Ch. 2.5 Implicit Differentiation Worksheet #1

Finding a Derivative In Exercises 1–16, find dy/dx by implicit differentiation.

Product Rule:

$$\frac{d}{dx}[f(x)g(x)] = f'g + fg'$$

1. $x^2 + y^2 = 9$

2. $x^2 - y^2 = 25$

4. $2x^3 + 3y^3 = 64$

5. $x^3 - xy + y^2 = 7$

6. $x^2y + y^2x = -2$

7. $x^3y^3 - y = x$

Finding and Evaluating a Derivative In Exercises 21–28, find dy/dx by implicit differentiation and evaluate the derivative at the given point.

21. $xy = 6$, $(-6, -1)$

22. $y^3 - x^2 = 4$, $(2, 2)$

24. $x^{2/3} + y^{2/3} = 5$, $(8, 1)$

25)

$(x^2 + 4)y = 8$

Point: $(2, 1)$

Find Equation of tangent line:

51. $\sqrt{x} + \sqrt{y} = 5$, $(9, 4)$

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Key

Finding a Derivative In Exercises 1–16, find dy/dx by implicit differentiation.

Product Rule:
$$\frac{d}{dx}[f(x)g(x)] = f'g + fg'$$

1. $x^2 + y^2 = 9$

$$2x + 2y\left(\frac{dy}{dx}\right) = 0$$
$$2y\left(\frac{dy}{dx}\right) = -2x$$
$$\frac{dy}{dx} = \frac{-2x}{2y}$$
$$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$$

2. $x^2 - y^2 = 25$

$$2x - 2y\left(\frac{dy}{dx}\right) = 0$$
$$-2y\left(\frac{dy}{dx}\right) = -2x$$
$$\frac{dy}{dx} = \frac{-2x}{-2y}$$
$$\boxed{\frac{dy}{dx} = \frac{x}{y}}$$

4. $2x^3 + 3y^3 = 64$

$$6x^2 + 9y^2\left(\frac{dy}{dx}\right) = 0$$
$$9y^2\left(\frac{dy}{dx}\right) = -6x^2$$
$$\frac{dy}{dx} = \frac{-6x^2}{9y^2}$$
$$\boxed{\frac{dy}{dx} = \frac{-2x^2}{3y^2}}$$

5. $x^3 - xy + y^2 = 7$ *product rule

$$x^3 - \overbrace{x}^{f}\overbrace{y}^{g} + y^2 = 7$$
$$3x^2 - \left(\overbrace{(1)}^{f'}\overbrace{(y)}^{g} + \overbrace{(x)}^{f}\overbrace{\left(\frac{dy}{dx}\right)}^{g'}\right) + 2y\left(\frac{dy}{dx}\right) = 0$$
$$3x^2 - y - x\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = 0$$
$$-x\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = y - 3x^2$$
$$\frac{dy}{dx}(-x + 2y) = y - 3x^2$$
$$\boxed{\frac{dy}{dx} = \frac{y - 3x^2}{-x + 2y}}$$

6. $x^2y + y^2x = -2$

$$\overbrace{x^2}^{f}\overbrace{y}^{g} + \overbrace{y^2}^{f} \cdot \overbrace{x}^{g} = -2$$
$$\overbrace{2x}^{f'} \cdot \overbrace{y}^{g} + \overbrace{x^2}^{f} \cdot \overbrace{\frac{dy}{dx}}^{g'} + \overbrace{2y\left(\frac{dy}{dx}\right)}^{f'} \cdot \overbrace{x}^{g} + \overbrace{y^2}^{f} \cdot \overbrace{(1)}^{g'} = 0$$
$$2xy + x^2\left(\frac{dy}{dx}\right) + 2xy\left(\frac{dy}{dx}\right) + y^2 = 0$$
$$x^2\left(\frac{dy}{dx}\right) + 2xy\left(\frac{dy}{dx}\right) = -2xy - y^2$$
$$\frac{dy}{dx}(x^2 + 2xy) = -2xy - y^2$$
$$\boxed{\frac{dy}{dx} = \frac{-2xy - y^2}{x^2 + 2xy}}$$

7. $x^3y^3 - y = x$

$$\overbrace{x^3}^{f}\overbrace{y^3}^{g} - y = x$$
$$\overbrace{3x^2}^{f'} \cdot \overbrace{y^3}^{g} + \overbrace{x^3}^{f} \cdot \overbrace{3y^2\left(\frac{dy}{dx}\right)}^{g'} - 1\left(\frac{dy}{dx}\right) = 1$$
$$3x^2y^3 + 3x^3y^2\left(\frac{dy}{dx}\right) - 1\left(\frac{dy}{dx}\right) = 1$$
$$3x^3y^2\left(\frac{dy}{dx}\right) - 1\left(\frac{dy}{dx}\right) = 1 - 3x^2y^3$$
$$\frac{dy}{dx}(3x^3y^2 - 1) = 1 - 3x^2y^3$$
$$\boxed{\frac{dy}{dx} = \frac{1 - 3x^2y^3}{3x^3y^2 - 1}}$$

Finding and Evaluating a Derivative In Exercises 21–28, find dy/dx by implicit differentiation and evaluate the derivative at the given point.

21. $xy = 6, \quad (-6, -1)$

$$\overbrace{x}^{f}\overbrace{y}^{g} = 6$$

$$\overbrace{(1)}^{f'}\overbrace{(y)}^{g} + \overbrace{(x)}^{f}\overbrace{\left(\frac{dy}{dx}\right)}^{g'} = 0$$

$$x\left(\frac{dy}{dx}\right) = -y$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

$$\left.\frac{dy}{dx}\right|_{(-6,-1)} = \frac{-(-1)}{(-6)}$$

$$\boxed{\left.\frac{dy}{dx}\right|_{(-6,-1)} = \frac{-1}{6}}$$

22. $y^3 - x^2 = 4, \quad (2, 2)$

$$3y^2\left(\frac{dy}{dx}\right) - 2x = 0$$

$$3y^2\left(\frac{dy}{dx}\right) = 2x$$

$$\frac{dy}{dx} = \frac{2x}{3y^2}$$

$$\left.\frac{dy}{dx}\right|_{(2,2)} = \frac{2(2)}{3(2)^2} = \boxed{\frac{1}{3}}$$

24. $x^{2/3} + y^{2/3} = 5, \quad (8, 1)$

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\left(\frac{dy}{dx}\right) = 0$$

$$\frac{2}{3x^{1/3}} + \frac{2}{3y^{1/3}}\left(\frac{dy}{dx}\right) = 0$$

$$\frac{2}{3y^{1/3}}\left(\frac{dy}{dx}\right) = \frac{-2}{3x^{1/3}}$$

$$\frac{dy}{dx} = \frac{-2}{3x^{1/3}} \cdot \frac{3y^{1/3}}{2}$$

$$\frac{dy}{dx} = \frac{-y^{1/3}}{x^{1/3}}$$

$$\left.\frac{dy}{dx}\right|_{(8,1)} = \frac{-(1)^{1/3}}{(8)^{1/3}}$$

$$\left.\frac{dy}{dx}\right|_{(8,1)} = \boxed{\frac{-1}{2}}$$

25)

$(x^2 + 4)y = 8$

Point: $(2, 1)$

$$\overbrace{(x^2+4)}^{f}\overbrace{(y)}^{g} = 8$$

$$\overbrace{2x}^{f'} \cdot \overbrace{y}^{g} + \overbrace{(x^2+4)}^{f}\overbrace{\left(\frac{dy}{dx}\right)}^{g'} = 0$$

$$2xy + (x^2+4)\left(\frac{dy}{dx}\right) = 0$$

$$(x^2+4)\left(\frac{dy}{dx}\right) = -2xy$$

$$\frac{dy}{dx} = \frac{-2xy}{x^2+4}$$

$$\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{-2xy}{x^2+4}$$

$$\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{-2(2)(1)}{2^2+4}$$

$$\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{-4}{8}$$

$$= \boxed{\frac{-1}{2}}$$

Find Equation of tangent line:

51. $\sqrt{x} + \sqrt{y} = 5, \quad (9, 4)$

$$x^{1/2} + y^{1/2} = 5$$

$$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}\left(\frac{dy}{dx}\right) = 0$$

$$\frac{1}{2x^{1/2}} + \frac{1}{2y^{1/2}}\left(\frac{dy}{dx}\right) = 0$$

$$\frac{1}{2y^{1/2}}\left(\frac{dy}{dx}\right) = \frac{-1}{2x^{1/2}}$$

$$\frac{dy}{dx} = \frac{-1}{2x^{1/2}} \cdot \frac{2y^{1/2}}{1}$$

$$\frac{dy}{dx} = \frac{-2y^{1/2}}{2x^{1/2}}$$

$$\frac{dy}{dx} = \frac{-y^{1/2}}{x^{1/2}}$$

$$\left.\frac{dy}{dx}\right|_{(9,4)} = \frac{-(4)^{1/2}}{(9)^{1/2}} = \frac{-2}{3}$$

point: $(9, 4)$

slope: $m = \frac{-2}{3}$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - 4 = \frac{-2}{3}(x - 9)}$$