2017

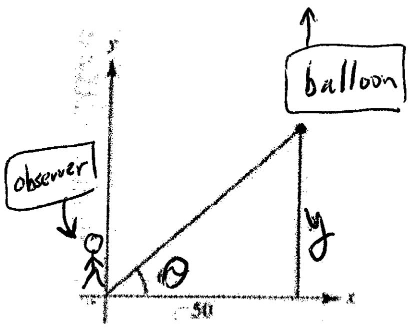
Key

## 2.6 Related Rates (Trig) pg. 155-156 #38, 39, 40, 41

**38. Angle of Elevation** A balloon rises at a rate of 4 meters per second from a point on the ground 50 meters from an observer. Find the rate of change of the angle of elevation of the balloon from the observer when the balloon is 50 meters above the ground.

$\frac{dy}{dt} = 4\,m/s$ Find $\frac{d\theta}{dt} = \_\_$

$y = 50$




$$\tan\theta = \frac{y}{50}$$

$$\tan\theta = \frac{1}{50}y$$

$$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{50}\left(\frac{dy}{dt}\right)$$

$$(\sqrt{2})^2\left(\frac{d\theta}{dt}\right) = \frac{1}{50}(4)$$

$$2\left(\frac{d\theta}{dt}\right) = \frac{1}{50}\cdot 4$$

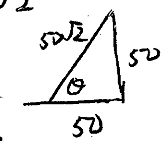
$$\frac{d\theta}{dt} = \frac{1}{2}\cdot\frac{1}{50}\cdot 4 = \boxed{\frac{1}{25}\ rad/sec}$$

$x^2 + y^2 = z^2$

$50^2 + 50^2 = z^2$

$5000 = z^2$

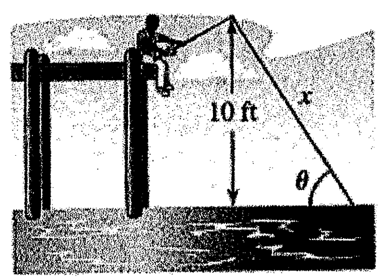
$z = 50\sqrt{2}$




$\sec\theta = \frac{50\sqrt{2}}{50}$

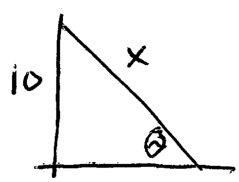
$\sec\theta = \sqrt{2}$

**39. Angle of Elevation** A fish is reeled in at a rate of 1 foot per second from a point 10 feet above the water (see figure). At what rate is the angle $\theta$ between the line and the water changing when there is a total of 25 feet of line from the end of the rod to the water?




$x = 25$ Find $\frac{d\theta}{dt} = \_\_$

$\frac{dx}{dt} = -1$ ft/s




$$\sin\theta = \frac{10}{x}$$

$$\sin\theta = 10x^{-1}$$

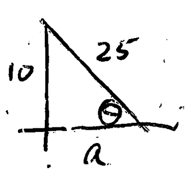
$$\cos\theta\left(\frac{d\theta}{dt}\right) = -10x^{-2}\left(\frac{dx}{dt}\right)$$

$$\left(\frac{\sqrt{21}}{5}\right)\left(\frac{d\theta}{dt}\right) = -10\left(\frac{1}{x^2}\right)\left(\frac{dx}{dt}\right)$$

$$\frac{d\theta}{dt} = \frac{5}{\sqrt{21}}(-10)\left(\frac{1}{25^2}\right)(-1)$$

$$= \frac{+5\cdot 10}{\sqrt{21}\cdot 25\cdot 25}$$

$$\boxed{\frac{d\theta}{dt} = \frac{2}{25\sqrt{21}} \approx 0.017\ rad/sec}$$

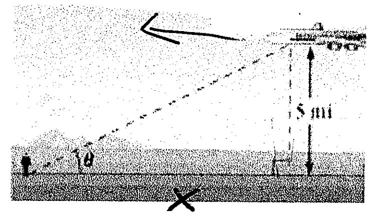



$a^2 + 10^2 = 25^2$

$a = \sqrt{525} = 5\sqrt{21}$

$\cos\theta = \frac{5\sqrt{21}}{25} = \frac{\sqrt{21}}{5}$

**40. Angle of Elevation** An airplane flies at an altitude of 5 miles toward a point directly over an observer (see figure). The speed of the plane is 600 miles per hour. Find the rates at which the angle of elevation $\theta$ is changing when the angle is (a) $\theta = 30°$, (b) $\theta = 60°$, and (c) $\theta = 75°$.




$\frac{dx}{dt} = -600 \text{ mph}$

a) Find $\frac{d\theta}{dt} = \_$ when $\theta = 30°$

$\tan\theta = \frac{5}{x}$

$\tan\theta = 5x^{-1}$

$\sec^2\theta \frac{d\theta}{dt} = -5x^{-2}\left(\frac{dx}{dt}\right)$

$\frac{d\theta}{dt} = \cos^2\theta \cdot \frac{-5}{x^2}\left(\frac{dx}{dt}\right)$

$= (\cos 30)^2 \cdot \frac{-5}{(5\sqrt{3})^2} \cdot (-600)$

$= \left(\frac{\sqrt{3}}{2}\right)^2 \left(\frac{-5}{25 \cdot 3}\right)(-600)$

a) $\frac{d\theta}{dt} = 30 \text{ rad/hr} \cdot \frac{1 \text{ hr}}{60 \text{ min}} = \boxed{\frac{1}{2} \text{ rad/min}}$

$\frac{90 \text{ rad}}{\text{hr}} \cdot \frac{\text{hr}}{60 \text{ min}}$

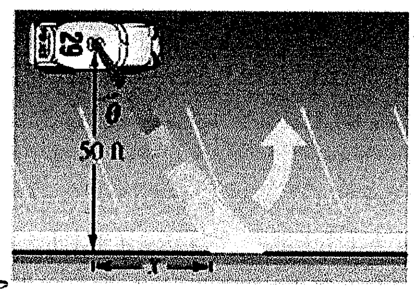
b) $\frac{d\theta}{dt} = (\cos 60)^2 \cdot \frac{-5}{(5/\sqrt{3})^2}(-600) = 90 \text{ rad/hr} = \boxed{\frac{3}{2} \text{ rad/min}}$

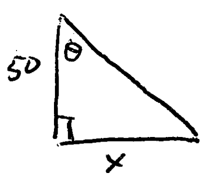
c) $\frac{d\theta}{dt} = (\cos 75)^2 \cdot \frac{-5}{5\tan 75} \cdot -600 \approx 111.96 \text{ rad/hr} \approx \boxed{1.87 \text{ rad/min}}$

~~cos 75~~

$\tan\theta = \frac{5}{x}$

**41. Linear vs. Angular Speed** A patrol car is parked 50 feet from a long warehouse (see figure). The revolving light on top of the car turns at a rate of 30 revolutions per minute. How fast is the light beam moving along the wall when the beam makes angles of (a) $\theta = 30°$, (b) $\theta = 60°$, and (c) $\theta = 70°$ with the perpendicular line from the light to the wall?



$\tan\theta = \frac{x}{50}$

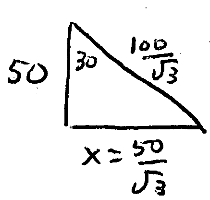
$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{50}\left(\frac{dx}{dt}\right)$

$\frac{d\theta}{dt} = \frac{30 \text{ rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{\text{revolution}} \cdot \frac{1 \text{ min}}{60 \text{ sec}}$

$\frac{d\theta}{dt} = \pi \text{ rad/sec}$

a) $\theta = 30°$

Find $\frac{dx}{dt}$



$\sec\theta = \frac{\frac{100}{\sqrt{3}}}{50} = \frac{2}{\sqrt{3}}$

$\sec^2\theta = \frac{4}{3}$

$\frac{4}{3} \cdot \pi = \frac{1}{50}\frac{dx}{dt}$

$\frac{200\pi}{3} \text{ ft/s} = \frac{dx}{dt}$

b) $\theta = 60°$, $\frac{dx}{dt} = 200\pi \text{ ft/s}$

c) $\theta = 70°$ $\frac{dx}{dt} = 427.43\pi \text{ ft/s}$

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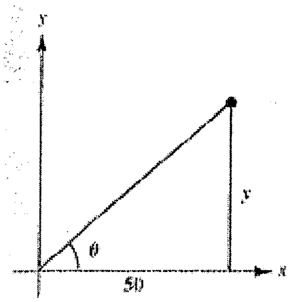


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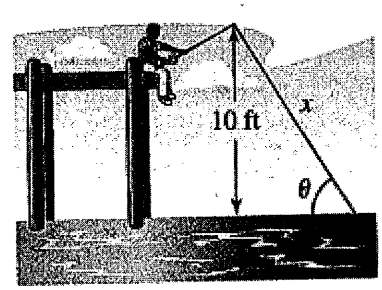

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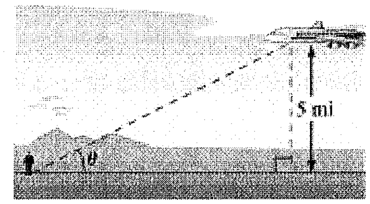


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