

Ch.2.5 Implicit Differentiation Vertical, Horizontal Tangent Lines HW Problems #57, #58

Vertical and Horizontal Tangent Lines In Exercises 57 and 58, find the points at which the graph of the equation has a vertical or horizontal tangent line.

(dy/dx)

*Find **Horizontal Tangent** lines by setting **numerator of derivative** equal to zero, solve for x

*Find **Vertical Tangent** lines by setting **denominator of derivative** equal to zero, solve for x

57. $25x^2 + 16y^2 + 200x - 160y + 400 = 0$

Apply implicit differentiation

$50x + 32y \left(\frac{dy}{dx}\right) + 200 - 160 \left(\frac{dy}{dx}\right) + 0 = 0$ horizontal tangent: $-50x - 200 = 0$

$32y \left(\frac{dy}{dx}\right) - 160 \left(\frac{dy}{dx}\right) = -50x - 200$

$\frac{dy}{dx} [32y - 160] = -50x - 200$

$\frac{dy}{dx} = \frac{-50x - 200}{32y - 160}$

plug into equation

$-50x = +200$
 $x = -4$
 $25(-4)^2 + 16y^2 + 200(-4) - 160y + 400 = 0$
 $400 + 16y^2 - 800 - 160y + 400 = 0$
 $16y^2 - 160y = 0$ $16y(y - 10) = 0$ $y = 0, 10$

Horizontal tangents: $(-4, 0)$ and $(-4, 10)$

vertical tangent: $32y - 160 = 0$ $32y = 160$
 $y = 5$

$25x^2 + 16(5)^2 + 200x - 160(5) + 400 = 0$

$25x^2 + 400 + 200x - 800 + 400 = 0$

$25x^2 + 200x = 0$

$25x(x + 8) = 0$

$x = 0, -8$

Vertical tangents:
 $(0, 5)$ and $(-8, 5)$

58. $4x^2 + y^2 - 8x + 4y + 4 = 0$

$8x + 2y \left(\frac{dy}{dx}\right) - 8 + 4 \left(\frac{dy}{dx}\right) + 0 = 0$

$2y \left(\frac{dy}{dx}\right) + 4 \left(\frac{dy}{dx}\right) = 8 - 8x$

$\frac{dy}{dx} (2y + 4) = 8 - 8x$

$\frac{dy}{dx} = \frac{8 - 8x}{2y + 4}$

horizontal tangent: $8 - 8x = 0$ $x = 1$

$4(1)^2 + y^2 - 8(1) + 4y + 4 = 0$

$y^2 + 4y = 0$

$y(y + 4) = 0$

$y = 0, -4$

horizontal tangents:
 $(1, 0)$ and $(1, -4)$

vertical tangents: $2y + 4 = 0$

$2y = -4$ $y = -2$

$4x^2 + (-2)^2 - 8x + 4(-2) + 4 = 0$

$4x^2 + 4 - 8x - 8 + 4 = 0$

$4x^2 - 8x = 0$

$4x(x - 2) = 0$

$x = 0, 2$

vertical tangents

$(0, -2)$ and $(2, -2)$

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*Find Vertical Tangent lines by setting denominator of derivative equal to zero, solve for x

57. $25x^2 + 16y^2 + 200x - 160y + 400 = 0$

58. $4x^2 + y^2 - 8x + 4y + 4 = 0$