2017

Ch. 2.5 Implicit Differentiation

p. 145-146
#1-9 odd, 17, 21, 22, 23, 24, 29, 31, 45, 47, 57, 51, 58

* Find derivative of each term, attach $\frac{dy}{dx}$ each time you apply derivative to the y variable.

*Apply product rule

5) $x^3 - xy + y^2 = 7$

$$3x^2 + \overbrace{(-1)}^{f'}\overbrace{(y)}^{g} + \overbrace{(-x)}^{f}\overbrace{\left(\frac{dy}{dx}\right)}^{g'} + 2y\left(\frac{dy}{dx}\right) = 0$$

$$3x^2 - y - x\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = 0$$

$$2y\left(\frac{dy}{dx}\right) - x\left(\frac{dy}{dx}\right) = y - 3x^2$$

$$\frac{dy}{dx}(2y - x) = y - 3x^2$$

$$\boxed{\frac{dy}{dx} = \frac{y - 3x^2}{2y - x}}$$

9) $x^3 - 3x^2y + 2xy^2 = 12$

$$3x^2 + \overbrace{(-6x)}^{f'}\overbrace{(y)}^{g} + \overbrace{(-3x^2)}^{f}\overbrace{\left(\frac{dy}{dx}\right)}^{g'} + \overbrace{(2)}^{f'}\overbrace{(y^2)}^{g} + \overbrace{(2x)}^{f}\overbrace{(2y)\frac{dy}{dx}}^{g'} = 0$$

$$3x^2 - 6xy - 3x^2\left(\frac{dy}{dx}\right) + 2y^2 + 4xy\left(\frac{dy}{dx}\right) = 0$$

$$4xy\left(\frac{dy}{dx}\right) - 3x^2\left(\frac{dy}{dx}\right) = 6xy - 3x^2 - 2y^2$$

$$\frac{dy}{dx}\left[4xy - 3x^2\right] = 6xy - 3x^2 - 2y^2$$

$$\boxed{\frac{dy}{dx} = \frac{6xy - 3x^2 - 2y^2}{4xy - 3x^2}}$$

17) $x^2 + y^2 = 64$

Explicit method:

$$y^2 = 64 - x^2$$

$$y = (64 - x^2)^{1/2}$$

$$y' = \tfrac{1}{2}(64 - x^2)^{-1/2}(-2x)$$

$$\boxed{y' = \frac{-x}{\sqrt{64 - x^2}}}$$

$$y' = \frac{-x}{y}$$

Implicit method:

$$2x + 2y\left(\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx}(2y) = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$$

2.5

Find derivative and evaluate derivative at a point:

21) $xy = 6 \quad (-6,-1)$ * Apply product rule, implicit differentiation

$(1)(y) + (x)\left(\frac{dy}{dx}\right) = 0$

$x\left(\frac{dy}{dx}\right) = -y$

$\frac{dy}{dx} = \frac{-y}{x}$

$\left.\frac{dy}{dx}\right|_{(-6,-1)} = \frac{-(-1)}{-6} = \boxed{\frac{-1}{6}}$

point: $(-6,-1)$

slope: $m = -1/6$

$y + 1 = -1/6(x+6)$

29) $(x^2+4)y = 8$ at point $(2,1)$

$(2x)(y) + (x^2+4)\left(\frac{dy}{dx}\right) = 0$

$\frac{dy}{dx}(x^2+4) = -2xy$

$\frac{dy}{dx} = \frac{-2xy}{x^2+4}$

$\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{-2(2)(1)}{2^2+4} = \frac{-4}{8} = \boxed{-1/2}$

31) $(x^2+y^2)^2 = 4x^2y$ at point $(1,1)$ product rule

$2(x^2+y^2)\left[2x + 2y\left(\frac{dy}{dx}\right)\right] = \overbrace{(8x)}^{f'}\overbrace{(y)}^{g} + \overbrace{(4x^2)}^{f}\overbrace{\left(\frac{dy}{dx}\right)}^{g'}$ ← start plugging in (1,1)

$2(1^2+1^2)\left[2 + 2\left(\frac{dy}{dx}\right)\right] = 8(1)(1) + 4(1)^2\left(\frac{dy}{dx}\right)$

$4\left(2 + 2\frac{dy}{dx}\right) = 8 + 4\left(\frac{dy}{dx}\right)$

$8 + 8\left(\frac{dy}{dx}\right) - 8 - 4\left(\frac{dy}{dx}\right) = 0$

$4\left(\frac{dy}{dx}\right) = 0$

$\frac{dy}{dx} = \frac{0}{4} = 0$

$\boxed{\left.\frac{dy}{dx}\right|_{(1,1)} = 0}$

2.5 continued...

Find $\frac{d^2y}{dx^2}$ (2nd derivative) ← * quotient rule

45) $x^2+y^2=4$

$2x+2y\left(\frac{dy}{dx}\right)=0$

$\frac{dy}{dx}=\frac{-2x}{2y}=\boxed{\frac{-x}{y}}$

$\frac{d^2y}{dx^2}=\frac{(-1)(y)-(-x)\left(\frac{dy}{dx}\right)}{y^2}$

$\frac{d^2y}{dx^2}=\frac{-y+x\left(\frac{dy}{dx}\right)}{y^2}$

$\frac{d^2y}{dx^2}=\frac{-y+x\left(\frac{-x}{y}\right)}{y^2}=\frac{-y-\frac{x^2}{y}}{y^2}\cdot\frac{y}{y}$

$\frac{d^2y}{dx^2}=\frac{-y^2-x^2}{y^3}=\frac{-(y^2+x^2)}{y^3}$

$\boxed{\frac{d^2y}{dx^2}=\frac{-4}{y^3}}$

47) $x^2-y^2=36$

$2x-2y\left(\frac{dy}{dx}\right)=0$

$-2y\frac{dy}{dx}=-2x$

$\frac{dy}{dx}=\frac{-2x}{-2y}=\frac{x}{y}$

$\frac{d^2y}{dx^2}=\frac{1(y)-x\left(\frac{dy}{dx}\right)}{y^2}$

$\frac{d^2y}{dx^2}=\frac{y-x\left(\frac{x}{y}\right)}{y^2}$

$\frac{d^2y}{dx^2}=\frac{y-\frac{x^2}{y}}{y^2}\cdot\frac{y}{y}$

$\frac{d^2y}{dx^2}=\frac{y^2-x^2}{y^3}=\frac{-(x^2-y^2)}{y^3}$

$\boxed{\frac{d^2y}{dx^2}=\frac{-36}{y^3}}$

51) Find tangent line equation: $\sqrt{x}+\sqrt{y}=5$ at $(9,4)$

$\frac{1}{2}x^{-1/2}+\frac{1}{2}y^{-1/2}\left(\frac{dy}{dx}\right)=0$

$\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\left(\frac{dy}{dx}\right)=0$

$\frac{dy}{dx}\left(\frac{1}{2\sqrt{y}}\right)=-\frac{1}{2\sqrt{x}}$

$\frac{dy}{dx}=\frac{2\sqrt{y}}{-2\sqrt{x}}=\frac{-\sqrt{y}}{\sqrt{x}}$

$\left.\frac{dy}{dx}\right|_{(9,4)}=\frac{-\sqrt{4}}{\sqrt{9}}=\frac{-2}{3}$

point: $(9,4)$

slope: $m=-\frac{2}{3}$

$\boxed{y-4=\frac{-2}{3}(x-9)}$