2017

Key

Ch. 2.6 Related Rates — selected AB/BC Homework Problems #14, 16, 20, 22, 24, 26, 28, 30, 41

14. Volume A spherical balloon is inflated with gas at the rate of 800 cubic centimeters per minute. How fast is the radius of the balloon increasing at the instant the radius is (a) 30 centimeters and (b) 60 centimeters?

context clues

$V = \frac{4}{3}\pi r^3$ $\quad \frac{dV}{dt} = 800\ cm^3/min$

Find $\frac{dr}{dt} =$ ___

a) $r = 30$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{\cancel{3}}\cancel{3}\pi r^2\left(\frac{dr}{dt}\right)$

$\frac{dV}{dt} = 4\pi r^2\left(\frac{dr}{dt}\right)$

$800 = 4\pi(30)^2\left(\frac{dr}{dt}\right)$

$\frac{800}{3600\pi}$

$\boxed{a)\ \frac{2}{9\pi}\ cm/min = \frac{dr}{dt}}$

b) $r = 60$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$800 = 4\pi(60)^2\left(\frac{dr}{dt}\right)$

$\frac{800}{14400\pi} = \frac{dr}{dt}$

$\boxed{\frac{1}{18\pi}\ cm/min = \frac{dr}{dt}}$

16. Surface Area All edges of a cube are expanding at a rate of 6 centimeters per second. How fast is the surface area changing when each edge is (a) 2 centimeters and (b) 10 centimeters?

Surface Area $= 6x^2$

$S = 6x^2$

$\frac{dx}{dt} = 6 cm/s$

Find $\frac{dS}{dt}$

a) $S = 6x^2$, $x = 2 cm$

$\frac{dS}{dt} = 12x\left(\frac{dx}{dt}\right)$

$\frac{dS}{dt} = 12(2)\left(\frac{dx}{dt}\right)$

$\boxed{a)\ \frac{dS}{dt} = 12(2)(6) = 144 cm^2/sec}$

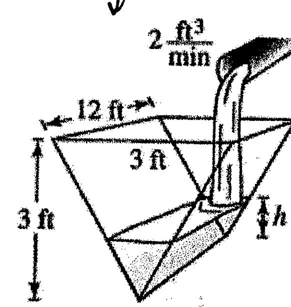
b) $x = 10 cm$

$\frac{dS}{dt} = 12x\left(\frac{dx}{dt}\right)$

$\frac{dS}{dt} = 12(10)(6)$

$\boxed{\frac{dS}{dt} = 720 cm^2/sec}$

20. Depth A trough is 12 feet long and 3 feet across the top (see figure). Its ends are isosceles triangles with altitudes of 3 feet.



$V = \frac{1}{2}lbh = \frac{1}{2}(12)bh$

$V = 6bh$, since $b = h$

$V = 6(h)(h)$

$V = 6h^2$

a) Water is pumped into trough at $2\ ft^3/min$. How fast is water level rising when depth h is 1 ft?

$V = 6h^2$ $\quad \frac{dV}{dt} = 2\ ft^3/min$ Find $\frac{dh}{dt}$ when $h = 1$

$\frac{dV}{dt} = 12h\left(\frac{dh}{dt}\right)$

$\frac{dV}{dt} = 12h\left(\frac{dh}{dt}\right)$

$2 = 12(1)\left(\frac{dh}{dt}\right)$

$\frac{2}{12} = \frac{dh}{dt} \rightarrow \boxed{\frac{dh}{dt} = \frac{1}{6}\ ft/min}$

b) Water is rising at rate of $\frac{3}{8}$ in/min when $h = 2$. Determine rate water is pumped into trough.

$\frac{dh}{dt} = \frac{3}{8}\ \cancel{in}/min \cdot \frac{1 ft}{12 \cancel{in}} = \frac{1}{32}\ ft/min$

$\frac{dV}{dt} = 12h\left(\frac{dh}{dt}\right)$

$\frac{dV}{dt} = 12(2)\left(\frac{1}{32}\right)$

$\boxed{\frac{dV}{dt} = \frac{24}{32} = \frac{3}{4}\ ft^3/min}$

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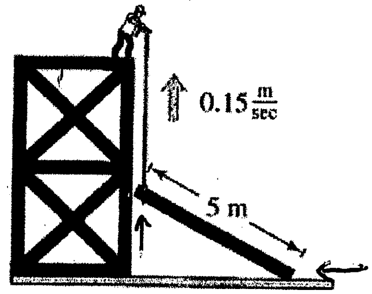
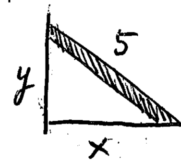
22. Construction A construction worker pulls a five-meter plank up the side of a building under construction by means of a rope tied to one end of the plank (see figure). Assume the opposite end of the plank follows a path perpendicular to the wall of the building and the worker pulls the rope at a rate of 0.15 meter per second. How fast is the end of the plank sliding along the ground when it is 2.5 meters from the wall of the building?





$x^2+y^2=5^2$

$2.5^2+y^2=5^2$

$y^2=25-2.5^2=18.75$

$y=\sqrt{18.75}$

$\frac{dy}{dt}=0.15\ m/s$ Find $\frac{dx}{dt}=$ ___ when $x=2.5$

$x^2+y^2=5^2$

$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=0$

$2(2.5)\left(\frac{dx}{dt}\right)+2(\sqrt{18.75})(0.15)=0$

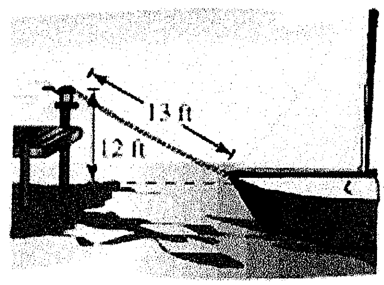
$5\left(\frac{dx}{dt}\right)=-1.299$

$\frac{dx}{dt}=\frac{-1.299}{5}=\boxed{-0.26\ m/sec}$

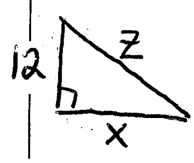
24. Boating A boat is pulled into a dock by means of a winch 12 feet above the deck of the boat (see figure).

(a) The winch pulls in rope at a rate of 4 feet per second. Determine the speed of the boat when there is 13 feet of rope out. What happens to the speed of the boat as it gets closer to the dock?

(b) Suppose the boat is moving at a constant rate of 4 feet per second. Determine the speed at which the winch pulls in rope when there is a total of 13 feet of rope out. What happens to the speed at which the winch pulls in rope as the boat gets closer to the dock?



z = length of rope



a) $\frac{dz}{dt}=-4\ ft/s$ $z=13$, Find $\frac{dx}{dt}=$ ___

$x^2+12^2=z^2$ $x^2+12^2=13^2$ $x=5$

$2x\left(\frac{dx}{dt}\right)+0=2z\left(\frac{dz}{dt}\right)$

$2(5)\left(\frac{dx}{dt}\right)=2(13)(-4)$

$\boxed{\frac{dx}{dt}=-10.4\ ft/s \quad \text{speed is } 10.4\ ft/s}$

b) $\frac{dx}{dt}=-4\ ft/s$, $z=13$, $x=5$, Find $\frac{dz}{dt}$

$2x\frac{dx}{dt}+0=2z\frac{dz}{dt}$

$2(5)(-4)=2(13)\left(\frac{dz}{dt}\right)$

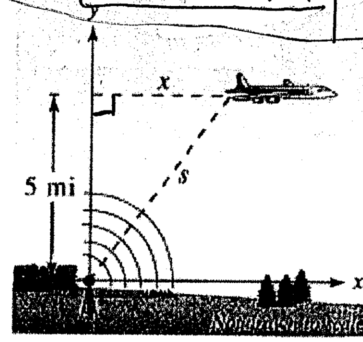
$-40=26\frac{dz}{dt}$

$\boxed{\frac{dz}{dt}=\frac{-20}{13}\ ft/s}$

$\lim_{z\to 12^+}\frac{\sqrt{z^2-144}}{z}(-4)$

$=\boxed{0}$

26. Air Traffic Control An airplane is flying at an altitude of 5 miles and passes directly over a radar antenna (see figure). When the plane is 10 miles away ($s=10$), the radar detects that the distance s is changing at a rate of 240 miles per hour. What is the speed of the plane?



$s=10$ $\frac{ds}{dt}=240\ mph$ Find $\frac{dx}{dt}=$ ___

$x^2+5^2=s^2 \rightarrow x^2+5^2=10^2$ $x^2=75$, $x=\sqrt{75}$

$2x\left(\frac{dx}{dt}\right)+0=2s\left(\frac{ds}{dt}\right)$

$\cancel{2}(\sqrt{75})\left(\frac{dx}{dt}\right)=\cancel{2}(10)(240)$

$\frac{dx}{dt}=\frac{2400}{\sqrt{75}}\approx\boxed{277.13\ mph}$

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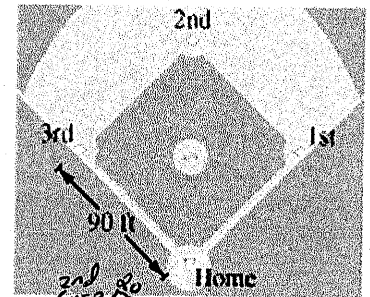
28. Sports For the baseball diamond in Exercise 27, suppose the player is running from first base to second base at a speed of 25 feet per second. Find the rate at which the distance from home plate is changing when the player is 20 feet from second base.



$\rightarrow 90-20$

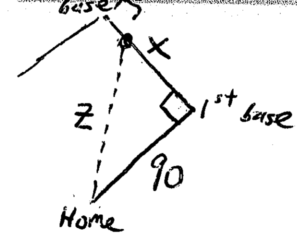
$\frac{dx}{dt}=25$ ft/s Find $\frac{dz}{dt}=$ __ when $x=\cancel{20}$ $x=70$

$x^2+90^2=z^2 \rightarrow 70^2+90^2=z^2$ $z=\sqrt{13000}$

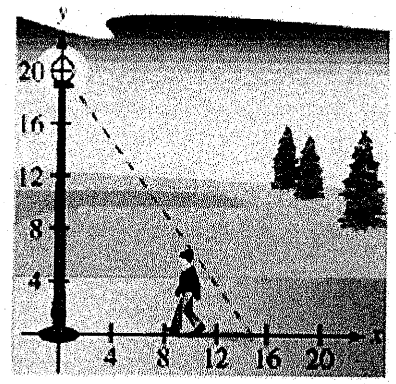
$2x\left(\frac{dx}{dt}\right)+0=2z\left(\frac{dz}{dt}\right)$

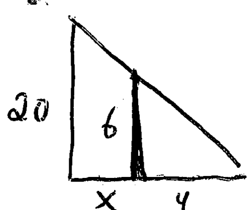
$\cancel{2}(70)(25)=\cancel{2}\sqrt{13000}\cdot\frac{dz}{dt}$

$\frac{dz}{dt}=\frac{70(25)}{\sqrt{13000}}\approx\boxed{15.35 \text{ ft/sec}}$



30. Shadow Length Repeat Exercise 29 for a man 6 feet tall walking at a rate of 5 feet per second *toward* a light that is 20 feet above the ground (see figure).





$\frac{dx}{dt}=-5$ ft/s

$\frac{6}{20}=\frac{y}{x+y}$

$6x+6y=20y$

$6x=14y$

a) Find rate of tip of shadow moving:

$6x=14y$

$6\left(\frac{dx}{dt}\right)=14\left(\frac{dy}{dt}\right)$

$6(-5)=14\left(\frac{dy}{dt}\right)$

$\frac{-30}{14}=\frac{-15}{7}=\frac{dy}{dt}$

$\frac{dx}{dt}+\frac{dy}{dt}=-5+\frac{-15}{7}$

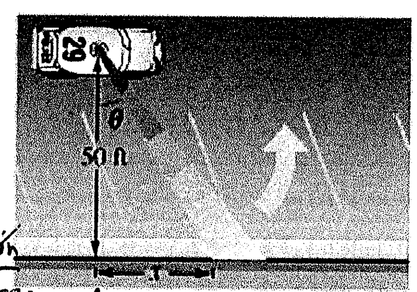
$\boxed{\text{Tip ROC}=\frac{-50}{7}\text{ ft/s}}$

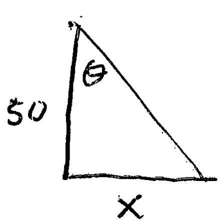
b) Length of shadow changing

$\boxed{\frac{dy}{dt}=\frac{-15}{7}\text{ ft/s}}$

* (Trig)

41. Linear vs. Angular Speed A patrol car is parked 50 feet from a long warehouse (see figure). The revolving light on top of the car turns at a rate of 30 revolutions per minute. How fast is the light beam moving along the wall when the beam makes angles of (a) $\theta = 30°$, (b) $\theta = 60°$, and (c) $\theta = 70°$ with the perpendicular line from the light to the wall?





$\tan\theta=\frac{x}{50}$

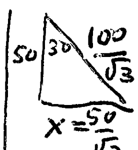
$\sec^2\theta\left(\frac{d\theta}{dt}\right)=\frac{1}{50}\left(\frac{dx}{dt}\right)$

$\frac{d\theta}{dt}=\frac{30\,\cancel{\text{rev}}}{\cancel{\text{min}}}\cdot\frac{2\pi\text{ rad}}{\cancel{\text{revolution}}}\cdot\frac{1\,\cancel{\text{min}}}{60\text{sec}}$

$\frac{d\theta}{dt}=\frac{60\pi}{60}=\pi$ rad/sec

a) $\theta=30°$

Find $\frac{dx}{dt}$



$\sec\theta=\frac{\frac{100}{\sqrt3}}{50}=\frac{2}{\sqrt3}$

$\sec^2\theta=\frac43$

$\frac43\cdot\pi=\frac{1}{50}\frac{dx}{dt}$

$\boxed{\frac{200\pi}{3}\text{ ft/s}=\frac{dx}{dt}}$

b) $\theta=60°$, $\frac{dx}{dt}=200\pi$ ft/s

c) $\theta=70°$, $\frac{dx}{dt}=427.43\pi$ ft/s

14. Volume A spherical balloon is inflated with gas at the rate of 800 cubic centimeters per minute. How fast is the radius of the balloon increasing at the instant the radius is (a) 30 centimeters and (b) 60 centimeters?

16. Surface Area All edges of a cube are expanding at a rate of 6 centimeters per second. How fast is the surface area changing when each edge is (a) 2 centimeters and (b) 10 centimeters?

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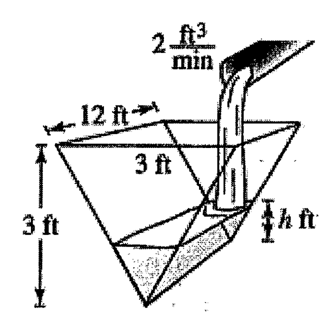
(a) Water is being pumped into the trough at 2 cubic feet per minute. How fast is the water level rising when the depth h is 1 foot?

(b) The water is rising at a rate of $\frac{3}{8}$ inch per minute when $h = 2$. Determine the rate at which water is being pumped into the trough.



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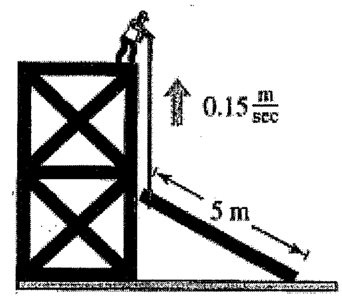

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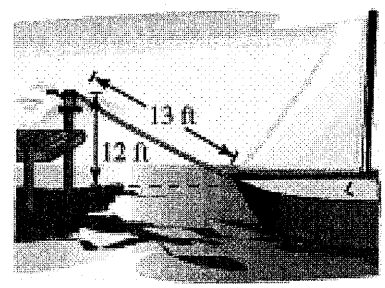

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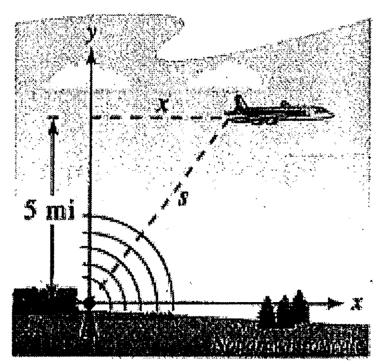

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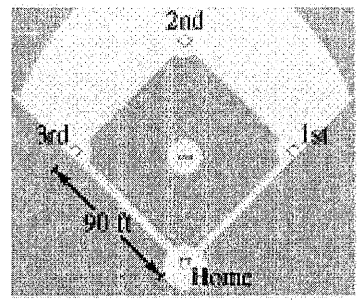

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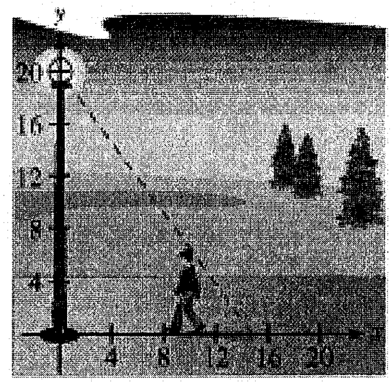
(a) When he is 10 feet from the base of the light, at what rate is the tip of his shadow moving?

(b) When he is 10 feet from the base of the light, at what rate is the length of his shadow changing?



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