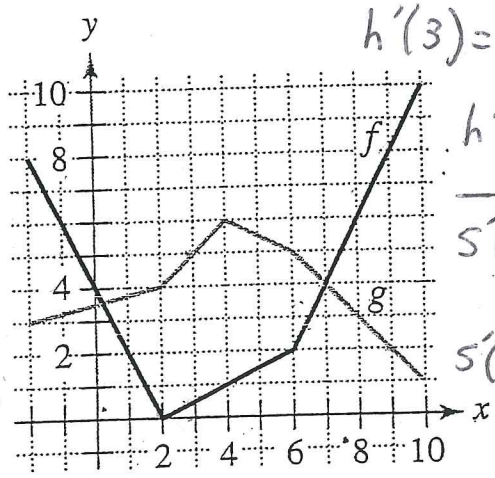
$g'(6) = \text{DNE}$

$h(x) = f(x)g(x)$

$s(x) = \frac{f(x)}{g(x)}$

100. (a) Find $h'(3)$.

(b) Find $s'(9)$.



$h'(x) = f'(x)g(x) + f(x)g'(x)$

$h'(3) = f'(3)g(3) + f(3)g'(3)$

$h'(3) = (\frac{1}{2})(5) + (\frac{1}{2})(1)$

$h'(3) = \frac{5}{2} + \frac{1}{2} = \frac{6}{2} = \boxed{3}$

$f(3) = \frac{1}{2}$

$f'(3) = \frac{1}{2}$

$g(3) = 5$

$g'(3) = 1$

$s'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

$s'(9) = \frac{f'(9)g(9) - f(9)g'(9)}{[g(9)]^2}$

$s'(9) = \frac{(2)(2) - (8)(-1)}{(2)^2} = \frac{4+8}{4} = \frac{12}{4} = 3$ $\boxed{s'(9) = 3}$

$f(9) = 8$

$f'(9) = 2$

$g(9) = 2$

$g'(9) = -1$