2017

# 2.4 Chain Rule

p. 136-138 # 9, 13, 19, 23, 27, 31, 65, 73a, 75a, 85, 102, 103, 115

* Chain Rule: $\frac{d}{dx} f[g(x)] = f'[g(x)] \cdot g'(x)$

9) $g(x) = 3(4-9x)^4$

$g'(x) = 3 \cdot 4(4-9x)^3 \cdot (-9) = \boxed{-108(4-9x)^3}$

13) $y = \sqrt[3]{6x^2+1}$

$y = (6x^2+1)^{1/3}$

$y' = \frac{1}{3}(6x^2+1)^{-2/3}(12x)$

$y' = 4x(6x^2+1)^{-2/3}$

$\boxed{y' = \frac{4x}{(6x^2+1)^{2/3}} = \frac{4x}{\sqrt[3]{(6x^2+1)^2}}}$

19) $f(t) = \left(\frac{1}{t-3}\right)^2 = \frac{1}{(t-3)^2}$

$f(t) = (t-3)^{-2}$

$f'(t) = -2(t-3)^{-3}(1)$

$\boxed{f'(t) = \frac{-2}{(t-3)^3}}$

23) $f(x) = x^2(x-2)^4$ * Apply product rule, chain rule

$f'(x) = \underbrace{2x}_{f'} \cdot \underbrace{(x-2)^4}_{g} + \underbrace{x^2}_{f} \cdot \underbrace{4(x-2)^3(1)}_{g'}$

$f'(x) = x(x-2)^3[\underbrace{2(x-2)+4x}_{2x-4+4x}]$

$f'(x) = x(x-2)^3[\underbrace{6x-4}_{2(3x-2)}]$

$\boxed{f'(x) = 2x(x-2)^3(3x-2)}$

27) $y = \frac{x}{\sqrt{x^2+1}} = \frac{x}{(x^2+1)^{1/2}}$ * Apply quotient rule, chain rule

$y' = \frac{(1)(x^2+1)^{1/2} - x \cdot \frac{1}{2}(x^2+1)^{-1/2}(2x)}{[(x^2+1)^{1/2}]^2}$

$y' = \frac{\sqrt{x^2+1} - \frac{x^2}{\sqrt{x^2+1}}}{x^2+1} \cdot \frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}$

$y' = \frac{x^2+1-x^2}{(x^2+1)\sqrt{x^2+1}}$

$y' = \frac{1}{(x^2+1)(x^2+1)^{1/2}}$

$\boxed{y' = \frac{1}{(x^2+1)^{3/2}}}$

31) $f(v) = \left(\frac{1-2v}{1+v}\right)^3$

*Apply chain rule first, then quotient rule

$$-2-2v-1+2v$$

$$f'(v) = 3\left(\frac{1-2v}{1+v}\right)^2 \cdot \left[\frac{(-2)(1+v)-(1-2v)(1)}{(1+v)^2}\right] = \frac{3(1-2v)^2(-3)}{(1+v)^2(1+v)^2} = \boxed{\frac{-9(1-2v)^2}{(1+v)^4}}$$

65) Evaluate derivative at a given point

*chain rule $\frac{d}{dx}f[g(x)] = f'[g(x)]\cdot g'(x)$

$y = \sqrt{x^2+8x}$ at $(1,3)$

$y = (x^2+8x)^{1/2}$

$y' = \frac{1}{2}(x^2+8x)^{-1/2}(2x+8)$

$y' = \frac{1}{2}(2x+8)(x^2+8x)^{-1/2}$

$y' = \frac{x+4}{\sqrt{x^2+8x}}$

$y'(1) = \frac{1+4}{\sqrt{1^2+8(1)}} = \frac{5}{\sqrt{9}}$

$\boxed{y'(1) = \frac{5}{3}}$

Find equation of tangent line

73) $f(x) = \sqrt{2x^2-7}$ at $(4,5)$

$f(x) = (2x^2-7)^{1/2}$

$f'(x) = \frac{1}{2}(2x^2-7)^{-1/2}(4x)$

$f'(x) = \frac{2x}{(2x^2-7)^{1/2}}$

$f'(x) = \frac{2x}{\sqrt{2x^2-7}}$

$f'(4) = \frac{2(4)}{\sqrt{2(4)^2-7}} = \frac{8}{\sqrt{32-7}}$

$f'(4) = \frac{8}{\sqrt{25}} = \frac{8}{5}$

point: $(4,5)$
slope: $m = 8/5$

$y - y_1 = m(x-x_1) \rightarrow \boxed{y-5 = \frac{8}{5}(x-4)}$

85) Find $f''(x)$

$f(x) = 5(2-7x)^4$

$f'(x) = 5\cdot 4(2-7x)^3(-7)$

$f'(x) = -140(2-7x)^3$

$f''(x) = -140\cdot 3(2-7x)^2(-7)$

$\boxed{f''(x) = 2940(2-7x)^2}$