2014

Key

Ch. 2.4 Chain Rule   HW Problems #102, #115

102. Using Relationships Given that $g(5) = -3$, $g'(5) = 6$, $h(5) = 3$, and $h'(5) = -2$, find $f'(5)$ for each of the following, if possible. If it is not possible, state what additional information is required.

Recall: product rule: $\frac{d}{dx} f(x)g(x) = f'(x)(g(x)) + f(x)g'(x)$

quotient rule: $\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

Chain rule: $\frac{d}{dx} f[g(x)] = f'(g(x)) \cdot g'(x)$

(a) $f(x) = g(x)h(x)$ *product rule

$f'(x) = g'(x)h(x) + g(x)h'(x)$

$f'(5) = g'(5)h(5) + g(5)h'(5)$

$f'(5) = 6(3) + (-3)(-2)$

$f'(5) = 18 + 6 = 24$ $\boxed{f'(5) = 24}$

(b) $f(x) = g(h(x))$ *chain rule

$f'(x) = g'[h(x)] \cdot h'(x)$

$f'(5) = g'[h(5)] \cdot h'(5)$

$= g'(3) \cdot h'(5)$

$= g'(3) \cdot -2$

$\boxed{f'(5) = -2g'(3)}$

(c) $f(x) = \dfrac{g(x)}{h(x)}$ *quotient rule

$f'(x) = \dfrac{g'(x)h(x) - g(x)h'(x)}{h(x)^2}$

$f'(5) = \dfrac{g'(5)h(5) - g(5)h'(5)}{h(5)^2}$

$= \dfrac{6(3) - (-3)(-2)}{3^2}$

$= \dfrac{18-6}{9} = \dfrac{12}{9} = 4/3$

$\boxed{f'(5) = 4/3}$

(d) $f(x) = [g(x)]^3$ *chain rule

$f'(x) = 3[g(x)]^2 \cdot g'(x)$

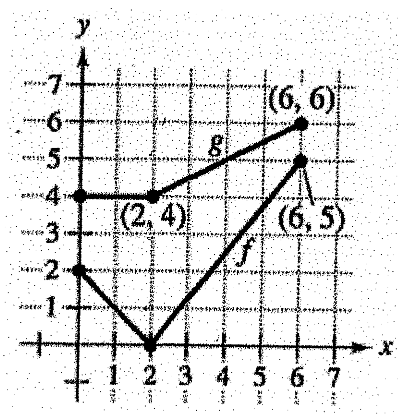
$f'(5) = 3[g(5)]^2 \cdot g'(5)$

$= 3[-3]^2 \cdot 6$

$= 3(9)(6) = 162$

$\boxed{f'(5) = 162}$

115. Think About It Let $r(x) = f(g(x))$ and $s(x) = g(f(x))$, where f and g are shown in the figure. Find (a) $r'(1)$ and (b) $s'(4)$.   ← Apply chain rule



a) $r'(x) = f'[g(x)] \cdot g'(x)$

$r'(1) = f'[g(1)] \cdot g'(1)$

$r'(1) = f'[4] \cdot 0$

$r'(1) = \frac{5}{4}(0) = \boxed{0}$

$\boxed{r'(1) = 0}$

b) $s'(x) = g'[f(x)] \cdot f'(x)$

$s'(4) = g'[f(4)] \cdot f'(4)$

$= g'[5/2] \cdot (5/4)$

$= (\frac{1}{2})(5/4) = 5/8$

$\boxed{s'(4) = 5/8}$

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Chain Rule: $\frac{d}{dx} f[g(x)] = f'[g(x)] * g'(x)$

(a) $f(x) = g(x)h(x)$

(b) $f(x) = g(h(x))$

(c) $f(x) = \dfrac{g(x)}{h(x)}$

(d) $f(x) = [g(x)]^3$

115. Think About It Let $r(x) = f(g(x))$ and $s(x) = g(f(x))$, where f and g are shown in the figure. Find (a) $r'(1)$ and (b) $s'(4)$.