Ch. 2.4 Exercise Problems: Product and Quotient Rule

p. 202-207 9, 23-37 odd, 69, 71, 81

9) $f(x) = \overbrace{x}^{f} \cdot \overbrace{e^x}^{g}$ $\qquad f'(x) = \overbrace{(1)}^{f'}\overbrace{(e^x)}^{g} + \overbrace{(x)}^{f} \cdot \overbrace{(e^x)}^{g'}$

$\boxed{f'(x) = e^x + xe^x}$

23) $f(x) = \dfrac{\overbrace{4x^2-2}^{f}}{\underbrace{3x+4}_{g}}$ $\qquad f'(x) = \dfrac{\overbrace{8x}^{f'}\overbrace{(3x+4)}^{g} - \overbrace{(4x^2-2)}^{f}\overbrace{(3)}^{g'}}{\underbrace{(3x+4)^2}_{g^2}}$

$f'(x) = \dfrac{24x^2+32x-12x^2+6}{(3x+4)^2}$ $\qquad \boxed{f'(x) = \dfrac{12x^2+32x+6}{(3x+4)^2}}$

25) $f(w) = \dfrac{1}{w^3-1}$ $\qquad f'(w) = \dfrac{\overbrace{(0)}^{f'}\overbrace{(w^3-1)}^{g} - \overbrace{(1)}^{f}\overbrace{(3w^2)}^{g'}}{\underbrace{(w^3-1)^2}_{g^2}}$

$\boxed{f'(w) = \dfrac{-3w^2}{(w^3-1)^2}}$

27) $s(t) = t^{-3}$ $\qquad s'(t) = -3t^{-4}$ $\qquad \boxed{s'(t) = \dfrac{-3}{t^4}}$

29) $f(x) = -\dfrac{4}{e^x} \rightarrow f'(x) = \dfrac{\overbrace{(0)}^{f'}\overbrace{e^x}^{g} - \overbrace{(-4)}^{f}\overbrace{(e^x)}^{g'}}{\underbrace{(e^x)^2}_{g^2}} \rightarrow \dfrac{+4e^x}{(e^x)^2}$

$\boxed{f'(x) = \dfrac{4}{e^x}}$

31) $f(x) = \frac{10}{x^4} + \frac{3}{x^2} \rightarrow 10x^{-4} + 3x^{-2}$

$f'(x) = -40x^{-5} + 3(-2)x^{-3}$ $\boxed{f'(x) = \frac{-40}{x^5} - \frac{6}{x^3}}$

33) $f(x) = 3x^3 - \frac{1}{3x^2}$

$f(x) = 3x^3 - \frac{1}{3}x^{-2}$

$f'(x) = 9x^2 - \frac{1}{3} \cdot -2x^{-3}$

$\boxed{f'(x) = 9x^2 + \frac{2}{3x^3}}$

35) $s(t) = \frac{1}{t} - \frac{1}{t^2} + \frac{1}{t^3}$

$s(t) = t^{-1} - t^{-2} + t^{-3}$

$s'(t) = -1t^{-2} - -2t^{-3} + -3t^{-4}$

$\boxed{s'(t) = \frac{-1}{t^2} + \frac{2}{t^3} - \frac{3}{t^4}}$

37) $f(x) = \frac{\overbrace{e^x}^{f}}{\underbrace{x^2}_{g}}$ $f'(x) = \frac{\overbrace{e^x}^{f'} \cdot \overbrace{x^2}^{g} - \overbrace{e^x}^{f} \cdot \overbrace{2x}^{g'}}{\underbrace{(x^2)^2}_{g^2}}$

$f'(x) = \frac{x^2e^x - 2xe^x}{x^4} \rightarrow f'(x) = \frac{x(xe^x - 2e^x)}{x^4} \rightarrow \boxed{\frac{xe^x - 2e^x}{x^3}}$

69) $f(x) = \frac{x^2}{x-1}$ at $(-1, -\frac{1}{2})$

a) find slope of tangent line

$f'(x) = \frac{\overbrace{2x}^{f'}\overbrace{(x-1)}^{g} - \overbrace{x^2}^{f}\overbrace{(1)}^{g'}}{\underbrace{(x-1)^2}_{g^2}}$

$f'(x) = \frac{2x^2 - 2x - x^2}{(x-1)^2} = \frac{x^2 - 2x}{(x-1)^2}$

$f'(-1) = \frac{(-1)^2 - 2(-1)}{(-1-1)^2} \rightarrow \frac{1+2}{4} \rightarrow \boxed{\frac{3}{4}}$

b) equation of tangent line:

point: $(-1, -\frac{1}{2})$ slope: $m = \frac{3}{4}$

$\boxed{y + \frac{1}{2} = \frac{3}{4}(x+1)}$

c) horizontal tangent line (slope of graph = 0)

set numerator of $f'(x) = 0$

$x^2 - 2x = 0$

$x(x-2) = 0$

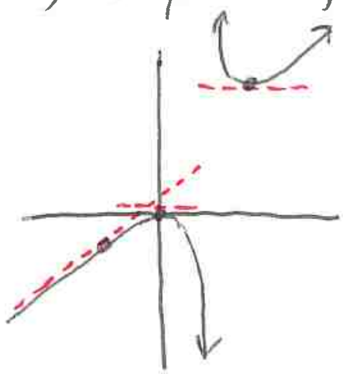
$x = 0, x = 2$

points are $f(0) = 0$ and $f(2) = 4$

$\boxed{(0,0) \text{ and } (2,4)}$

$\boxed{2.4}$ (continued...)

69) d) Graph f(x) and tangent line found in (b) and (c)



71) $f(x) = \dfrac{x^3}{x+1}$ $\quad (1, \tfrac{1}{2})$

a) find slope of tangent line

$$f'(x) = \frac{\overbrace{3x^2}^{f'}\overbrace{(x+1)}^{g} - \overbrace{x^3}^{f}\overbrace{(1)}^{g'}}{\underbrace{(x+1)^2}_{g^2}}$$

$$f'(x) = \frac{3x^3 + 3x^2 - x^3}{(x+1)^2}$$

$$\boxed{f'(x) = \frac{2x^3 + 3x^2}{(x+1)^2}}$$

$$\boxed{f'(1) = \frac{2+3}{(1+1)^2} \rightarrow \frac{5}{4}}$$

b) equation of tangent line

point: $(1, \tfrac{1}{2})$ slope: $m = \tfrac{5}{4}$

$$\boxed{y - \tfrac{1}{2} = \tfrac{5}{4}(x-1)}$$

c) Find points where graph f(x) has horizontal tangent (set $f'(x) = 0$ numerator)

$$f'(x) = \frac{2x^3 + 3x^2}{(x+1)^2} \rightarrow 2x^3 + 3x^2 = 0$$

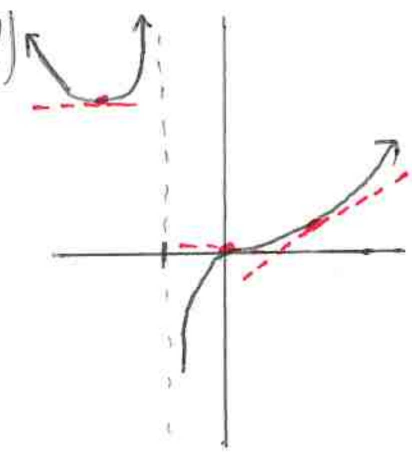
$$x^2(2x+3) = 0$$

$$x = 0, \; x = -\tfrac{3}{2}$$

$f(0) = 0 \qquad f(-\tfrac{3}{2}) = \tfrac{27}{4}$

$$\boxed{(0,0) \text{ and } (-\tfrac{3}{2}, \tfrac{27}{4})}$$

d)



2.4 Exercise Problems

In Problems 81 and 82, use the graphs to determine each derivative.

81. Let $u(x) = f(x) \cdot g(x)$ and $v(x) = \dfrac{g(x)}{f(x)}$.

$m = \frac{2-5}{-1-(-4)}$

$m = \frac{-3}{3} = -1$

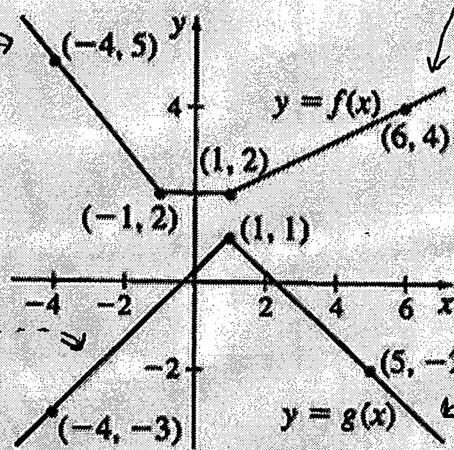
$y - 2 = -1(x+1)$

$\boxed{y = -1x + 1}$

$m = \frac{4-2}{6-1} = \frac{2}{5}$

$y - 2 = \frac{2}{5}(x-1)$

$\boxed{y = \frac{2}{5}x + 8/5}$



$m = \frac{-2-1}{5-1} = \frac{-3}{4}$

$y - 1 = \frac{-3}{4}(x-1)$

$\boxed{y = \frac{-3}{4}x + 7/4}$

$m = \frac{1-(-3)}{1-(-4)}$

$m = \frac{4}{5}$

$y - 1 = 4/5(x-1)$

$\boxed{y = \frac{4}{5}x + 1/5}$

(a) $u'(0)$ **(b)** $u'(4)$

(c) $v'(-2)$ **(d)** $v'(6)$

(e) $\dfrac{d}{dx}\dfrac{1}{f(x)}$ at $x = -2$ **(f)** $\dfrac{d}{dx}\dfrac{1}{g(x)}$ at $x = 4$

a) $u(x) = f(x)g(x)$

$u'(0) = f'(0)g(0) + f(0)g'(0)$

$u'(0) = (0)(1/5) + (2)(4/5) = \boxed{8/5}$

b) $u'(x) = f'(x)g(x) + f(x)g'(x)$

$u'(4) = f'(4)g(4) + f(4)g'(4)$

$u'(4) = \left(\frac{2}{5}\right)(-5/4) + \left(\frac{16}{5}\right)(-3/4) = \boxed{\frac{-29}{10}}$

c) $v'(x) = \dfrac{g'(x)f(x) - g(x)f'(x)}{f(x)^2}$

$V'(-2) = \dfrac{g'(-2)f(-2) - g(-2)f'(-2)}{(f(-2))^2} \rightarrow \dfrac{(4/5)(3) - (-7/5)(-1)}{3^2} = \boxed{1/9}$

d) $v'(6) = \dfrac{g'(6)f(6) - g(6)f'(6)}{f(6)^2}$

$v'(6) = \dfrac{(-3/4)(4) - (-1/4)(2/5)}{4^2} = \boxed{\frac{-19}{160}}$

e) $\frac{d}{dx}\left[\frac{1}{f(x)}\right] = \dfrac{0(f(x)) - 1f'(x)}{f(x)^2}$

$\rightarrow \dfrac{0f(-2) - f'(-2)}{f(-2)^2} \rightarrow \dfrac{-1}{3^2} = \boxed{1/9}$

f) $\frac{d}{dx}\left[\frac{1}{g(x)}\right] = \dfrac{0g(x) - 1g'(x)}{g(x)^2}$

$\rightarrow \dfrac{0g(4) - 1g'(4)}{g(4)^2} \rightarrow \dfrac{-(-3/4)}{(-5/4)^2}$

$\frac{3}{4} \cdot \frac{16}{25} \rightarrow \boxed{\frac{12}{25}}$