Related Rates Worksheet #3 **Morning Review**

1) An 25-foot ladder is leaning against a wall. The bottom of the ladder is being pushed towards the wall at the rate of 5 feet per second. How fast is the top of the ladder moving up against the wall at this time when the bottom of the ladder is 7 feet from the wall?

2)

A man is driving north at a rate of 17 m/s. He sees a railroad track 20m ahead of him that is perpendicular to the road. There is a train going east on the track crossing the road and the man determines with a radar gun that the engine is 35 m from him and the distance between his car and the engine is increasing at the rate of 5 m/s. What is the speed of the train?

b. What is the rate at which the area of the triangle is changing?

3) $V = \frac{4}{3}\pi r^3$ $S = 4\pi r^2$

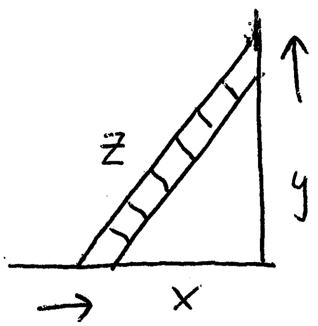
A spherical balloon is inflated so that its volume is increasing at the rate of 40 cubic feet per minute. How fast is the surface area of the balloon increasing when the radius is 5 feet? (Note: $S = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$)

4) $V = \frac{\pi}{3} r^2 h$

A conical paper cup is 70 cm tall with a diameter of 60 cm. The cup is being filled with water at a rate of $\frac{19\pi}{5}$ cm^3/sec. How fast is the water level rising when the water level is 20 cm?

Related Rates Worksheet #3 **Morning Review** Key

1) An 25-foot ladder is leaning against a wall. The bottom of the ladder is being pushed towards the wall at the rate of 5 feet per second. How fast is the top of the ladder moving up against the wall at this time when the bottom of the ladder is 7 feet from the wall?



$x^2+y^2=z^2$

$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=2z\left(\frac{dz}{dt}\right)$

$x=7$ $\qquad \frac{dx}{dt}=-5$

$y=\underline{24}$ $\qquad \frac{dy}{dt}=\underline{\quad}?$

$z=25$ $\qquad \frac{dz}{dt}=0$

$7^2+y^2=25^2$

$y=24$

$2(7)(-5)+2(24)\left(\frac{dy}{dt}\right)=2(25)(0)$

$-70+48\left(\frac{dy}{dt}\right)=0$

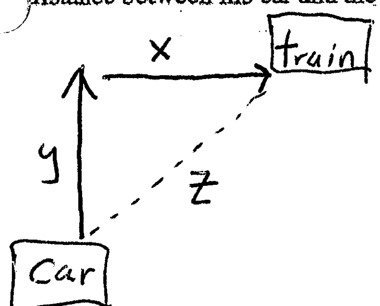
$48\left(\frac{dy}{dt}\right)=70$

$\frac{dy}{dt}=\frac{70}{48}$

$\boxed{\frac{dy}{dt}=1.458\ ft/sec}$

2)

A man is driving north at a rate of 17 m/s. He sees a railroad track 20m ahead of him that is perpendicular to the road. There is a train going east on the track crossing the road and the man determines with a radar gun that the engine is 35 m from him and the distance between his car and the engine is increasing at the rate of 5 m/s. What is the speed of the train?



$x^2+y^2=z^2$

$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=2z\left(\frac{dz}{dt}\right)$

$x=\underline{28.723}$ $\qquad \frac{dx}{dt}=\underline{\quad}?$

$y=20$ $\qquad \frac{dy}{dt}=-17$

$z=35$ $\qquad \frac{dz}{dt}=5$

$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=2z\left(\frac{dz}{dt}\right)$

$2(28.723)\left(\frac{dx}{dt}\right)+2(20)(-17)=2(35)(5)$

$57.446\left(\frac{dx}{dt}\right)-680=350$

$\left(\frac{dx}{dt}\right)57.446=1030$

$\frac{dx}{dt}=\frac{1030}{57.446}=\boxed{17.929\ m/s}$

b. What is the rate at which the area of the triangle is changing?

$A=\frac{1}{2}xy$

$A=\overbrace{\tfrac{1}{2}x}^{f}\overbrace{y}^{g}$

$A=\overbrace{\tfrac{1}{2}x}^{f}\overbrace{y}^{g}$

$\frac{dA}{dt}=\overbrace{\frac{1}{2}\left(\frac{dx}{dt}\right)}^{f'}\cdot\overbrace{y}^{g}+\overbrace{\frac{1}{2}x}^{f}\cdot\overbrace{\frac{dy}{dt}}^{g'}$

$\frac{dA}{dt}=\frac{1}{2}(17.929)(20)+\frac{1}{2}(28.723)(-17)$

$\boxed{\frac{dA}{dt}=-64.84\ m^2/s}$

3) $V = \frac{4}{3}\pi r^3$ $\quad S = 4\pi r^2$

A spherical balloon is inflated so that its volume is increasing at the rate of 40 cubic feet per minute. How fast is the surface area of the balloon increasing when the radius is 5 feet? (Note: $S = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$)

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2\left(\frac{dr}{dt}\right)$

$\frac{dV}{dt} = 4\pi r^2\left(\frac{dr}{dt}\right)$

$S = 4\pi r^2$

$\frac{dS}{dt} = 4\pi \cdot 2r\left(\frac{dr}{dt}\right)$

$\frac{dS}{dt} = 8\pi r\left(\frac{dr}{dt}\right)$

$\frac{dV}{dt} = 40 \quad \frac{dS}{dt} = \_\_? \quad r = 5$

$\frac{dV}{dt} = 4\pi r^2\left(\frac{dr}{dt}\right)$

$40 = 4\pi(5)^2\left(\frac{dr}{dt}\right)$

$40 = 100\pi\left(\frac{dr}{dt}\right)$

$\frac{2}{5\pi} = \frac{dr}{dt}$

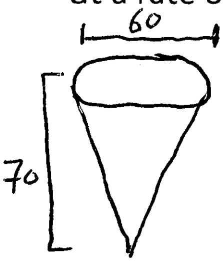
$\frac{dS}{dt} = 8\pi r\left(\frac{dr}{dt}\right)$

$\frac{dS}{dt} = 8\pi(5)\left(\frac{2}{5\pi}\right)$

$\boxed{\frac{dS}{dt} = 16\ ft^2/min}$

4) $V = \frac{\pi}{3}r^2 h$

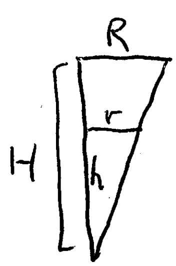
A conical paper cup is 70 cm tall with a diameter of 60 cm. The cup is being filled with water at a rate of $\frac{19\pi}{5}$ cm³/sec. How fast is the water level rising when the water level is 20 cm?



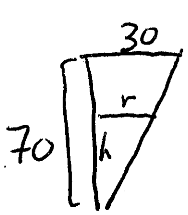
$\frac{dV}{dt} = \frac{19\pi}{5}$

$\boxed{h = 20\,cm}$ *save until after derivative

$\frac{dh}{dt} = \_\_?$



$\frac{r}{R} = \frac{h}{H}$



$\frac{r}{30} = \frac{h}{70}$

$70r = 30h$

$r = \frac{30}{70}h \quad r = \frac{3}{7}h$

$V = \frac{\pi}{3}r^2 h$

$r = \frac{3}{7}h$

$V = \frac{\pi}{3}\left(\frac{3}{7}h\right)^2 h$

$V = \frac{\pi}{3}\cdot\frac{9}{49}h^2\cdot h$

$V = \frac{3\pi}{49}h^3$

$V = \frac{3\pi}{49}h^3$

$\frac{dV}{dt} = \frac{3\pi}{49}\cdot 3h^2\left(\frac{dh}{dt}\right)$

$\frac{dV}{dt} = \frac{9\pi}{49}h^2\left(\frac{dh}{dt}\right)$

$\frac{19\pi}{5} = \frac{9\pi}{49}(20)^2\left(\frac{dh}{dt}\right)$

$\frac{19\pi}{5} = \frac{3600\pi}{49}\left(\frac{dh}{dt}\right)$

$\frac{19\pi}{5}\cdot\frac{49}{3600\pi} = \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = 0.0517\ cm/sec}$