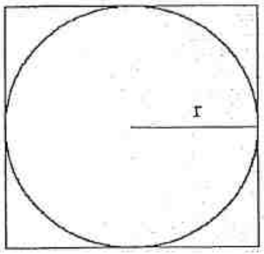
AP Calculus AB Related Rates WS #3 Name________________________ Date______

1

1994 AB5, BC2

1) A circle is inscribed in a square as shown in the figure. The circumference of the circle is increasing at a constant rate of 6 inches per minute. As the circle expands, the square expands to maintain the condition of tangency. (Note: A circle with radius r has circumference $C = 2\pi r$ and area $A = \pi r^2$.)



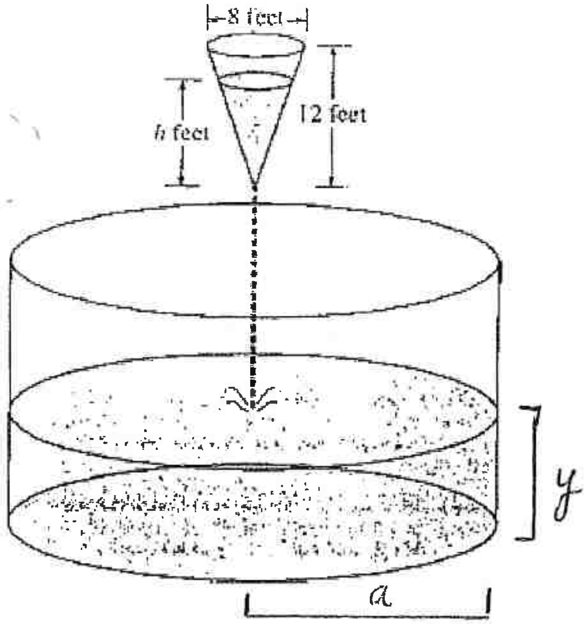
a) Find the rate at which the perimeter of the square is increasing. Indicate units of measure.

b) At the instant when the area of the circle is 25π square inches, find the rate of increase in the area enclosed between the circle and the square. Indicate units of measure.

2. Suppose that a spherical balloon grows in such a way that after t seconds, $V = 4\sqrt{t}$ in^3. How fast is the radius changing after 64 seconds? $(V = \frac{4}{3}\pi r^3)$

②

3. 1995 AB 5



As shown in the figure above, water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth h, in feet, of the water in the conical tank is changing at the rate of $(h-12)$ feet per minute. (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

(a) Write an expression for the volume of water in the conical tank as a function of h.

(b) At what rate is the volume of water in the conical tank changing when $h = 3$? Indicate units of measure.

(c) Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when $h = 3$? Indicate units of measure.

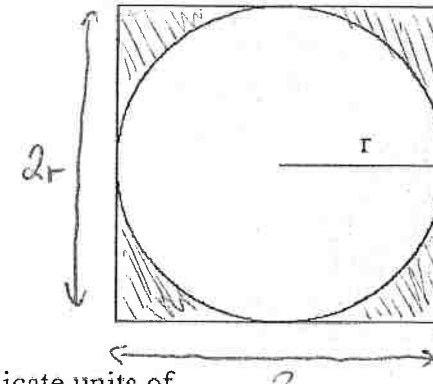
$V = \pi a^2 y$

AP Calculus AB   Related Rates WS #4   Name Key   Date______

3

1994 AB5, BC2

1) A circle is inscribed in a square as shown in the figure. The circumference of the circle is increasing at a constant rate of 6 inches per minute. As the circle expands, the square expands to maintain the condition of tangency. (Note: A circle with radius r has circumference $C = 2\pi r$ and area $A = \pi r^2$.)



$\frac{dC}{dt} = 6 \text{ in/min}$ $\frac{dC}{dt} = 2\pi\left(\frac{dr}{dt}\right)$

a) Find the rate at which the perimeter of the square is increasing. Indicate units of measure.

$P = 8r$

$\frac{dP}{dt} = 8\frac{dr}{dt}$

$6 = 2\pi\left(\frac{dr}{dt}\right)$

$\frac{6}{2\pi} = \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{3}{\pi} \text{ in/min}$

$\frac{dP}{dt} = 8\left(\frac{3}{\pi}\right) = \boxed{\frac{24}{\pi} \text{ in/min}}$

A_e = Area enclosed
A_c = Area circle
A_s = Area square

b) At the instant when the area of the circle is 25π square inches, find the rate of increase in the area enclosed between the circle and the square. Indicate units of measure.

$A = 25\pi$

$\frac{dA_e}{dt} = \_\_\_$

$A = \pi r^2$

$25\pi = \pi r^2$

$25 = r^2$

$\boxed{5 = r}$

$A_e = A_s - A_c$

$A_s = (2r)^2$

$A_c = \pi r^2$

$A_e = 4r^2 - \pi r^2$

$\frac{dA_e}{dt} = 8r\left(\frac{dr}{dt}\right) - 2\pi r\left(\frac{dr}{dt}\right)$

$\frac{dr}{dt} = \frac{3}{\pi} \text{ in/min}$

$\frac{dA}{dt} = 8(5)\left(\frac{3}{\pi}\right) - 2\pi(5)\left(\frac{3}{\pi}\right)$

$\boxed{\frac{dA}{dt} = \frac{120}{\pi} - 30 \text{ in}^2\text{/min}}$

$V = 4(t)^{1/2}$

2. Suppose that a spherical balloon grows in such a way that after t seconds, $V = 4\sqrt{t}$ in^3. How fast is the radius changing after 64 seconds? ($V = \frac{4}{3}\pi r^3$)

$\longrightarrow 32 = \frac{4}{3}\pi(r^3)$ $\frac{24}{\pi} = r^3$

$r = \sqrt[3]{24/\pi}$

$t = 64$

$V = 4\sqrt{64} = 4 \cdot 8 = 32 \text{ in}^3$

$\frac{dV}{dt} = 4 \cdot \frac{1}{2} t^{-1/2}\left(\cancel{\frac{dt}{dt}}\right)$

$\frac{dV}{dt} = \frac{2}{\sqrt{t}} = \frac{2}{\sqrt{64}} = \frac{2}{8} = \frac{1}{4} \text{ in}^3\text{/s}$

Find $\frac{dr}{dt} = \_\_\_$

$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3}r^2 \cdot \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

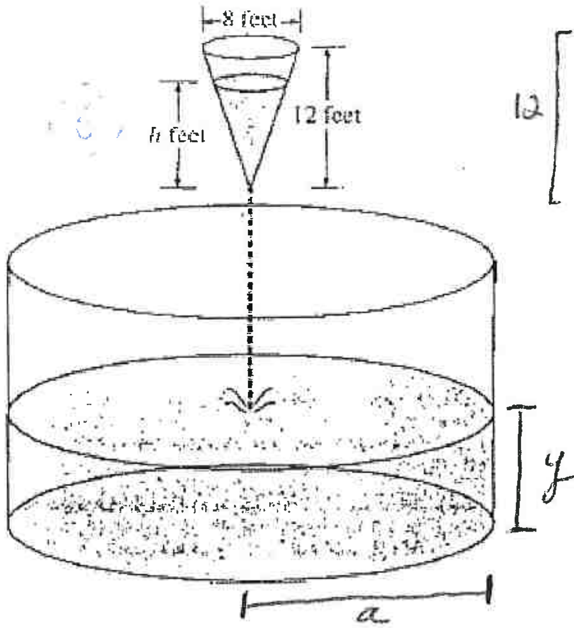
$\frac{1}{4} = 4\pi\left(\sqrt[3]{\frac{24}{\pi}}\right)^2 \frac{dr}{dt}$

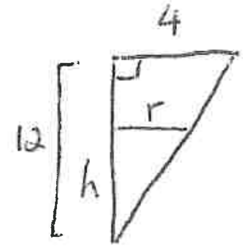
$\frac{1}{4 \cdot 4\pi\left(\sqrt[3]{\frac{24}{\pi}}\right)^2} = \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{194.974} \approx \boxed{0.005 \text{ in/s}}$

4

3. 1995 AB 5





$\frac{r}{4} = \frac{h}{12}$

$12r = 4h$

$r = \frac{4}{12}h$

$\boxed{r = \frac{h}{3}}$

$\frac{dh}{dt} = h-12$ ft/min

As shown in the figure above, water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth h, in feet, of the water in the conical tank is changing at the rate of $(h-12)$ feet per minute. (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

(a) Write an expression for the volume of water in the conical tank as a function of h.

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}\left(\frac{h}{3}\right)^2 h$

$V = \frac{\pi}{3} \cdot \frac{h^2}{9} \cdot h$

$\boxed{V = \frac{\pi}{27} h^3}$

(b) At what rate is the volume of water in the conical tank changing when $h=3$? Indicate units of measure.

$V = \frac{\pi}{27} h^3$

$\frac{dV}{dt} = \frac{\pi}{27} \cdot 3h^2 \left(\frac{dh}{dt}\right)$

$\frac{dV}{dt} = \frac{\pi}{9} h^2 \cdot \left(\frac{dh}{dt}\right)$

$\frac{dV}{dt} = \frac{\pi}{9} h^2 \cdot (h-12)$

$\frac{dV}{dt} = \frac{\pi}{9}(3)^2(3-12)$

$= \frac{\pi}{\cancel{9}} \cdot \cancel{9}(-9)$

$\boxed{\frac{dV}{dt} = -9\pi \text{ ft}^3/\text{min}}$

(c) Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when $h=3$? Indicate units of measure. $(V = \pi a^2 y)$

$\frac{dV}{dt} = 9\pi$ ft^3/min

Area(base) $= 400\pi$

$A = \pi r^2$

$400\pi = \pi r^2$

$400 = r^2$

$20 = r$

$r = 20$ ft

$a = 20$

$V = \pi (20)^2 y$

$V = 400\pi y$

$\frac{dV}{dt} = 400\pi\left(\frac{dy}{dt}\right)$

$9\pi = 400\pi\left(\frac{dy}{dt}\right)$

$\frac{9}{400} = \frac{dy}{dt}$

$\boxed{\frac{dy}{dt} = \frac{9}{400} \text{ ft/min}}$

5

Calculus AB Related Rates Test Review

1. A balloon is rising vertically above a level, straight road at a constant rate of 1 ft/sec. Just when the balloon is 65 ft above the ground, a bicycle moving at a constant rate of 17 ft/sec passes under it. How fast is the distance between the bicycle and balloon increasing 3 seconds later?

2) a) Suppose $f(x) = 3x^2 - 2x$.
Write the equation of the line tangent to $f(x)$ at $x = 2$. Then use local linear approximation to estimate $f(1.9)$.

b) Suppose $h(5) = 3$ and $h'(5) = -2$. Use local linear approximation to estimate $h(5.2)$

3. Find the limit:

a) $\lim_{x \to 3} \frac{4x^2 - 5x}{1 - 3x^2}$

b) $\lim_{x \to 1} \frac{5x^4 - 4x^2 - 1}{10 - x - 9x^3}$

4. Water is leaking out of a cylindrical container at a rate of 5 cm³/hr. The container has a diameter of 12 cm and height of 16 cm. At what rate is the height changing when water level has height of 3 cm? $V = \pi r^2 h$

6

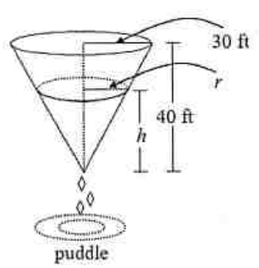
6. A boat is pulled toward a dock by a rope from the bow through a ring on the dock 6 ft above the bow. The rope is hauled in at a rate of 2 ft/sec

a) How fast is the boat approaching the dock when 10 ft of rope are out?

b) At what rate is area of triangle changing at that moment?

7) A conical water tank with a height of 40 ft and a radius of 30 ft is leaking at the rate of 2 ft^3 / min. When the height (h) of the water in the tank is 30 ft, at what rate is the height of the water changing?

(Volume of a cone = $\frac{1}{3}\pi r^2 h$)



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8. The volume of a cube is decreasing at a rate of 10 m^3 / hour. How fast is the total surface area decreasing when the surface area is 54 m^2? ($V = x^3$ $S = 6x^2$)

9. Jet A travels due east from San Francisco toward St. Louis at 500 mph. Jet B travels due north from New Orleans toward St. Louis at 600mph. Find the rate of change of the distance between the two jets when they are 300 miles apart, and jet A is 100 miles from St. Louis (round answer to 3 decimal places) *Be sure to draw diagram, and watch your signs!)

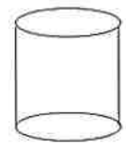
10. A man 6 ft tall walks at a rate of 5 ft/sec toward a streetlight that is 16 ft above the ground. A) At what rate is the length of his shadow changing when he is 10 ft from the base of the light? B) At what rate is the tip of his shadow moving?

8

Additional Practice Problem:

11)

Water is leaking out of a full cylindrical container at a rate of 2 in^3/hr. The container has a diameter of 8 in. and height of 12 in.
At what rate is the height of the container changing when the container is half full?



$V = \pi r^2 h$

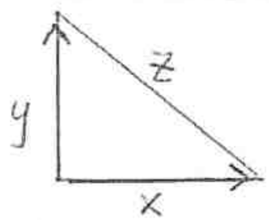
12) Two boats leave the same port at the same time with one boat traveling north at 15 knots per hour and the other boat traveling west at 12 knots per hour. How fast is the distance between the 2 boats changing after 2 hours?

13) A man is driving north at a rate of 17 m/s. He sees a railroad track 20m ahead of him that is perpendicular to the road. There is a train going east on the track crossing the road and the man determines with a radar gun that the engine is 35 m from him and the distance between his car and the engine is increasing at the rate of 5 m/s. What is the speed of the train?

Key 9

Calculus AB Related Rates Test Review

1. A balloon is rising vertically above a level, straight road at a constant rate of 1 ft/sec. Just when the balloon is 65 ft above the ground, a bicycle moving at a constant rate of 17 ft/sec passes under it. How fast is the distance between the bicycle and balloon increasing 3 seconds later?



$x^2+y^2=z^2$

$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=2z\left(\frac{dz}{dt}\right)$

$x=17(3)=51$

$y=65+3=68$

$z=85$

$x^2+y^2=z^2$

$51^2+68^2=z^2$

$dx/dt = 17$ ft/s

$dy/dt = 1$ ft/s

$dz/dt =$ ___

$2(51)(17)+2(68)(1)=2(85)\left(\frac{dz}{dt}\right)$

$1734+136=170\left(\frac{dz}{dt}\right)$

$1870=170\left(\frac{dz}{dt}\right)$

$\boxed{\frac{dz}{dt}=11 \text{ ft/s}}$

2) Suppose $f(x)=3x^2-2x$.

a) Write the equation of the line tangent to $f(x)$ at $x=2$. Then use local linear approximation to estimate $f(1.9)$.

$f(2)=3(2)^2-2(2)=12-4=8$

$f'(x)=6x-2$

$f'(2)=6(2)-2=10$

point: $(2,8)$

slope $m=10$

$y-y_1=m(x-x_1)$

$y-8=10(x-2)$

$y=10(x-2)+8$

$y(1.9)=10(1.9-2)+8$

$\boxed{y(1.9)=7}$

b) Suppose $h(5)=3$ and $h'(5)=-2$. Use local linear approximation to estimate $h(5.2)$

point: $(5,3)$

slope: $m=-2$

$y-3=-2(x-5)$

$y=-2(x-5)+3$

$y(5.2)=-2(5.2-5)+3$

$\boxed{y(5.2)=2.6}$

3. Find the limit:

a) $\lim_{x\to 3}\frac{4x^2-5x}{1-3x^2} \to \frac{4(3)^2-5(3)}{1-3(3)^2}$ $\boxed{\frac{21}{-26}}$

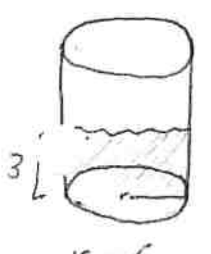
b) $\lim_{x\to 1}\frac{5x^4-4x^2-1}{10-x-9x^3} = \frac{0}{0}$ Apply L'Hopital's Rule $\frac{f'(c)}{g'(c)}$

$\lim_{x\to 1}\frac{20x^3-8x}{-1-27x^2}=\frac{20-8}{-1-27}=\frac{12}{-28}=\boxed{\frac{-3}{7}}$

4. Water is leaking out of a cylindrical container at a rate of 5 cm³/hr. The container has a diameter of 12 cm and height of 16 cm. At what rate is the height changing when water has height of 3 cm? $V=\pi r^2 h$

*No need for similar triangle: radius is a constant

Find $\frac{dh}{dt}$



$r=6$ cm

$\frac{dV}{dt}=-5 \text{ cm}^3/\text{hr}$

$V=\pi r^2 h$

Replace r since radius of water level is constant

$V=\pi(6)^2 h$

$V=36\pi h$

$\frac{dV}{dt}=36\pi\left(\frac{dh}{dt}\right)$

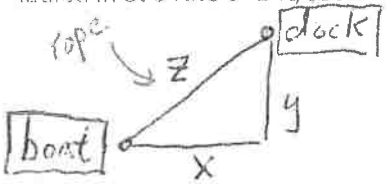
$-5=36\pi\left(\frac{dh}{dt}\right)$

$\frac{-5}{36\pi}=\frac{dh}{dt}$

$\boxed{\frac{dh}{dt}=\frac{-5}{36\pi} \text{ cm/hr.}}$

10

6. A boat is pulled toward a dock by a rope from the bow through a ring on the dock 6 ft above the bow. The rope is hauled in at a rate of 2 ft/sec



z = rope length y is constant

$x^2+y^2=z^2$

a) How fast is the boat approaching the dock when 10 ft of rope are out?

$x^2+y^2=z^2$

$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=2z\left(\frac{dz}{dt}\right)$

$x = \underline{8}$ $dx/dt = \underline{\quad}$

$y = 6$ $dy/dt = \underline{0}$

$z = 10$ $dz/dt = -2$

$2(8)\left(\frac{dx}{dt}\right)+2(6)(0)=2(10)(-2)$

$16\left(\frac{dx}{dt}\right)+0=-40$

$\boxed{\frac{dx}{dt}=\frac{-40}{16}=-2.5\ ft/s}$

b) At what rate is area of triangle changing at that moment?

$A=\frac{1}{2}xy$ *product Rule $f'g+fg'$

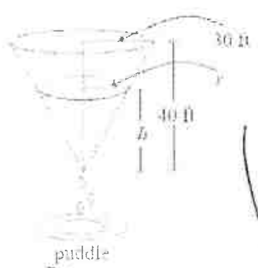
$\frac{dA}{dt}=\frac{1}{2}\left(\frac{dx}{dt}\right)\cdot y+\frac{1}{2}x\cdot\frac{dy}{dt}$

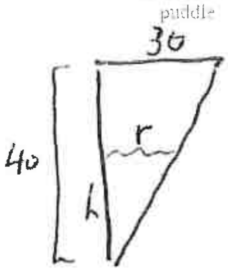
$\frac{dA}{dt}=\frac{1}{2}(-2.5)(6)+\frac{1}{2}(8)(0)$

$\boxed{\frac{dA}{dt}=-7.5\ ft^2/sec}$

7) A conical water tank with a height of 40 ft and a radius of 30 ft is leaking at the rate of 2 ft³ / min. When the height (h) of the water in the tank is 30 ft, at what rate is the height of the water changing?

(Volume of a cone = $\frac{1}{3}\pi r^2 h$)





$V=\frac{\pi}{3}r^2h$

$\frac{dV}{dt}=-2$ $\frac{dh}{dt}=\underline{\quad}?$ $\boxed{h=30}$ Do not use until end of problem

$\frac{r}{30}=\frac{h}{40}$

$40r=30h$

$r=\frac{30h}{40}$

$r=\frac{3h}{4}$

$V=\frac{\pi}{3}\left(\frac{3h}{4}\right)^2h$

$V=\frac{\pi}{3}\cdot\frac{9h^2}{16}\cdot h$

$V=\frac{3\pi}{16}h^3$

$\frac{dV}{dt}=\frac{3\pi}{16}\cdot 3h^2\left(\frac{dh}{dt}\right)$

$\frac{dV}{dt}=\frac{9\pi}{16}h^2\left(\frac{dh}{dt}\right)$

$-2=\frac{9\pi}{16}(30)^2\left(\frac{dh}{dt}\right)$

$-2=\frac{8100\pi}{16}\left(\frac{dh}{dt}\right)$

$-2=\frac{2025\pi}{4}\left(\frac{dh}{dt}\right)$

$-2\cdot\frac{4}{2025\pi}=\frac{dh}{dt}$

$\boxed{\frac{dh}{dt}=\frac{-8}{2025\pi}\ ft/min}$

8. The volume of a cube is decreasing at a rate of 10 m^3 / hour. How fast is the total surface area decreasing when the surface area is 54 m^2? ($V = x^3$ $S = 6x^2$)

$V = x^3$ | $S = 6x^2$ | $\frac{dS}{dt} = \_\_?$ $\frac{dV}{dt} = -10\ m^3/hr$

$\frac{dV}{dt} = 3x^2\left(\frac{dx}{dt}\right)$ | $\frac{dS}{dt} = 12x\left(\frac{dx}{dt}\right)$ | $S = 54m^2$

$S = 6x^2$
$54 = 6x^2$
$9 = x^2$
$3 = x$

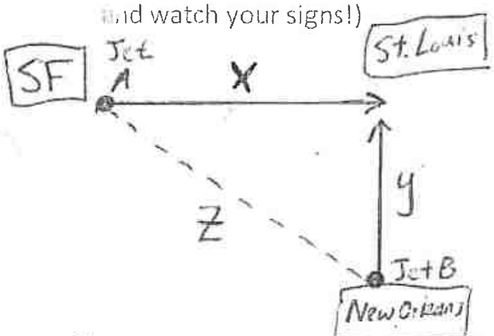
$-10 = 3(3)^2\left(\frac{dx}{dt}\right)$
$-10 = 27\left(\frac{dx}{dt}\right)$

$\frac{-10}{27} = \frac{dx}{dt}$

$\frac{dS}{dt} = 12(3)\left(\frac{-10}{27}\right)$

$\boxed{\frac{dS}{dt} = \frac{-40}{3}\ m^2/hr}$

9. Jet A travels due east from San Francisco toward St. Louis at 500 mph. Jet B travels due north from New Orleans toward St. Louis at 600mph. Find the rate of change of the distance between the two jets when they are 300 miles apart, and jet A is 100 miles from St. Louis (round answer to 3 decimal places) *Be sure to draw diagram, and watch your signs!)



$x^2 + y^2 = z^2$

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right)$

$x = 100$
$y = 282.843$
$z = 300$

$100^2 + y^2 = 300^2$
$y \approx 282.843$

$dx/dt = -500$
$dy/dt = -600$
$dz/dt = \_\_\_$

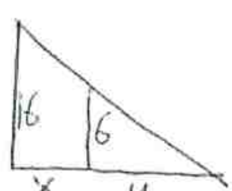
$2(100)(-500) + 2(282.843)(-600) = 2(300)(dz/dt)$

$-100{,}000 - 339411.6 = 600\left(\frac{dz}{dt}\right)$

$-439411.6 = 600\left(\frac{dz}{dt}\right)$

$\boxed{\frac{dz}{dt} = -732.353\ mph}$

10. A man 6 ft tall walks at a rate of 5 ft/sec toward a streetlight that is 16 ft above the ground. A) At what rate is the length of his shadow changing when he is 10 ft from the base of the light? B) At what rate is the tip of his shadow moving?



$\frac{6}{16} = \frac{y}{x+y}$

$6x + 6y = 16y$
$6x = 10y$
$6\left(\frac{dx}{dt}\right) = 10\left(\frac{dy}{dt}\right)$

$\frac{dx}{dt} = -5\ ft/sec$ $x = 10ft$ $\frac{dy}{dt} = \_\_?$

$6(-5) = 10\left(\frac{dy}{dt}\right)$

$-3 = \frac{dy}{dt}$

$\boxed{a)\ \frac{dy}{dt} = -3\ ft/sec}$

b) ROC tip of shadow = $\frac{dx}{dt} + \frac{dy}{dt} \rightarrow -5 - 3 =$

$\boxed{-8\ ft/sec}$

(12)

Additional Practice Problem:

11)

Water is leaking out of a full cylindrical container at a rate of 2 in³/hr. The container has a diameter of 8 in. and height of 12 in.
At what rate is the height of the container changing when the container is half full?

$r = 4\text{ in.}$

*radius is constant: $\left(\frac{dh}{dt}\right)$ $h = 6\text{ in.}$ (not used)

$r = 4\text{ in.}$ → $V = \pi r^2 h$

$\frac{dV}{dt} = -2\text{ in}^3/\text{hr.}$

$\frac{dh}{dt} = \_\_\_?$

$V = \pi(4)^2 h$

$V = 16\pi h$

$\frac{dV}{dt} = 16\pi\left(\frac{dh}{dt}\right)$

$-2 = 16\pi\left(\frac{dh}{dt}\right)$

$\frac{-2}{16\pi} = \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{-1}{8\pi}\text{ in/hr.}}$

12) Two boats leave the same port at the same time with one boat traveling north at 15 knots per hour and the other boat traveling west at 12 knots per hour. How fast is the distance between the 2 boats changing after 2 hours?

$x^2 + y^2 = z^2$

$\frac{dx}{dt} = 12$

$dy/dt = 15$

$\frac{dz}{dt} = \_\_\_?$

$x = 12\text{ knots/hr} \cdot 2\text{ hr} = 24\text{ knots}$

$y = 15\text{ knots/hr} \cdot 2\text{ hr} = 30\text{ knots}$

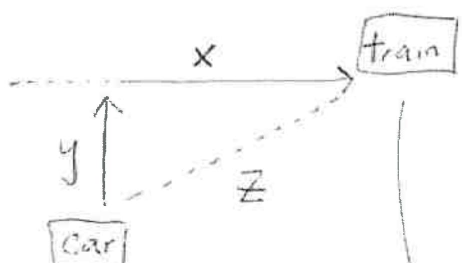
$z = 38.419 \leftarrow x^2 + y^2 = z^2$, $24^2 + 30^2 = z^2$

$\boxed{\frac{dz}{dt} = 19.209\text{ knots/hr}}$

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right) \rightarrow$ $2(24)(12) + 2(30)(15) = 2(38.419)\left(\frac{dz}{dt}\right)$

$576 + 900 = 76.839\left(\frac{dz}{dt}\right)$

13) A man is driving north at a rate of 17 m/s. He sees a railroad track 20m ahead of him that is perpendicular to the road. There is a train going east on the track crossing the road and the man determines with a radar gun that the engine is 35 m from him and the distance between his car and the engine is increasing at the rate of 5 m/s. What is the speed of the train?



$x^2 + 20^2 = 35^2$

$x = 28.723$ $dx/dt = \_\_\_?$

$y = 20$ $dy/dt = -17$

$z = 35$ $dz/dt = 5$

$57.446\left(\frac{dx}{dt}\right) = 1030$

$\boxed{\frac{dx}{dt} = 17.929\text{ m/s}}$

$x^2 + y^2 = z^2$

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right)$

$2(28.723)\left(\frac{dx}{dt}\right) + 2(20)(-17) = 2(35)(5)$

$57.446\left(\frac{dx}{dt}\right) - 680 = 350$

Related Rates Test Review (2)

(13)

1. Helium is pumped into a spherical balloon at the constant rate of 25 cubic feet/minute. At what rate is the surface area of the balloon increasing at the moment when its radius is 8 feet?

2. A spherical snowball with an outer layer of ice melts so that the volume of the snowball decreases at a rate of 2 $\frac{cm^3}{min}$. How fast is the radius changing when diameter of the snowball is 10 cm?

3. Two cars start moving from the same point. One travels south at 60 m/hr and the other travels west at 25 m/hr. At what rate is the distance between the cars increasing two hours later?

b) Find ROC of Area

14

4. A street light is mounted on a 15 foot pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 feet from the pole? Write down the exact answer. b) Find ROC of Area

5. At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/hr and ship B is sailing north at 25 km/hr. How fast is the distance between the ships changing at 4:00 P.M.?

b) Find ROC of Area

6) Find $\lim_{x \to 1} \frac{x^3 - 1}{4x^3 - x - 3}$

7) Use linear approximation to approximate $\frac{1}{\sqrt[3]{8.04}}$

/

Related Rates Test Review $V=\frac{4}{3}\pi r^3$ $S=4\pi r^2$

Key

15

1. Helium is pumped into a spherical balloon at the constant rate of 25 cubic feet/minute. At what rate is the surface area of the balloon increasing at the moment when its radius is 8 feet?

$\frac{dV}{dt}=25\ ft^3/min.$

$\frac{dS}{dt}=$ ___

$r=8$ ft.

$\frac{dV}{dt}=4\pi r^2\left(\frac{dr}{dt}\right)$

$25=4\pi(8)^2\left(\frac{dr}{dt}\right)$

$\frac{25}{256\pi}=\frac{dr}{dt}$

$\frac{dS}{dt}=8\pi r\left(\frac{dr}{dt}\right)$

$\frac{dS}{dt}=8\pi(8)\left(\frac{25}{256\pi}\right)$

$\boxed{\frac{dS}{dt}=\frac{25}{4}\ ft^2/min}$

2. A spherical snowball with an outer layer of ice melts so that the volume of the snowball decreases at a rate of 2 $\frac{cm^3}{min}$. How fast is the radius changing when diameter of the snowball is 10 cm?

$r=5$

$\frac{dV}{dt}=-2\ cm^3/min$

$V=\frac{4}{3}\pi r^3$

$\frac{dV}{dt}=4\pi r^2\left(\frac{dr}{dt}\right)$

$-2=4\pi(5)^2\left(\frac{dr}{dt}\right)$

$\frac{-2}{25(4)\pi}=\frac{dr}{dt}$

$\boxed{\frac{dr}{dt}=\frac{-1}{50\pi}\ ft/min}$

3b) Find Rate of Change in Area

$A=\frac{1}{2}xy$

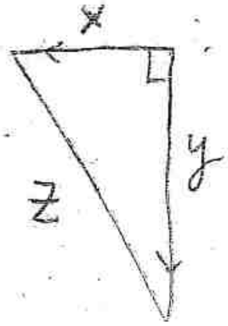
$\frac{dA}{dt}=\frac{1}{2}\frac{dx}{dt}y+\frac{1}{2}x\left(\frac{dy}{dt}\right)$

$\frac{dA}{dt}=\frac{1}{2}(25)(120)+\frac{1}{2}(50)(60)$

$=1500+1500$

$=\boxed{3000\ m^2/hr.}$

3. Two cars start moving from the same point. One travels south at 60 m/hr and the other travels west at 25 m/hr. At what rate is the distance between the cars increasing two hours later?



$x^2+y^2=z^2$

$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=2z\left(\frac{dz}{dt}\right)$

$2(50)(25)+2(120)(60)=2(130)\left(\frac{dz}{dt}\right)$

$\boxed{\frac{dz}{dt}=65\ mph}$

$\frac{dx}{dt}=25\ mph$ $x=25(2)=50$

$\frac{dy}{dt}=60\ mph$ $y=60(2)=120$

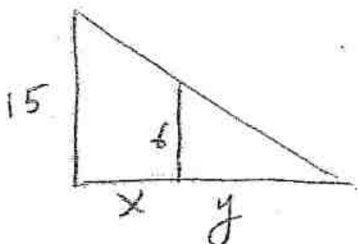
$x^2+y^2=z^2$

$50^2+120^2=z^2$

$z=130$

(16)

4. A street light is mounted on a 15 foot pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 feet from the pole? Write down the exact answer.



$\frac{dx}{dt} = 5$ ft/s

$x = 40$ ft

$\frac{6}{15} = \frac{y}{x+y}$

$6x+6y = 15y$

$6x = 9y$

$6\left(\frac{dx}{dt}\right) = 9\left(\frac{dy}{dt}\right)$

$6(5) = 9\left(\frac{dy}{dt}\right)$

$\frac{30}{9} = \frac{10}{3}$ ft/s

tip $= \frac{10}{3} + 5 = \boxed{\frac{25}{3} \text{ ft/s}}$

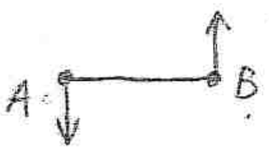
$\boxed{4B}$ How fast is area changing? (large triangle)

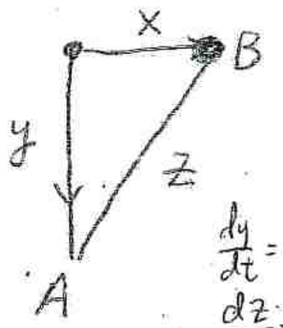
$A = \frac{1}{2}(15)(x+y)$

$\frac{dA}{dt} = \frac{15}{2}\left(\frac{dx}{dt} + \frac{dy}{dt}\right) = \frac{15}{2}\left(\frac{25}{3}\right)$

$= \boxed{62.5 \text{ ft}^2/\text{s}}$

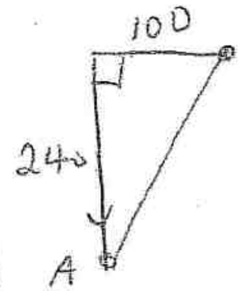
5. At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/hr and ship B is sailing north at 25 km/hr. How fast is the distance between the ships changing at 4:00 P.M.?





$\frac{dy}{dt} = 60$ km/hr

$\frac{dz}{dt} =$



$y = 60(4) = 240$ km

$100^2 + 240^2 = z^2$

$z = 260$

$x^2 + y^2 = z^2$

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right)$

$\cancel{2(100)(0)} + 2(240)(60) = 2(260)\left(\frac{dz}{dt}\right)$

$\frac{dz}{dt} = 55.385$ km/hr

$\boxed{5B}$ $A = \frac{1}{2}(100)(y)$

$\frac{dA}{dt} = 50\left(\frac{dy}{dt}\right)$

$\frac{dA}{dt} = 50(60)$

$\frac{dA}{dt} = 3000 \text{ km}^2/\text{hr}$

6) $\lim_{x\to1} \frac{x^3-1}{4x^3-x-3} = \frac{0}{0} \to \lim_{x\to1} \frac{3x^2}{12x^2-1}$ $\boxed{\frac{3}{11}}$

7) $\frac{1}{\sqrt[3]{8.04}}$ $y = \frac{1}{\sqrt[3]{x}} = x^{-1/3}$

$(8, \frac{1}{2})$ $y' = -\frac{1}{3}x^{-4/3}$ $y' = \frac{-1}{3x^{4/3}}$

$y'(8) = \frac{-1}{3(8)^{4/3}} = \frac{-1}{48}$

$m = -1/48$

$y - \frac{1}{2} = \frac{-1}{48}(x-8)$

$y = \frac{-1}{48}(x-8) + \frac{1}{2}$

$y = \frac{-1}{48}(8.04-8) + \frac{1}{2} = \boxed{0.4992}$

17

Related Rates Morning Review WS #3

1. Two cyclists leave from the same position. Cyclist A travels due North at 10 mph. **One hour later**, the cyclist B leaves from the position and travels due East at 20 mph. At what rate is the distance between the two cyclists changing **2 hours after cyclist B leaves**?

2) Water is being pumped into a conical tank that is 8 feet tall and has a diameter of 10 feet. If the water is being pumped in at a constant rate of 3/5 cubic feet per hour, at what rate is the depth of the water in the tank changing when the tank is half full? $\left(V = \frac{\pi}{3} r^2 h\right)$

18

3) The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

$(A = \pi r^2 \quad C = 2\pi r)$

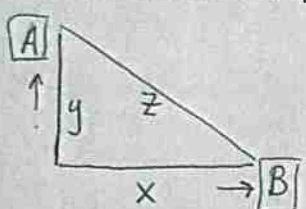
4) A cylindrical tank has a height of 16 feet with the area of the circular base being 25π ft^2 .

Water flows at 8 cubic feet per minute into the tank. How fast is the water level rising when the tank is half full? $(Area\ of\ circle = \pi r^2)$ $(Volume\ of\ cylinder = \pi r^2 h)$

Key

Related Rates Morning Review WS #3

1. Two cyclists leave from the same position. Cyclist A travels due North at 10 mph. **One hour later,** the cyclist B leaves from the position and travels due East at 20 mph. At what rate is the distance between the two cyclists changing **2 hours after cyclist B leaves**?



$x = 20\,\text{mph} \times 2\,\text{hrs} = 40\,\text{mi}$ | $dx/dt = 20\,\text{mph}$

$y = 10\,\text{mph} \times 3\,\text{hrs} = 30\,\text{mi}$ | $dy/dt = 10\,\text{mph}$

$z = 50$ ($30^2 + 40^2 = z^2$) | $dz/dt = \_\_\_$?

$$x^2 + y^2 = z^2$$

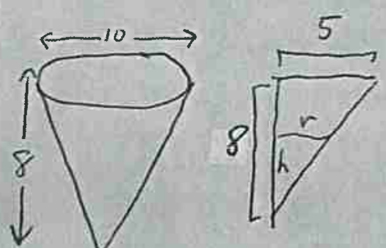
$$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right) \rightarrow 2(40)(20) + 2(30)(10) = 2(50)\left(\frac{dz}{dt}\right)$$

$$1600 + 600 = 100\left(\frac{dz}{dt}\right)$$

$$2200 = 100\left(\frac{dz}{dt}\right)$$

$$\boxed{\frac{dz}{dt} = 22\,\text{mph}}$$

2) Water is being pumped into a conical tank that is 8 feet tall and has a diameter of 10 feet. If the water is being pumped in at a constant rate of 3/5 cubic feet per hour, at what rate is the depth of the water in the tank changing when the tank is half full? $\left(V = \frac{\pi}{3} r^2 h\right)$



$\frac{dV}{dt} = \frac{3}{5}\ \text{ft}^3/\text{hr}$ $\boxed{h = 4\ \text{ft}}$ save until after derivative to plug in.

$\frac{dh}{dt} = \_\_\_$?

Rewrite Volume equation in terms of h.

$$\frac{r}{5} = \frac{h}{8}$$

$$8r = 5h$$

$$r = \boxed{\frac{5}{8}h}$$

$$V = \frac{\pi}{3} r^2 h$$

$$V = \frac{\pi}{3}\left(\frac{5h}{8}\right)^2 h$$

$$V = \frac{\pi}{3} \cdot \frac{25h^2}{64} \cdot h$$

$$V = \frac{25\pi}{192} h^3$$

$$\frac{dV}{dt} = \frac{25\pi}{192} \cdot 3h^2 \cdot \frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{75\pi}{192} h^2 \left(\frac{dh}{dt}\right)$$

$$\frac{3}{5} = \frac{75\pi}{192} \cdot (4)^2 \left(\frac{dh}{dt}\right)$$

$$\frac{3}{5} = \frac{1200\pi}{192}\left(\frac{dh}{dt}\right)$$

$$\frac{3}{5} = \frac{25\pi}{4}\left(\frac{dh}{dt}\right)$$

$$\frac{3}{5} \cdot \frac{4}{25\pi} = \frac{dh}{dt}$$

$$\frac{12}{125\pi} = \frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt} = \frac{12}{125\pi}\ \text{ft/hr}}$$

3) The radius of a circle is increasing at a constant rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?

$(A = \pi r^2 \quad C = 2\pi r)$

$\frac{dr}{dt} = 0.2\, m/s \qquad \frac{dA}{dt} = \_\_\_ ? \qquad C = 20\pi$ meters

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r\left(\frac{dr}{dt}\right)$

$C = 2\pi r$

$\frac{dC}{dt} = 2\pi\left(\frac{dr}{dt}\right)$

$C = 2\pi r$

$\frac{20\pi}{2\pi} = \frac{2\pi r}{2\pi}$

$\underline{\underline{10 = r}}$

$\frac{dA}{dt} = 2\pi r\left(\frac{dr}{dt}\right)$

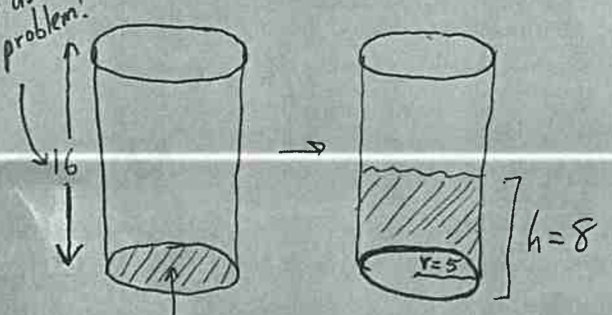
$\frac{dA}{dt} = 2\pi(10)(0.2)$

$\boxed{\frac{dA}{dt} = 4\pi \ m^2/sec}$

4) A cylindrical tank has a height of 16 feet with the area of the circular base being 25π ft^2.

Water flows at 8 cubic feet per minute into the tank. How fast is the water level rising when the tank is half full? $(Area\ of\ circle = \pi r^2)$ $(Volume\ of\ cylinder = \pi r^2 h)$

not used in problem!



$\frac{dV}{dt} = 8\ ft^3/min \qquad h = 8$ ← * not used!

$\frac{dh}{dt} = \_\_\_ ?$

$A = 25\pi$

$A = \pi r^2$

$25\pi = \pi r^2$

$25 = r^2$

$\underline{\underline{r = 5}}$

* Radius r is constant:

$V = \pi r^2 h$

$V = \pi(5)^2 h$

$V = 25\pi h$

$\frac{dV}{dt} = 25\pi\left(\frac{dh}{dt}\right)$

$8 = 25\pi\left(\frac{dh}{dt}\right)$

$\frac{8}{25\pi} = \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{8}{25\pi}\ ft/min}$