136. (a) $(fg' - f'g)' = fg'' + f'g' - f'g' - f''g$
$= fg'' - f''g$ True

(b) $(fg)'' = (fg' + f'g)'$
$= fg'' + f'g' + f'g' + f''g$
$= fg'' + 2f'g' + f''g$
$\neq fg'' + f''g$ False

137. $\frac{d}{dx}[f(x)g(x)h(x)] = \frac{d}{dx}[(f(x)g(x))h(x)]$
$= \frac{d}{dx}[f(x)g(x)]h(x) + f(x)g(x)h'(x)$
$= [f(x)g'(x) + g(x)f'(x)]h(x) + f(x)g(x)h'(x)$
$= f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$

Section 2.4 The Chain Rule

$y = f(g(x))$	$u = g(x)$	$y = f(u)$
1. $y = (5x - 8)^4$	$u = 5x - 8$	$y = u^4$
2. $y = \frac{1}{\sqrt{x + 1}}$	$u = x + 1$	$y = u^{-1/2}$
3. $y = \sqrt{x^3 - 7}$	$u = x^3 - 7$	$y = \sqrt{u}$
4. $y = 3\tan(\pi x^2)$	$u = \pi x^2$	$y = 3\tan u$
5. $y = \csc^3 x$	$u = \csc x$	$y = u^3$
6. $y = \sin\frac{5x}{2}$	$u = \frac{5x}{2}$	$y = \sin u$

7. $y = (4x - 1)^3$
$y' = 3(4x - 1)^2(4) = 12(4x - 1)^2$

8. $y = 5(2 - x^3)^4$
$y' = 5(4)(2 - x^3)^3(-3x^2) = -60x^2(2 - x^3)^3$
$= 60x^2(x^3 - 2)^3$

9. $g(x) = 3(4 - 9x)^4$
$g'(x) = 12(4 - 9x)^3(-9) = -108(4 - 9x)^3$

10. $f(t) = (9t + 2)^{2/3}$
$f'(t) = \frac{2}{3}(9t + 2)^{-1/3}(9) = \frac{6}{\sqrt[3]{9t + 2}}$

11. $f(t) = \sqrt{5 - t} = (5 - t)^{1/2}$
$f'(t) = \frac{1}{2}(5 - t)^{-1/2}(-1) = \frac{-1}{2\sqrt{5 - t}}$

12. $g(x) = \sqrt{4 - 3x^2} = (4 - 3x^2)^{1/2}$
$g'(x) = \frac{1}{2}(4 - 3x^2)^{-1/2}(-6x) = -\frac{3x}{\sqrt{4 - 3x^2}}$

13. $y = \sqrt[3]{6x^2 + 1} = (6x^2 + 1)^{1/3}$
$y' = \frac{1}{3}(6x^2 + 1)^{-2/3}(12x) = \frac{4x}{(6x^2 + 1)^{2/3}} = \frac{4x}{\sqrt[3]{(6x^2 + 1)^2}}$

14. $f(x) = \sqrt{x^2 - 4x + 2} = (x^2 - 4x + 2)^{1/2}$
$f'(x) = \frac{1}{2}(x^2 - 4x + 2)^{-1/2}(2x - 4) = \frac{x - 2}{\sqrt{x^2 - 4x + 2}}$

15. $y = 2\sqrt[4]{9 - x^2} = 2(9 - x^2)^{1/4}$
$y' = 2\left(\frac{1}{4}\right)(9 - x^2)^{-3/4}(-2x)$
$= \frac{-x}{(9 - x^2)^{3/4}} = \frac{-x}{\sqrt[4]{(9 - x^2)^3}}$

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16. $f(x) = \sqrt[3]{12x - 5} = (12x - 5)^{1/3}$

$f'(x) = \frac{1}{3}(12x - 5)^{-2/3}(12) = \frac{4}{(12x - 5)^{2/3}}$

17. $y = (x - 2)^{-1}$

$y' = -1(x - 2)^{-2}(1) = \frac{-1}{(x - 2)^2}$

18. $s(t) = \frac{1}{4 - 5t - t^2} = (4 - 5t - t^2)^{-1}$

$s'(t) = -(4 - 5t - t^2)^{-2}(-5 - 2t)$

$= \frac{5 + 2t}{(4 - 5t - t^2)^2} = \frac{2t + 5}{(t^2 + 5t - 4)^2}$

19. $f(t) = (t - 3)^{-2}$

$f'(t) = -2(t - 3)^{-3}(1) = \frac{-2}{(t - 3)^3}$

20. $y = -\frac{3}{(t - 2)^4} = -3(t - 2)^{-4}$

$y' = 12(t - 2)^{-5} = \frac{12}{(t - 2)^5}$

21. $y = \frac{1}{\sqrt{3x + 5}} = (3x + 5)^{-1/2}$

$y' = -\frac{1}{2}(3x + 5)^{-3/2}(3)$

$= \frac{-3}{2(3x + 5)^{3/2}}$

$= -\frac{3}{2\sqrt{(3x + 5)^3}}$

22. $g(t) = \frac{1}{\sqrt{t^2 - 2}} = (t^2 - 2)^{-1/2}$

$g'(t) = -\frac{1}{2}(t^2 - 2)^{-3/2}(2t)$

$= \frac{-t}{(t^2 - 2)^{3/2}}$

$= -\frac{t}{\sqrt{(t^2 - 2)^3}}$

23. $f(x) = x^2(x - 2)^4$

$f'(x) = x^2\left[4(x - 2)^3(1)\right] + (x - 2)^4(2x)$

$= 2x(x - 2)^3\left[2x + (x - 2)\right]$

$= 2x(x - 2)^3(3x - 2)$

24. $f(x) = x(2x - 5)^3$

$f'(x) = x(3)(2x - 5)^2(2) + (2x - 5)^3(1)$

$= (2x - 5)^2\left[6x + (2x - 5)\right]$

$= (2x - 5)^2(8x - 5)$

25. $y = x\sqrt{1 - x^2} = x(1 - x^2)^{1/2}$

$y' = x\left[\frac{1}{2}(1 - x^2)^{-1/2}(-2x)\right] + (1 - x^2)^{1/2}(1)$

$= -x^2(1 - x^2)^{-1/2} + (1 - x^2)^{1/2}$

$= (1 - x^2)^{-1/2}\left[-x^2 + (1 - x^2)\right]$

$= \frac{1 - 2x^2}{\sqrt{1 - x^2}}$

26. $y = \frac{1}{2}x^2\sqrt{16 - x^2}$

$y' = \frac{1}{2}x^2\left(\frac{1}{2}(16 - x^2)^{-1/2}(-2x)\right) + x(16 - x^2)^{1/2}$

$= \frac{-x^3}{2\sqrt{16 - x^2}} + x\sqrt{16 - x^2} = \frac{-x(3x^2 - 32)}{2\sqrt{16 - x^2}}$

27. $y = \frac{x}{\sqrt{x^2 + 1}} = \frac{x}{(x^2 + 1)^{1/2}}$

$y' = \frac{(x^2 + 1)^{1/2}(1) - x\left(\frac{1}{2}\right)(x^2 + 1)^{-1/2}(2x)}{\left[(x^2 + 1)^{1/2}\right]^2}$

$= \frac{(x^2 + 1)^{1/2} - x^2(x^2 + 1)^{-1/2}}{x^2 + 1}$

$= \frac{(x^2 + 1)^{-1/2}\left[x^2 + 1 - x^2\right]}{x^2 + 1}$

$= \frac{1}{(x^2 + 1)^{3/2}} = \frac{1}{\sqrt{(x^2 + 1)^3}}$

28. $y = \frac{x}{\sqrt{x^4 + 4}}$

$y' = \frac{(x^4 + 4)^{1/2}(1) - x\frac{1}{2}(x^4 + 4)^{-1/2}(4x^3)}{x^4 + 4}$

$= \frac{x^4 + 4 - 2x^4}{(x^4 + 4)^{3/2}} = \frac{4 - x^4}{(x^4 + 4)^{3/2}} = \frac{4 - x^4}{\sqrt{(x^4 + 4)^3}}$

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29. $g(x) = \left(\dfrac{x+5}{x^2+2}\right)^2$

$$g'(x) = 2\left(\frac{x+5}{x^2+2}\right)\left(\frac{(x^2+2)-(x+5)(2x)}{(x^2+2)^2}\right)$$
$$= \frac{2(x+5)(2-10x-x^2)}{(x^2+2)^3}$$
$$= \frac{-2(x+5)(x^2+10x-2)}{(x^2+2)^3}$$

30. $h(t) = \left(\dfrac{t^2}{t^3+2}\right)^2$

$$h'(t) = 2\left(\frac{t^2}{t^3+2}\right)\left(\frac{(t^3+2)(2t)-t^2(3t^2)}{(t^3+2)^2}\right)$$
$$= \frac{2t^2(4t-t^4)}{(t^3+2)^3} = \frac{2t^3(4-t^3)}{(t^3+2)^3}$$

31. $f(v) = \left(\dfrac{1-2v}{1+v}\right)^3$

$$f'(v) = 3\left(\frac{1-2v}{1+v}\right)^2\left(\frac{(1+v)(-2)-(1-2v)}{(1+v)^2}\right)$$
$$= \frac{-9(1-2v)^2}{(1+v)^4}$$

32. $g(x) = \left(\dfrac{3x^2-2}{2x+3}\right)^3$

$$g'(x) = 3\left(\frac{3x^2-2}{2x+3}\right)^2\left(\frac{(2x+3)(6x)-(3x^2-2)(2)}{(2x+3)^2}\right)$$
$$= \frac{3(3x^2-2)^2(6x^2+18x+4)}{(2x+3)^4}$$
$$= \frac{6(3x^2-2)^2(3x^2+9x+2)}{(2x+3)^4}$$

33. $f(x) = \left((x^2+3)^5 + x\right)^2$

$$f'(x) = 2\left((x^2+3)^5 + x\right)\left(5(x^2+3)^4(2x) + 1\right)$$
$$= 2\left[10x(x^2+3)^9 + (x^2+3)^5 + 10x^2(x^2+3)^4 + x\right] = 20x(x^2+3)^9 + 2(x^2+3)^5 + 20x^2(x^2+3)^4 + 2x$$

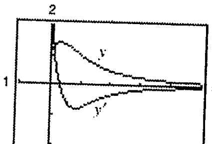
34. $g(x) = \left(2 + (x^2+1)^4\right)^3$

$$g'(x) = 3\left(2+(x^2+1)^4\right)^2\left(4(x^2+1)^3(2x)\right) = 24x(x^2+1)^3\left(2+(x^2+1)^4\right)^2$$

35. $y = \dfrac{\sqrt{x}+1}{x^2+1}$

$$y' = \frac{1-3x^2-4x^{3/2}}{2\sqrt{x}(x^2+1)^2}$$

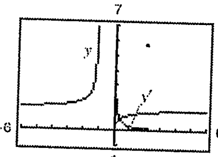
The zero of y' corresponds to the point on the graph of y where the tangent line is horizontal.



36. $y = \sqrt{\dfrac{2x}{x+1}}$

$$y' = \frac{1}{\sqrt{2x}(x+1)^{3/2}}$$

y' has no zeros.



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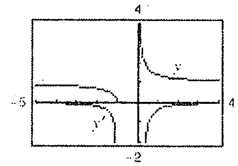
37. $y = \sqrt{\dfrac{x+1}{x}}$

$y' = -\dfrac{\sqrt{(x+1)/x}}{2x(x+1)}$

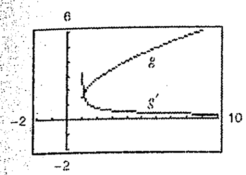


y' has no zeros.

38. $g(x) = \sqrt{x-1} + \sqrt{x+1}$

$g'(x) = \dfrac{1}{2\sqrt{x-1}} + \dfrac{1}{2\sqrt{x+1}}$

g' has no zeros.

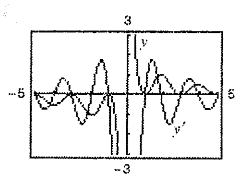


39. $y = \dfrac{\cos \pi x + 1}{x}$

$\dfrac{dy}{dx} = \dfrac{-\pi x \sin \pi x - \cos \pi x - 1}{x^2}$

$= -\dfrac{\pi x \sin \pi x + \cos \pi x + 1}{x^2}$

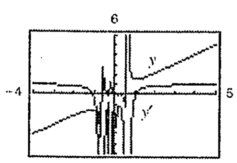
The zeros of y' correspond to the points on the graph of y where the tangent lines are horizontal.



40. $y = x^2 \tan \dfrac{1}{x}$

$\dfrac{dy}{dx} = 2x \tan \dfrac{1}{x} - \sec^2 \dfrac{1}{x}$

The zeros of y' correspond to the points on the graph of y where the tangent lines are horizontal.



41. (a) $y = \sin x$

$y' = \cos x$

$y'(0) = 1$

1 cycle in $[0, 2\pi]$

(b) $y = \sin 2x$

$y' = 2 \cos 2x$

$y'(0) = 2$

2 cycles in $[0, 2\pi]$

The slope of $\sin ax$ at the origin is a.

42. (a) $y = \sin 3x$

$y' = 3 \cos 3x$

$y'(0) = 3$

3 cycles in $[0, 2\pi]$

(b) $y = \sin\left(\dfrac{x}{2}\right)$

$y' = \left(\dfrac{1}{2}\right)\cos\left(\dfrac{x}{2}\right)$

$y'(0) = \dfrac{1}{2}$

Half cycle in $[0, 2\pi]$

The slope of $\sin ax$ at the origin is a.

43. $y = \cos 4x$

$\dfrac{dy}{dx} = -4 \sin 4x$

44. $y = \sin \pi x$

$\dfrac{dy}{dx} = \pi \cos \pi x$

45. $g(x) = 5 \tan 3x$

$g'(x) = 15 \sec^2 3x$

46. $h(x) = \sec(x^2)$

$h'(x) = 2x \sec(x^2)\tan(x^2)$

47. $y = \sin(\pi x)^2 = \sin(\pi^2 x^2)$

$y' = \cos(\pi^2 x^2)[2\pi^2 x] = 2\pi^2 x \cos(\pi^2 x^2)$

$= 2\pi^2 x \cos(\pi x)^2$

48. $y = \cos(1 - 2x)^2 = \cos((1 - 2x)^2)$

$y' = -\sin(1 - 2x)^2(2(1 - 2x)(-2))$

$= 4(1 - 2x)\sin(1 - 2x)^2$

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49. $h(x) = \sin 2x \cos 2x$

$h'(x) = \sin 2x(-2\sin 2x) + \cos 2x(2\cos 2x)$

$= 2\cos^2 2x - 2\sin^2 2x$

$= 2\cos 4x$

Alternate solution: $h(x) = \frac{1}{2}\sin 4x$

$h'(x) = \frac{1}{2}\cos 4x(4) = 2\cos 4x$

50. $g(\theta) = \sec\frac{1}{2}\theta\tan\frac{1}{2}\theta$

$g'(\theta) = \sec\left(\frac{1}{2}\theta\right)\sec^2\left(\frac{1}{2}\theta\right)\frac{1}{2} + \tan\left(\frac{1}{2}\theta\right)\sec\left(\frac{1}{2}\theta\right)\tan\left(\frac{1}{2}\theta\right)\frac{1}{2}$

$= \frac{1}{2}\sec\left(\frac{1}{2}\theta\right)\left[\sec^2\left(\frac{1}{2}\theta\right) + \tan^2\left(\frac{1}{2}\theta\right)\right]$

51. $f(x) = \dfrac{\cot x}{\sin x} = \dfrac{\cos x}{\sin^2 x}$

$f'(x) = \dfrac{\sin^2 x(-\sin x) - \cos x(2\sin x\cos x)}{\sin^4 x}$

$= \dfrac{-\sin^2 x - 2\cos^2 x}{\sin^3 x} = \dfrac{-1-\cos^2 x}{\sin^3 x}$

52. $g(v) = \dfrac{\cos v}{\csc v} = \cos v \cdot \sin v$

$g'(v) = \cos v(\cos v) + \sin v(-\sin v)$

$= \cos^2 v - \sin^2 v = \cos 2v$

53. $y = 4\sec^2 x$

$y' = 8\sec x \cdot \sec x\tan x = 8\sec^2 x\tan x$

54. $g(t) = 5\cos^2\pi t = 5(\cos\pi t)^2$

$g'(t) = 10\cos\pi t(-\sin\pi t)(\pi)$

$= -10\pi(\sin\pi t)(\cos\pi t)$

$= -5\pi\sin 2\pi t$

55. $f(\theta) = \tan^2 5\theta = (\tan 5\theta)^2$

$f'(\theta) = 2(\tan 5\theta)(\sec^2 5\theta)5 = 10\tan 5\theta\sec^2 5\theta$

56. $g(\theta) = \cos^2 8\theta = (\cos 8\theta)^2$

$g'(\theta) = 2(\cos 8\theta)(-\sin 8\theta)8 = -16\cos 8\theta\sin 8\theta$

57. $f(\theta) = \frac{1}{4}\sin^2 2\theta = \frac{1}{4}(\sin 2\theta)^2$

$f'(\theta) = 2\left(\frac{1}{4}\right)(\sin 2\theta)(\cos 2\theta)(2)$

$= \sin 2\theta\cos 2\theta = \frac{1}{2}\sin 4\theta$

58. $h(t) = 2\cot^2(\pi t + 2)$

$h'(t) = 4\cot(\pi t + 2)(-\csc^2(\pi t + 2)(\pi))$

$= -4\pi\cot(\pi t + 2)\csc^2(\pi t + 2)$

59. $f(t) = 3\sec^2(\pi t - 1)$

$f'(t) = 6\sec(\pi t - 1)\sec(\pi t - 1)\tan(\pi t - 1)(\pi)$

$= 6\pi\sec^2(\pi t - 1)\tan(\pi t - 1) = \dfrac{6\pi\sin(\pi t - 1)}{\cos^3(\pi t - 1)}$

60. $y = 3x - 5\cos(\pi x)^2 = 3x - 5\cos(\pi^2 x^2)$

$\dfrac{dy}{dx} = 3 + 5\sin(\pi^2x^2)(2\pi^2 x) = 3 + 10\pi^2 x\sin(\pi x)^2$

61. $y = \sqrt{x} + \dfrac{1}{4}\sin(2x)^2 = \sqrt{x} + \dfrac{1}{4}\sin(4x^2)$

$\dfrac{dy}{dx} = \dfrac{1}{2}x^{-1/2} + \dfrac{1}{4}\cos(4x^2)(8x) = \dfrac{1}{2\sqrt{x}} + 2x\cos(2x)^2$

62. $y = \sin x^{1/3} + (\sin x)^{1/3}$

$y' = \cos x^{1/3}\left(\dfrac{1}{3}x^{-2/3}\right) + \dfrac{1}{3}(\sin x)^{-2/3}\cos x$

$= \dfrac{1}{3}\left[\dfrac{\cos x^{1/3}}{x^{2/3}} + \dfrac{\cos x}{(\sin x)^{2/3}}\right]$

63. $y = \sin(\tan 2x)$

$y' = \cos(\tan 2x)(\sec^2 2x)(2) = 2\cos(\tan 2x)\sec^2 2x$

64. $y = \cos\sqrt{\sin(\tan\pi x)}$

$y' = -\sin\sqrt{\sin(\tan\pi x)} \cdot \dfrac{1}{2}(\sin(\tan\pi x))^{-1/2}\cos(\tan\pi x)\sec^2\pi x(\pi) = \dfrac{-\pi\sin\sqrt{\sin(\tan\pi x)}\cos(\tan\pi x)\sec^2\pi x}{2\sqrt{\sin(\tan\pi x)}}$

65. $y = \sqrt{x^2 + 8x} = (x^2 + 8x)^{1/2}$, $(1, 3)$

$y' = \dfrac{1}{2}(x^2 + 8x)^{-1/2}(2x + 8) = \dfrac{2(x+4)}{2(x^2+8x)^{1/2}} = \dfrac{x+4}{\sqrt{x^2+8x}}$

$y'(1) = \dfrac{1+4}{\sqrt{1^2 + 8(1)}} = \dfrac{5}{\sqrt{9}} = \dfrac{5}{3}$

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66. $y = \left(3x^3 + 4x\right)^{1/5}, \quad (2, 2)$

$$y' = \frac{1}{5}\left(3x^3 + 4x\right)^{-4/5}\left(9x^2 + 4\right)$$
$$= \frac{9x^2 + 4}{5\left(3x^3 + 4x\right)^{4/5}}$$
$$y'(2) = \frac{1}{2}$$

67. $f(x) = \frac{5}{x^3 - 2} = 5\left(x^3 - 2\right)^{-1}, \quad \left(-2, -\frac{1}{2}\right)$

$$f'(x) = -5\left(x^3 - 2\right)^{-2}\left(3x^2\right) = \frac{-15x^2}{\left(x^3 - 2\right)^2}$$
$$f'(-2) = -\frac{60}{100} = -\frac{3}{5}$$

68. $f(x) = \frac{1}{\left(x^2 - 3x\right)^2} = \left(x^2 - 3x\right)^{-2}, \quad \left(4, \frac{1}{16}\right)$

$$f'(x) = -2\left(x^2 - 3x\right)^{-3}(2x - 3) = \frac{-2(2x - 3)}{\left(x^2 - 3x\right)^3}$$
$$f'(4) = -\frac{5}{32}$$

69. $f(t) = \frac{3t + 2}{t - 1}, \quad (0, -2)$

$$f'(t) = \frac{(t - 1)(3) - (3t + 2)(1)}{(t - 1)^2}$$
$$= \frac{3t - 3 - 3t - 2}{(t - 1)^2}$$
$$= \frac{-5}{(t - 1)^2}$$
$$f'(0) = -5$$

70. $f(x) = \frac{x + 4}{2x - 5}, \quad (9, 1)$

$$f'(x) = \frac{(2x - 5)(1) - (x + 4)(2)}{(2x - 5)^2}$$
$$= \frac{2x - 5 - 2x - 8}{(2x - 5)^2}$$
$$= -\frac{13}{(2x - 5)^2}$$
$$f'(9) = -\frac{13}{(18 - 5)^2} = -\frac{1}{13}$$

71. $y = 26 - \sec^3 4x, \quad (0, 25)$

$$y' = -3\sec^2 4x \sec 4x \tan 4x\ 4$$
$$= -12\sec^3 4x \tan 4x$$
$$y'(0) = 0$$

72. $y = \frac{1}{x} + \sqrt{\cos x} = x^{-1} + (\cos x)^{1/2}, \quad \left(\frac{\pi}{2}, \frac{2}{\pi}\right)$

$$y' = -x^{-2} + \frac{1}{2}(\cos x)^{-1/2}(-\sin x) = -\frac{1}{x^2} - \frac{\sin x}{2\sqrt{\cos x}}$$

$y'(\pi/2)$ is undefined.

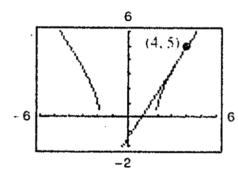
73. (a) $f(x) = \left(2x^2 - 7\right)^{1/2}, \quad (4, 5)$

$$f'(x) = \frac{1}{2}\left(2x^2 - 7\right)^{-1/2}(4x) = \frac{2x}{\sqrt{2x^2 - 7}}$$
$$f'(4) = \frac{8}{5}$$

Tangent line:

$$y - 5 = \frac{8}{5}(x - 4) \Rightarrow 8x - 5y - 7 = 0$$

(b)



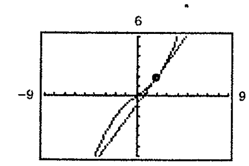
74. (a) $f(x) = \frac{1}{3}x\sqrt{x^2 + 5} = \frac{1}{3}x\left(x^2 + 5\right)^{1/2}, (2, 2)$

$$f'(x) = \frac{1}{3}x\left[\frac{1}{2}\left(x^2 + 5\right)^{-1/2}(2x)\right] + \frac{1}{3}\left(x^2 + 5\right)^{1/2}$$
$$= \frac{x^2}{2\sqrt{x^2 + 5}} + \frac{1}{3}\sqrt{x^2 + 5}$$
$$f'(2) = \frac{4}{3(3)} + \frac{1}{3}(3) = \frac{13}{9}$$

Tangent line:

$$y - 2 = \frac{13}{9}(x - 2) \Rightarrow 13x - 9y - 8 = 0$$

(b)



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75. (a) $y = \left(4x^3 + 3\right)^2, \quad (-1, 1)$

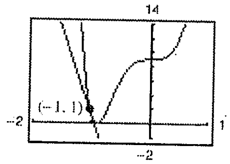
$$y' = 2\left(4x^3 + 3\right)\left(12x^2\right) = 24x^2\left(4x^3 + 3\right)$$

$$y'(-1) = -24$$

Tangent line:

$$y - 1 = -24(x + 1) \Rightarrow 24x + y + 23 = 0$$

(b)



76. (a) $f(x) = \left(9 - x^2\right)^{2/3}, \quad (1, 4)$

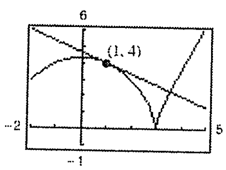
$$f'(x) = \frac{2}{3}\left(9 - x^2\right)^{-1/3}(-2x) = \frac{-4x}{3\left(9 - x^2\right)^{1/3}}$$

$$f'(1) = \frac{-4}{3(8)^{1/3}} = -\frac{2}{3}$$

Tangent line:

$$y - 4 = -\frac{2}{3}(x - 1) \Rightarrow 2x + 3y - 14 = 0$$

(b)



77. (a) $f(x) = \sin 2x, \quad (\pi, 0)$

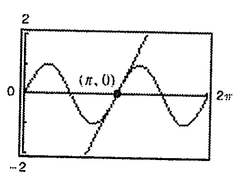
$$f'(x) = 2\cos 2x$$

$$f'(\pi) = 2$$

Tangent line:

$$y = 2(x - \pi) \Rightarrow 2x - y - 2\pi = 0$$

(b)



78. (a) $y = \cos 3x, \quad \left(\frac{\pi}{4}, -\frac{\sqrt{2}}{2}\right)$

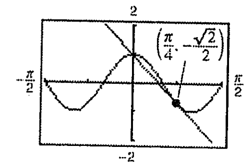
$$y' = -3\sin 3x$$

$$y'\left(\frac{\pi}{4}\right) = -3\sin\left(\frac{3\pi}{4}\right) = \frac{-3\sqrt{2}}{2}$$

Tangent line: $y + \frac{\sqrt{2}}{2} = \frac{-3\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$

$$y = \frac{-3\sqrt{2}}{2}x + \frac{3\sqrt{2}\pi}{8} - \frac{\sqrt{2}}{2}$$

(b)



79. (a) $f(x) = \tan^2 x, \quad \left(\frac{\pi}{4}, 1\right)$

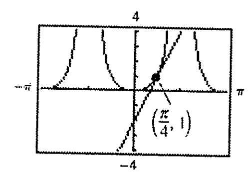
$$f'(x) = 2\tan x \sec^2 x$$

$$f'\left(\frac{\pi}{4}\right) = 2(1)(2) = 4$$

Tangent line:

$$y - 1 = 4\left(x - \frac{\pi}{4}\right) \Rightarrow 4x - y + (1 - \pi) = 0$$

(b)



80. (a) $y = 2\tan^3 x, \quad \left(\frac{\pi}{4}, 2\right)$

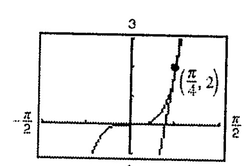
$$y' = 6\tan^2 x \cdot \sec^2 x$$

$$y'\left(\frac{\pi}{4}\right) = 6(1)(2) = 12$$

Tangent line:

$$y - 2 = 12\left(x - \frac{\pi}{4}\right) \Rightarrow 12x - y + (2 - 3\pi) = 0$$

(b)



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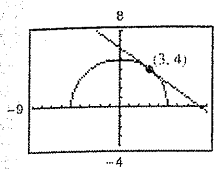
81. $f(x) = \sqrt{25 - x^2} = (25 - x^2)^{1/2}, \quad (3, 4)$

$f'(x) = \frac{1}{2}(25 - x^2)(-2x) = \frac{-x}{\sqrt{25 - x^2}}$

$f'(3) = -\frac{3}{4}$

Tangent line:

$y - 4 = -\frac{3}{4}(x - 3) \Rightarrow 3x + 4y - 25 = 0$

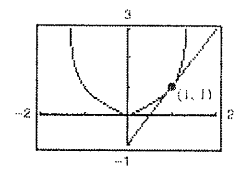


82. $f(x) = \frac{|x|}{\sqrt{2 - x^2}} = |x|(2 - x^2)^{-1/2}, \quad (1, 1)$

$f'(x) = \frac{2}{(2 - x^2)^{3/2}}$ for $x > 0$

$f'(1) = 2$

Tangent line: $y - 1 = 2(x - 1) \Rightarrow 2x - y - 1 = 0$



83.

$$f(x) = 2\cos x + \sin 2x, \quad 0 < x < 2\pi$$

$$f'(x) = -2\sin x + 2\cos 2x$$

$$= -2\sin x + 2 - 4\sin^2 x = 0$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(\sin x + 1)(2\sin x - 1) = 0$$

$$\sin x = -1 \Rightarrow x = \frac{3\pi}{2}$$

$$\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$$

Horizontal tangents at $x = \frac{\pi}{6}, \frac{3\pi}{2}, \frac{5\pi}{6}$

Horizontal tangent at the points $\left(\frac{\pi}{6}, \frac{3\sqrt{3}}{2}\right), \left(\frac{3\pi}{2}, 0\right)$, and $\left(\frac{5\pi}{6}, -\frac{3\sqrt{3}}{2}\right)$

84.

$$f(x) = \frac{x}{\sqrt{2x - 1}}$$

$$f'(x) = \frac{(2x - 1)^{1/2} - x(2x - 1)^{-1/2}}{2x - 1}$$

$$= \frac{2x - 1 - x}{(2x - 1)^{3/2}}$$

$$= \frac{x - 1}{(2x - 1)^{3/2}}$$

$$\frac{x - 1}{(2x - 1)^{3/2}} = 0 \Rightarrow x = 1$$

Horizontal tangent at $(1, 1)$

85. $f(x) = 5(2 - 7x)^4$

$f'(x) = 20(2 - 7x)^3(-7) = -140(2 - 7x)^3$

$f''(x) = -420(2 - 7x)^2(-7) = 2940(2 - 7x)^2$

86. $f(x) = 6(x^3 + 4)^3$

$f'(x) = 18(x^3 + 4)^2(3x^2) = 54x^2(x^3 + 4)^2$

$f''(x) = 54x^2(2)(x^3 + 4)(3x^2) + 108x(x^3 + 4)^2$

$= 108x(x^3 + 4)[3x^3 + x^3 + 4]$

$= 432x(x^3 + 4)(x^3 + 1)$

87. $f(x) = \frac{1}{x - 6} = (x - 6)^{-1}$

$f'(x) = -(x - 6)^{-2}$

$f''(x) = 2(x - 6)^{-3} = \frac{2}{(x - 6)^3}$

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88. $f(x) = \dfrac{8}{(x-2)^2} = 8(x-2)^{-2}$

$f'(x) = -16(x-2)^{-3}$

$f''(x) = 48(x-2)^{-4} = \dfrac{48}{(x-2)^4}$

89. $f(x) = \sin x^2$

$f'(x) = 2x\cos x^2$

$f''(x) = 2x\left[2x\left(-\sin x^2\right)\right] + 2\cos x^2$

$= 2\left(\cos x^2 - 2x^2 \sin x^2\right)$

90. $f(x) = \sec^2 \pi x$

$f'(x) = 2\sec \pi x(\pi \sec \pi x \tan \pi x)$

$= 2\pi \sec^2 \pi x \tan \pi x$

$f''(x) = 2\pi \sec^2 \pi x\left(\sec^2 \pi x\right)(\pi) + 2\pi \tan \pi x\left(2\pi \sec^2 \pi x \tan \pi x\right)$

$= 2\pi^2 \sec^4 \pi x + 4\pi^2 \sec^2 \pi x \tan^2 \pi x$

$= 2\pi^2 \sec^2 \pi x\left(\sec^2 \pi x + 2\tan^2 \pi x\right)$

$= 2\pi^2 \sec^2 \pi x\left(3\sec^2 \pi x - 2\right)$

91. $h(x) = \frac{1}{9}(3x+1)^3, \quad \left(1, \frac{64}{9}\right)$

$h'(x) = \frac{1}{9}3(3x+1)^2(3) = (3x+1)^2$

$h''(x) = 2(3x+1)(3) = 18x + 6$

$h''(1) = 24$

92. $f(x) = \dfrac{1}{\sqrt{x+4}} = (x+4)^{-1/2}, \quad \left(0, \dfrac{1}{2}\right)$

$f'(x) = -\dfrac{1}{2}(x+4)^{-3/2}$

$f''(x) = \dfrac{3}{4}(x+4)^{-5/2} = \dfrac{3}{4(x+4)^{5/2}}$

$f''(0) = \dfrac{3}{128}$

93. $f(x) = \cos x^2, \quad (0, 1)$

$f'(x) = -\sin\left(x^2\right)(2x) = -2x\sin\left(x^2\right)$

$f''(x) = -2x\cos\left(x^2\right)(2x) - 2\sin\left(x^2\right)$

$= -4x^2\cos\left(x^2\right) - 2\sin\left(x^2\right)$

$f''(0) = 0$

94. $g(t) = \tan 2t, \quad \left(\dfrac{\pi}{6}, \sqrt{3}\right)$

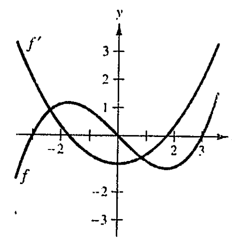
$g'(t) = 2\sec^2(2t)$

$g''(t) = 4\sec(2t)\cdot\sec(2t)\tan(2t)2$

$= 8\sec^2(2t)\tan(2t)$

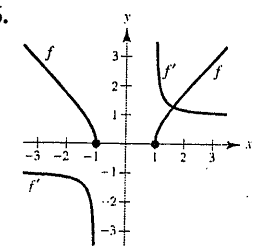
$g''\left(\dfrac{\pi}{6}\right) = 32\sqrt{3}$

95.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

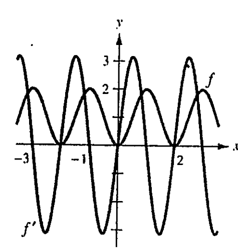
96.



f is decreasing on $(-\infty, -1)$ so f' must be negative there.

f is increasing on $(1, \infty)$ so f' must be positive there.

97.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

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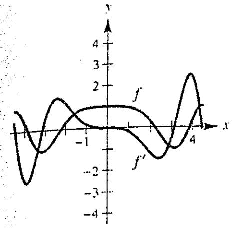
98.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

99. $g(x) = f(3x)$

$g'(x) = f'(3x)(3) \Rightarrow g'(x) = 3f'(3x)$

100. $g(x) = f(x^2)$

$g'(x) = f'(x^2)(2x) \Rightarrow g'(x) = 2xf'(x^2)$

101. (a) $g(x) = f(x) - 2 \Rightarrow g'(x) = f'(x)$

(b) $h(x) = 2f(x) \Rightarrow h'(x) = 2f'(x)$

(c) $r(x) = f(-3x) \Rightarrow r'(x) = f'(-3x)(-3) = -3f'(-3x)$

So, you need to know $f'(-3x)$.

$r'(0) = -3f'(0) = (-3)\left(-\frac{1}{3}\right) = 1$

$r'(-1) = -3f'(3) = (-3)(-4) = 12$

(d) $s(x) = f(x + 2) \Rightarrow s'(x) = f'(x + 2)$

So, you need to know $f'(x + 2)$.

$s'(-2) = f'(0) = -\frac{1}{3}$, etc.

x	−2	−1	0	1	2	3
$f'(x)$	4	$\frac{2}{3}$	$-\frac{1}{3}$	−1	−2	−4
$g'(x)$	4	$\frac{2}{3}$	$-\frac{1}{3}$	−1	−2	−4
$h'(x)$	8	$\frac{4}{3}$	$-\frac{2}{3}$	−2	−4	−8
$r'(x)$		12	1			
$s'(x)$	$-\frac{1}{3}$	−1	−2	−4		

102. (a) $f(x) = g(x)h(x)$

$f'(x) = g(x)h'(x) + g'(x)h(x)$

$f'(5) = (-3)(-2) + (6)(3) = 24$

(b) $f(x) = g(h(x))$

$f'(x) = g'(h(x))h'(x)$

$f'(5) = g'(3)(-2) = -2g'(3)$

Not possible, you need $g'(3)$ to find $f'(5)$.

(c) $f(x) = \dfrac{g(x)}{h(x)}$

$$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2}$$

$$f'(5) = \frac{(3)(6) - (-3)(-2)}{(3)^2} = \frac{12}{9} = \frac{4}{3}$$

(d) $f(x) = [g(x)]^3$

$f'(x) = 3[g(x)]^2 g'(x)$

$f'(5) = 3(-3)^2(6) = 162$

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103. (a) $h(x) = f(g(x)), g(1) = 4, g'(1) = -\frac{1}{2}, f'(4) = -1$

$h'(x) = f'(g(x))g'(x)$

$h'(1) = f'(g(1))g'(1) = f'(4)g'(1) = (-1)(-\frac{1}{2}) = \frac{1}{2}$

(b) $s(x) = g(f(x)), f(5) = 6, f'(5) = -1, g'(6)$ does not exist.

$s'(x) = g'(f(x))f'(x)$

$s'(5) = g'(f(5))f'(5) = g'(6)(-1)$

$s'(5)$ does not exist because g is not differentiable at 6.

104. (a) $h(x) = f(g(x))$

$h'(x) = f'(g(x))g'(x)$

$h'(3) = f'(g(3))g'(3) = f'(5)(1) = \frac{1}{2}$

(b) $s(x) = g(f(x))$

$s'(x) = g'(f(x))f'(x)$

$s'(9) = g'(f(9))f'(9) = g'(8)(2) = (-1)(2) = -2$

105. (a) $F = 132{,}400(331 - v)^{-1}$

$F' = (-1)(132{,}400)(331 - v)^{-2}(-1) = \dfrac{132{,}400}{(331 - v)^2}$

When $v = 30, F' \approx 1.461$.

(b) $F = 132{,}400(331 + v)^{-1}$

$F' = (-1)(132{,}400)(331 + v)^{-2}(-1) = \dfrac{-132{,}400}{(331 + v)^2}$

When $v = 30, F' \approx -1.016$.

106. $y = \frac{1}{3}\cos 12t - \frac{1}{4}\sin 12t$

$v = y' = \frac{1}{3}[-12 \sin 12t] - \frac{1}{4}[12 \cos 12t]$

$= -4 \sin 12t - 3 \cos 12t$

When $t = \pi/8, y = 0.25$ ft and $v = 4$ ft/sec.

107. $\theta = 0.2 \cos 8t$

The maximum angular displacement is $\theta = 0.2$ (because $-1 \le \cos 8t \le 1$).

$\dfrac{d\theta}{dt} = 0.2[-8 \sin 8t] = -1.6 \sin 8t$

When $t = 3, d\theta/dt = -1.6 \sin 24 \approx 1.4489$ rad/sec.

108. $y = A \cos \omega t$

(a) Amplitude: $A = \dfrac{3.5}{2} = 1.75$

$y = 1.75 \cos \omega t$

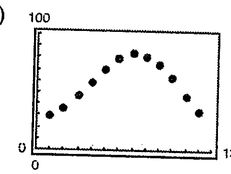
Period: $10 \Rightarrow \omega = \dfrac{2\pi}{10} = \dfrac{\pi}{5}$

$y = 1.75 \cos \dfrac{\pi t}{5}$

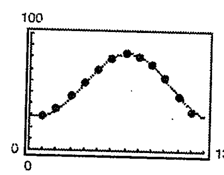
(b) $v = y' = 1.75\left[-\dfrac{\pi}{5} \sin \dfrac{\pi t}{5}\right] = -0.35\pi \sin \dfrac{\pi t}{5}$

109. (a) Using a graphing utility, you obtain a model similar to $T(t) = 56.1 + 27.6 \sin(0.48t - 1.86)$.

(a)

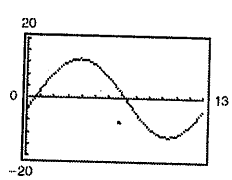


(b)



The model is a good fit.

(c)



$T'(t) \approx 13.25 \cos(0.48t - 1.86)$

(d) The temperature changes most rapidly around spring (March–May), and fall (Oct–Nov).

110. (a) According to the graph $C'(4) > C'(1)$.

(b) Answers will vary.

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111. $N = 400\left[1 - \frac{3}{(t^2+2)^2}\right] = 400 - 1200(t^2+2)^{-2}$

$N'(t) = 2400(t^2+2)^{-3}(2t) = \frac{4800t}{(t^2+2)^3}$

(a) $N'(0) = 0$ bacteria/day

(b) $N'(1) = \frac{4800(1)}{(1+2)^3} = \frac{4800}{27} \approx 177.8$ bacteria/day

(c) $N'(2) = \frac{4800(2)}{(4+2)^3} = \frac{9600}{216} \approx 44.4$ bacteria/day

(d) $N'(3) = \frac{4800(3)}{(9+2)^3} = \frac{14{,}400}{1331} \approx 10.8$ bacteria/day

(e) $N'(4) = \frac{4800(4)}{(16+2)^3} = \frac{19{,}200}{5832} \approx 3.3$ bacteria/day

(f) The rate of change of the population is decreasing as $t \to \infty$.

112. (a) $V = \frac{k}{\sqrt{t+1}}$

$V(0) = 10{,}000 = \frac{k}{\sqrt{0+1}} = k$

$V = \frac{10{,}000}{\sqrt{t+1}} = 10{,}000(t+1)^{-1/2}$

(b) $\frac{dV}{dt} = 10{,}000\left(-\frac{1}{2}\right)(t+1)^{-3/2} = \frac{-5000}{(t+1)^{3/2}}$

$V'(1) = \frac{-5000}{2^{3/2}} \approx -1767.77$ dollars/year

(c) $V'(3) = \frac{-5000}{4^{3/2}} = \frac{-5000}{8} = -625$ dollars/year

113. $f(x) = \sin \beta x$

(a) $f'(x) = \beta \cos \beta x$

$f''(x) = -\beta^2 \sin \beta x$

$f'''(x) = -\beta^3 \cos \beta x$

$f^{(4)} = \beta^4 \sin \beta x$

(b) $f''(x) + \beta^2 f(x) = -\beta^2 \sin \beta x + \beta^2(\sin \beta x) = 0$

(c) $f^{(2k)}(x) = (-1)^k \beta^{2k} \sin \beta x$

$f^{(2k-1)}(x) = (-1)^{k+1} \beta^{2k-1} \cos \beta x$

114. (a) Yes, if $f(x+p) = f(x)$ for all x, then $f'(x+p) = f'(x)$, which shows that f' is periodic as well.

(b) Yes, if $g(x) = f(2x)$, then $g'(x) = 2f'(2x)$. Because f' is periodic, so is g'.

115. (a) $r'(x) = f'(g(x))g'(x)$

$r'(1) = f'(g(1))g'(1)$

Note that $g(1) = 4$ and $f'(4) = \frac{5-0}{6-2} = \frac{5}{4}$.

Also, $g'(1) = 0$. So, $r'(1) = 0$.

(b) $s'(x) = g'(f(x))f'(x)$

$s'(4) = g'(f(4))f'(4)$

Note that $f(4) = \frac{5}{2}$, $g'\left(\frac{5}{2}\right) = \frac{6-4}{6-2} = \frac{1}{2}$ and

$f'(4) = \frac{5}{4}$. So, $s'(4) = \frac{1}{2}\left(\frac{5}{4}\right) = \frac{5}{8}$.

116. (a) $g(x) = \sin^2 x + \cos^2 x = 1 \Rightarrow g'(x) = 0$

$g'(x) = 2 \sin x \cos x + 2 \cos x(-\sin x) = 0$

(b) $\tan^2 x + 1 = \sec^2 x$

$g(x) + 1 = f(x)$

Taking derivatives of both sides, $g'(x) = f'(x)$.

Equivalently,

$f'(x) = 2 \sec x \cdot \sec x \tan x = 2 \sec^2 x \tan x$ and

$g'(x) = 2 \tan x \cdot \sec^2 x = 2 \sec^2 x \tan x$, which are the same.

117. (a) If $f(-x) = -f(x)$, then

$\frac{d}{dx}[f(-x)] = \frac{d}{dx}[-f(x)]$

$f'(-x)(-1) = -f'(x)$

$f'(-x) = f'(x)$.

So, $f'(x)$ is even.

(b) If $f(-x) = f(x)$, then

$\frac{d}{dx}[f(-x)] = \frac{d}{dx}[f(x)]$

$f'(-x)(-1) = f'(x)$

$f'(-x) = -f'(x)$.

So, f' is odd.

118. $|u| = \sqrt{u^2}$

$\frac{d}{dx}[|u|] = \frac{d}{dx}\left[\sqrt{u^2}\right] = \frac{1}{2}(u^2)^{-1/2}(2uu')$

$= \frac{uu'}{\sqrt{u^2}} = u'\frac{u}{|u|}, \quad u \neq 0$

119. $g(x) = |3x - 5|$

$g'(x) = 3\left(\frac{3x-5}{|3x-5|}\right), \quad x \neq \frac{5}{3}$

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120. $f(x) = |x^2 - 9|$

$$f'(x) = 2x\left(\frac{x^2 - 9}{|x^2 - 9|}\right), \quad x \neq \pm 3$$

121. $h(x) = |x|\cos x$

$$h'(x) = -|x|\sin x + \frac{x}{|x|}\cos x, \quad x \neq 0$$

122. $f(x) = |\sin x|$

$$f'(x) = \cos x\left(\frac{\sin x}{|\sin x|}\right), x \neq k\pi$$

123. (a) $f(x) = \tan x \qquad f(\pi/4) = 1$

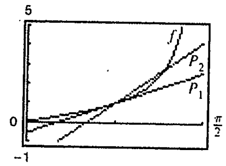
$f'(x) = \sec^2 x \qquad f'(\pi/4) = 2$

$f''(x) = 2\sec^2 x \tan x \qquad f''(\pi/4) = 4$

$P_1(x) = 2(x - \pi/4) + 1$

$$P_2(x) = \frac{1}{2}(4)(x - \pi/4)^2 + 2(x - \pi/4) + 1$$
$$= 2(x - \pi/4)^2 + 2(x - \pi/4) + 1$$

(b)



(c) P_2 is a better approximation than P_1.

(d) The accuracy worsens as you move away from $x = \pi/4$.

124. (a) $f(x) = \sec x \qquad f(\pi/6) = \dfrac{2}{\sqrt{3}}$

$f'(x) = \sec x \tan x \qquad f'(\pi/6) = \dfrac{2}{3}$

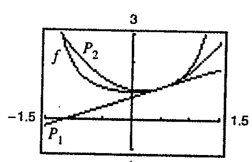
$f''(x) = \sec x(\sec^2 x) + \tan x(\sec x \tan x) \qquad f''(\pi/6) = \dfrac{10\sqrt{3}}{9}$

$= \sec^3 x + \sec x \tan^2 x$

$$P_1(x) = \frac{2}{3}(x - \pi/6) + \frac{2}{\sqrt{3}}$$

$$P_2(x) = \frac{1}{2}\cdot\left(\frac{10}{3\sqrt{3}}\right)\left(x - \frac{\pi}{6}\right)^2 + \frac{2}{3}\left(x - \frac{\pi}{6}\right) + \frac{2}{\sqrt{3}}$$
$$= \left(\frac{5}{3\sqrt{3}}\right)\left(x - \frac{\pi}{6}\right)^2 + \frac{2}{3}\left(x - \frac{\pi}{6}\right) + \frac{2}{\sqrt{3}}$$

(b)



(c) P_2 is a better approximation than P_1.

(d) The accuracy worsens as you move away from $x = \pi/6$.

125. False. If $y = (1 - x)^{1/2}$, then $y' = \frac{1}{2}(1 - x)^{-1/2}(-1)$.

126. False. If $f(x) = \sin^2 2x$, then $f'(x) = 2(\sin 2x)(2\cos 2x)$.

127. True

128. True

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