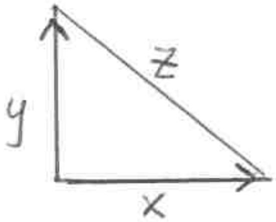
Key

Calculus AB Related Rates Test Review

1. A balloon is rising vertically above a level, straight road at a constant rate of 1 ft/sec. Just when the balloon is 65 ft above the ground, a bicycle moving at a constant rate of 17 ft/sec passes under it. How fast is the distance between the bicycle and balloon increasing $\boxed{\text{3 seconds later?}}$



$x^2+y^2=z^2$

$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=2z\left(\frac{dz}{dt}\right)$

$x=17(3)=51$

$y=65+3=68$

$z=85$

$x^2+y^2=z^2$

$51^2+68^2=z^2$

$dx/dt=17$ ft/s

$dy/dt=1$ ft/s

$dz/dt=$ ___

$2(51)(17)+2(68)(1)=2(85)\left(\frac{dz}{dt}\right)$

$1734+136=170\left(\frac{dz}{dt}\right)$

$1870=170\left(\frac{dz}{dt}\right)$

$\boxed{\frac{dz}{dt}=11\text{ ft/s}}$

2) Suppose $f(x)=3x^2-2x$.

a) Write the equation of the line tangent to $f(x)$ at $x=2$. Then use local linear approximation to estimate $f(1.9)$.

$f(2)=3(2)^2-2(2)=12-4=8$

$f'(x)=6x-2$

$f'(2)=6(2)-2=10$

point: $(2,8)$

slope $m=10$

$y-y_1=m(x-x_1)$

$y-8=10(x-2)$

$y=10(x-2)+8$

$y(1.9)=10(1.9-2)+8$

$\boxed{y(1.9)=7}$

b) Suppose $h(5)=3$ and $h'(5)=-2$. Use local linear approximation to estimate $h(5.2)$

point: $(5,3)$

slope: $m=-2$

$y-3=-2(x-5)$

$y=-2(x-5)+3$

$y(5.2)=-2(5.2-5)+3$

$\boxed{y(5.2)=2.6}$

3. Find the limit:

a) $\lim_{x\to 3}\frac{4x^2-5x}{1-3x^2} \to \frac{4(3)^2-5(3)}{1-3(3)^2}$ $\boxed{\frac{21}{-26}}$

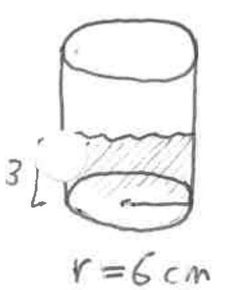
b) $\lim_{x\to 1}\frac{5x^4-4x^2-1}{10-x-9x^3}=\frac{0}{0}$ Apply L'Hopital's Rule $\frac{f'(c)}{g'(c)}$

$\lim_{x\to 1}\frac{20x^3-8x}{-1-27x^2}=\frac{20-8}{-1-27}=\frac{12}{-28}=\boxed{\frac{-3}{7}}$

4. Water is leaking out of a cylindrical container at a rate of 5 cm^3/hr. The container has a diameter of 12 cm and height of 16 cm. At what rate is the height changing when water has height of 3 cm? $V=\pi r^2 h$

*No need for similar triangle: radius is a constant

Find $\frac{dh}{dt}$



$\frac{dV}{dt}=-5\text{ cm}^3/\text{hr}$

$V=\pi r^2 h$

Replace r since radius of water level is constant

$V=\pi(6)^2h$

$V=36\pi h$

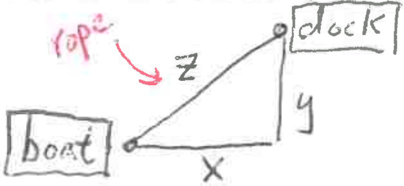
$\frac{dV}{dt}=36\pi\left(\frac{dh}{dt}\right)$

$-5=36\pi\left(\frac{dh}{dt}\right)$

$\frac{-5}{36\pi}=\frac{dh}{dt}$

$\boxed{\frac{dh}{dt}=\frac{-5}{36\pi}\text{ cm/hr.}}$

6. A boat is pulled toward a dock by a rope from the bow through a ring on the dock 6 ft above the bow. The rope is hauled in at a rate of 2 ft/sec



z = rope length y is constant

$x^2+y^2=z^2$

a) How fast is the boat approaching the dock when 10 ft of rope are out?

$x^2+y^2=z^2$

$2x\left(\frac{dx}{dt}\right)+2y\left(\frac{dy}{dt}\right)=2z\left(\frac{dz}{dt}\right)$

$x = 8$ $dx/dt =$ ___

$y = 6$ $dy/dt = 0$

$z = 10$ $dz/dt = -2$

$2(8)\left(\frac{dx}{dt}\right)+2(6)(0)=2(10)(-2)$

$16\left(\frac{dx}{dt}\right)+0=-40$

$\boxed{\frac{dx}{dt}=\frac{-40}{16}=-2.5\text{ ft/s}}$

b) At what rate is area of triangle changing at that moment?

$A=\underbrace{\tfrac{1}{2}x}_{f}\,\underbrace{y}_{g}$ *product Rule $f'g+fg'$

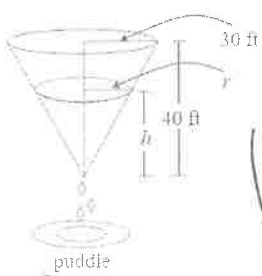
$\frac{dA}{dt}=\underbrace{\tfrac{1}{2}\left(\frac{dx}{dt}\right)}_{f'}\cdot\underbrace{y}_{g}+\underbrace{\tfrac{1}{2}x}_{f}\cdot\underbrace{\frac{dy}{dt}}_{g'}$

$\frac{dA}{dt}=\frac{1}{2}(-2.5)(6)+\frac{1}{2}(8)(0)$

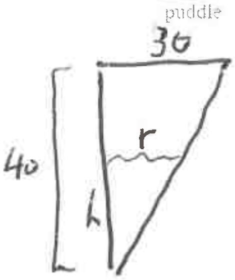
$\boxed{\frac{dA}{dt}=-7.5\text{ ft}^2/\text{sec}}$

7) A conical water tank with a height of 40 ft and a radius of 30 ft is leaking at the rate of 2 ft³ / min. When the height (h) of the water in the tank is 30 ft, at what rate is the height of the water changing?

(Volume of a cone = $\frac{1}{3}\pi r^2 h$)



$\frac{dV}{dt}=-2$ $\frac{dh}{dt}=$ ___ ? $\boxed{h=30}$ Do not use until end of problem



$\frac{r}{30}=\frac{h}{40}$

$40r=30h$

$r=\frac{30h}{40}$

$r=\frac{3h}{4}$

$V=\frac{\pi}{3}r^2h$

$V=\frac{\pi}{3}\left(\frac{3h}{4}\right)^2h$

$V=\frac{\pi}{3}\cdot\frac{9h^2}{16}\cdot h$

$V=\frac{3\pi}{16}h^3$

$\frac{dV}{dt}=\frac{3\pi}{16}\cdot 3h^2\left(\frac{dh}{dt}\right)$

$\frac{dV}{dt}=\frac{9\pi}{16}h^2\left(\frac{dh}{dt}\right)$

$-2=\frac{9\pi}{16}(30)^2\left(\frac{dh}{dt}\right)$

$-2=\frac{8100\pi}{16}\left(\frac{dh}{dt}\right)$

$-2=\frac{2025\pi}{4}\left(\frac{dh}{dt}\right)$

$-2\cdot\frac{4}{2025\pi}=\frac{dh}{dt}$

$\boxed{\frac{dh}{dt}=\frac{-8}{2025\pi}\text{ ft/min}}$

8. The volume of a cube is decreasing at a rate of 10 m^3 / hour. How fast is the total surface area decreasing when the surface area is 54 m^2? ($V = x^3$ $S = 6x^2$)

$V = x^3$ | $S = 6x^2$ | $\frac{dS}{dt} = \_\_?$ $\frac{dV}{dt} = -10\ m^3/hr$

$\frac{dV}{dt} = 3x^2\left(\frac{dx}{dt}\right)$ | $\frac{dS}{dt} = 12x\left(\frac{dx}{dt}\right)$ | $S = 54m^2$

$S = 6x^2$
$54 = 6x^2$
$9 = x^2$
$3 = x$

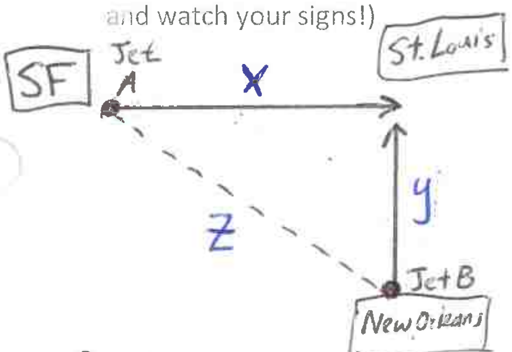
$-10 = 3(3)^2\left(\frac{dx}{dt}\right)$
$-10 = 27\left(\frac{dx}{dt}\right)$

$\frac{-10}{27} = \frac{dx}{dt}$

$\frac{dS}{dt} = 12(3)\left(\frac{-10}{27}\right)$

$\boxed{\frac{dS}{dt} = \frac{-40}{3}\ m^2/hr}$

9. Jet A travels due east from San Francisco toward St. Louis at 500 mph. Jet B travels due north from New Orleans toward St. Louis at 600mph. Find the rate of change of the distance between the two jets when they are 300 miles apart, and jet A is 100 miles from St. Louis (round answer to 3 decimal places) *Be sure to draw diagram, and watch your signs!)



$x^2 + y^2 = z^2$

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right)$

$x = 100$
$y = 282.843$
$z = 300$

$100^2 + y^2 = 300^2$
$y \approx 282.843$

$dx/dt = -500$
$dy/dt = -600$
$dz/dt = \_\_\_$

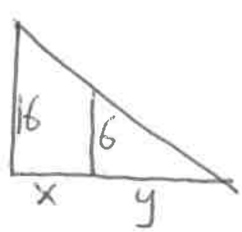
$2(100)(-500) + 2(282.843)(-600) = 2(300)(dz/dt)$

$-100{,}000 - 339411.6 = 600\left(\frac{dz}{dt}\right)$

$-439411.6 = 600\left(\frac{dz}{dt}\right)$

$\boxed{\frac{dz}{dt} = -732.353\ mph}$

10. A man 6 ft tall walks at a rate of 5 ft/sec toward a streetlight that is 16 ft above the ground. A) At what rate is the length of his shadow changing when he is 10 ft from the base of the light? B) At what rate is the tip of his shadow moving?



$\frac{6}{16} = \frac{y}{x+y}$

$6x + 6y = 16y$
$6x = 10y$
$6\left(\frac{dx}{dt}\right) = 10\left(\frac{dy}{dt}\right)$

$\frac{dx}{dt} = -5\ ft/sec$ $x = 10\ ft$ $\frac{dy}{dt} = \_\_?$

$6(-5) = 10\left(\frac{dy}{dt}\right)$

$-3 = \frac{dy}{dt}$

$\boxed{a)\ \frac{dy}{dt} = -3\ ft/sec}$

b) ROC tip of shadow =
$\frac{dx}{dt} + \frac{dy}{dt} \rightarrow -5 - 3 =$

$\boxed{-8\ ft/sec}$

Additional Practice Problem:

11)

Water is leaking out of a full cylindrical container at a rate of 2 in^3/hr. The container has a diameter of 8 in. and height of 12 in.
At what rate is the height of the container changing when the container is half full?

$r = 4$ in.

$V = \pi r^2 h$

*radius is constant:

$\frac{dh}{dt}$

$h = 6$ in.
(not used)

$r = 4$ in.

$\frac{dV}{dt} = -2\ in^3/hr.$

$\frac{dh}{dt} = \_\_\_\_ ?$

$V = \pi r^2 h$

$V = \pi (4)^2 h$

$V = 16\pi h$

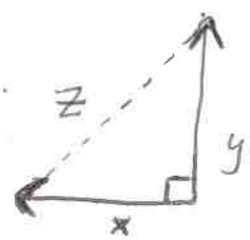
$\frac{dV}{dt} = 16\pi\left(\frac{dh}{dt}\right)$

$-2 = 16\pi\left(\frac{dh}{dt}\right)$

$\frac{-2}{16\pi} = \frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{-1}{8\pi}\ in/hr.}$

12) Two boats leave the same port at the same time with one boat traveling north at 15 knots per hour and the other boat traveling west at 12 knots per hour. How fast is the distance between the 2 boats changing after 2 hours?



$x^2 + y^2 = z^2$

$\frac{dx}{dt} = 12$

$dy/dt = 15$

$\frac{dz}{dt} = \_\_\_\_ ?$

$x = 12\ knots/hr \cdot 2\ hr = 24\ knots$

$y = 15\ knots/hr \cdot 2\ hr = 30\ knots$

$z = 38.419 \leftarrow x^2 + y^2 = z^2$
$24^2 + 30^2 = z^2$

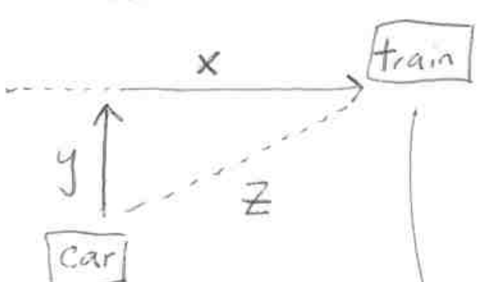
$\boxed{\frac{dz}{dt} = 19.209\ knots/hr}$

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right) \rightarrow$

$2(24)(12) + 2(30)(15) = 2(38.419)\left(\frac{dz}{dt}\right)$

$576 + 900 = 76.839\left(\frac{dz}{dt}\right)$

13) A man is driving north at a rate of 17 m/s. He sees a railroad track 20m ahead of him that is perpendicular to the road. There is a train going east on the track crossing the road and the man determines with a radar gun that the engine is 35 m from him and the distance between his car and the engine is increasing at the rate of 5 m/s. What is the speed of the train?



$x^2 + 20^2 = 35^2$

$x = 28.723$ $\quad dx/dt = \_\_\_\_ ?$

$y = 20$ $\quad dy/dt = -17$

$z = 35$ $\quad dz/dt = 5$

$x^2 + y^2 = z^2$

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right)$

$2(28.723)\left(\frac{dx}{dt}\right) + 2(20)(-17) = 2(35)(5)$

$57.446\left(\frac{dx}{dt}\right) - 680 = 350$

$57.446\left(\frac{dx}{dt}\right) = 1030$

$\boxed{\frac{dx}{dt} = 17.929\ m/s}$