Ch. 2.5 Exercise Problems Derivatives of Trig Functions

p. 212-214 #17-35 odd, 49, 55

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x \qquad \frac{d}{dx}\csc x = -\csc x \cot x$$

17) $y = \overbrace{\pi\sec u}^{f}\,\overbrace{\tan u}^{g}$ *product Rule

$$y' = \overbrace{\pi\sec u\tan u}^{f'} \cdot \overbrace{\tan u}^{g} + \overbrace{\pi\sec u}^{f} \cdot \overbrace{\sec^2 u}^{g'}$$

$$\boxed{y' = \pi\sec u\tan^2 u + \pi\sec^3 u}$$

19) $y = \dfrac{\cot x}{x}$ *quotient Rule

$$y' = \frac{\overbrace{-\csc^2 x}^{f'} \cdot \overbrace{x}^{g} - \overbrace{\cot x}^{f} \cdot \overbrace{(1)}^{g'}}{\underbrace{(x)^2}_{g^2}}$$

$$\boxed{y' = \frac{-x\csc^2 x - \cot x}{x^2}}$$

21) $y = \overbrace{x^2}^{f}\,\overbrace{\sin x}^{g}$ *product Rule

$$y' = \overbrace{2x}^{f'} \cdot \overbrace{\sin x}^{g} + \overbrace{x^2}^{f} \cdot \overbrace{\cos x}^{g'} \rightarrow \boxed{y' = 2x\sin x + x^2\cos x}$$

23) $y = \overbrace{t}^{f}\,\overbrace{\tan(t)}^{g} - \sqrt{3}\sec(t)$

$$y' = \overbrace{(1)}^{f'}\,\overbrace{\tan(t)}^{g} + \overbrace{t}^{f} \cdot \overbrace{\sec^2(t)}^{g'} - \sqrt{3}\sec(t)\tan(t)$$

$$\boxed{y' = \tan(t) + t\sec^2(t) - \sqrt{3}\sec(t)\tan(t)}$$

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25) $y = \dfrac{\overbrace{\sin\theta}^{f}}{\underbrace{1-\cos\theta}_{g}}$ $\qquad y' = \dfrac{\overbrace{\cos\theta}^{f'} \cdot \overbrace{(1-\cos\theta)}^{g} - \overbrace{\sin\theta}^{f} \cdot \overbrace{(--\sin\theta)}^{g'}}{\underbrace{(1-\cos\theta)^2}_{g^2}}$

$\cos^2\theta + \sin^2\theta = 1$

$$y' = \frac{\cos\theta - \cos^2\theta - \sin^2\theta}{(1-\cos\theta)^2} \rightarrow \frac{\cos\theta - (\cos^2\theta + \sin^2\theta)}{(1-\cos\theta)^2}$$

$$y' = \frac{\cancel{(\cos\theta - 1)}(-1)}{\cancel{(1-\cos\theta)}(1-\cos\theta)} \qquad \boxed{y' = \frac{-1}{1-\cos\theta} \text{ or } y' = \frac{1}{\cos\theta - 1}}$$

27) $y = \dfrac{\overbrace{\sin(t)}^{f}}{\underbrace{1+t}_{g}}$ $\qquad y' = \dfrac{\overbrace{\cos(t)}^{f'}\overbrace{(1+t)}^{g} - \overbrace{\sin(t)}^{f}\overbrace{(1)}^{g'}}{\underbrace{(1+t)^2}_{g^2}}$

$$\boxed{y' = \frac{\cos(t) + t\cos(t) - \sin(t)}{(1+t)^2}}$$

29) $y = \dfrac{\sin x}{e^x}$ $\qquad y' = \dfrac{\overbrace{\cos x}^{f'} \cdot \overbrace{e^x}^{g} - \overbrace{\sin x}^{f} \cdot \overbrace{e^x}^{g'}}{\underbrace{(e^x)^2}_{g^2}}$ $\qquad * \dfrac{d}{dx} e^x = e^x$

$$y' = \frac{e^x\cos x - e^x \sin x}{e^{2x}} \rightarrow \frac{\cancel{e^x}(\cos x - \sin x)}{(\cancel{e^x})(e^x)} \qquad \boxed{y' = \frac{\cos x - \sin x}{e^x}}$$

31) $y = \dfrac{\overbrace{\sin\theta + \cos\theta}^{f}}{\underbrace{\sin\theta - \cos\theta}_{g}}$

$$y' = \frac{\overbrace{(\cos\theta - \sin\theta)}^{f'}\overbrace{(\sin\theta - \cos\theta)}^{g} - \overbrace{(\sin\theta + \cos\theta)}^{f}\overbrace{(\cos\theta + \sin\theta)}^{g'}}{\underbrace{(\sin\theta - \cos\theta)^2}_{g^2}}$$

$-(\cos^2\theta + \sin^2\theta)$
$= -(1) = -1$

$$y' = \frac{\cancel{\cos\theta\sin\theta} - \cos^2\theta - \sin^2\theta + \cancel{\sin\theta\cos\theta} - \cancel{\sin\theta\cos\theta} - \sin^2\theta - \cos^2\theta - \cancel{\cos\theta\sin\theta}}{(\sin\theta - \cos\theta)^2}$$

$$y' = \frac{-1-1}{(\sin\theta - \cos\theta)^2} \qquad \boxed{y' = \frac{-2}{(\sin\theta - \cos\theta)^2}}$$

$\boxed{2.5}$

33) $y = \frac{\overbrace{\sec(t)}^{f}}{\underbrace{1+t\sin t}_{g}} \rightarrow y' = \frac{\overbrace{\sec t \tan t}^{f'} \cdot \overbrace{(1+t\sin t)}^{g} - \overbrace{\sec(t)}^{f}\overbrace{[0+1\sin(t)+t\cos(t)]}^{g'}}{\underbrace{(1+t\sin t)^2}_{g^2}}$

product Rule within g'

$$y' = \frac{\sec(t)\tan(t) + t\sec(t)\sin(t)\tan(t) - \sec(t)\sin t - t\cos(t)\sec(t)}{(1+t\sin t)^2}$$

$$y' = \frac{\sec(t)\tan(t) + t\cdot\left(\frac{1}{\cos t}\right)\sin t\cdot\tan(t) - \frac{1}{\cos(t)}\cdot\sin(t) - t\cdot\cancel{\cos(t)}\cdot\frac{1}{\cancel{\cos(t)}}}{(1+t\sin t)^2}$$

$$\boxed{y' = \frac{\sec(t)\tan(t) + t\tan^2(t) - \tan(t) - t}{(1+t\sin t)^2}}$$

35) $y = \overbrace{\csc\theta}^{f}\overbrace{\cot\theta}^{g}$ $\quad y' = \overbrace{-\csc\theta\cot\theta}^{f'} \cdot \overbrace{\cot\theta}^{g} + \overbrace{\csc\theta}^{f} \cdot \overbrace{-\csc^2\theta}^{g'}$

$$\boxed{y' = -\csc\theta\cot^2\theta - \csc^3\theta}$$

49) $y = a\sin x + b\cos x$

* treat a and b as constant coefficients. No need for product Rule.

$y' = a\cos x + b(-\sin x)$

$y'' = a(-\sin x) + b(-\cos x)$

$$\boxed{y'' = -a\sin x - b\cos x}$$

55) Find equation of tangent line at given point

$f(x) = \sin x + \cos x$ at $(\pi/4, \sqrt{2})$

$f'(x) = \cos x - \sin x$ $\qquad$ $f'(\pi/4) = \cos(\pi/4) - \sin(\pi/4)$

$f'(\pi/4) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = 0$

point: $(\pi/4, \sqrt{2})$

slope: $m = 0$ $\rightarrow$ $\boxed{y - \sqrt{2} = 0(x - \pi/4) \quad \text{OR} \quad y = \sqrt{2}}$