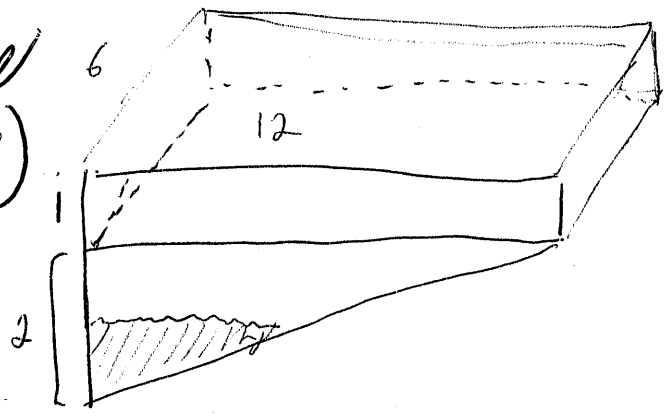
HW

19)

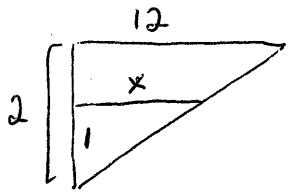



BC selected Problems 2.6 Related Rates

Rectangular prism $1 \cdot 6 \cdot 12 = 72$

Triangular prism $= \frac{1}{2} \cdot 2 \cdot 12 \cdot 6 = 72$

Total $= 144\ m^3$




$\frac{1}{2} = \frac{x}{12}$

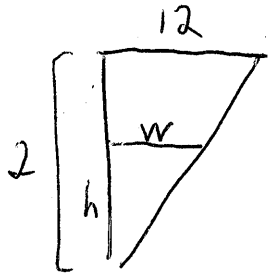
$x = 6$

Volume of water $= \frac{1}{2} \cdot 1 \cdot 6 \cdot 6 = 18\ m^3$

% Filled $= \frac{18}{144} = \frac{1}{8}$ or $12.5\%$

b) $\frac{dV}{dt} = \frac{1}{4}\ m^3/min$

$h = 1$




$\frac{w}{12} = \frac{h}{2}$

$2w = 12h$

$w = 6h$

$V = \frac{1}{2} h \cdot l \cdot w$

$V = \frac{1}{2} \cdot h \cdot 6 \cdot w$

$V = 3hw$

$V = 3h(6h)$

$V = 18h^2$

$\frac{dV}{dt} = 36h\left(\frac{dh}{dt}\right)$

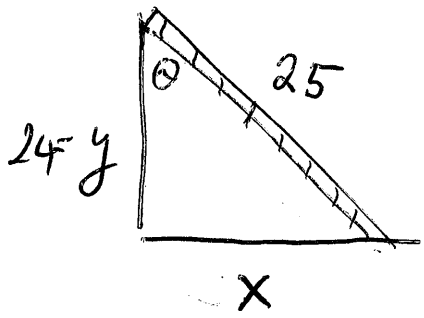
$\frac{1}{4} = 36(1)\left(\frac{dh}{dt}\right)$

$\frac{1}{36} \cdot \frac{1}{4} = \frac{dh}{dt}$

$$\frac{dh}{dt} = \frac{1}{144}\ m/min$$

21 c) Find rate of change of angle b/t wall and ladder

$\frac{d\theta}{dt} =$ ____




$\tan\theta = \frac{x}{y}$

$\frac{\sec^2\theta}{\sec^2\theta}\left(\frac{d\theta}{dt}\right) = \frac{\left(\frac{dx}{dt}\right)y - x\left(\frac{dy}{dt}\right)}{y^2} \cdot \frac{1}{\sec^2\theta}$

$\frac{d\theta}{dt} = \frac{(\cos\theta)^2\left[\frac{dx}{dt}y - x\left(\frac{dy}{dt}\right)\right]}{y^2}$

$\frac{d\theta}{dt} = \frac{\left(\frac{24}{25}\right)^2\left[2(24) - 7\left(-\frac{7}{12}\right)\right]}{24^2}$

$x = 7$

$y = 24$

$\frac{dx}{dt} = 2$

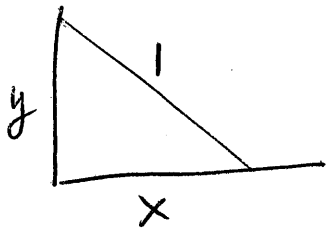
$\frac{dy}{dt} = -7/12$

$\cos\theta = \frac{24}{25}$

$= \frac{1}{12}$ rad/sec

# 2.6 Related Rates

31) $x(t) = \frac{1}{2}\sin\left(\frac{\pi t}{6}\right)$ , $x^2 + y^2 = 1$




a) Period $= \frac{2\pi}{B} = \frac{2\pi}{\pi/6} = 12$ seconds

b) Greatest value for $x(t) = 1/2$, since largest value for $\sin\left(\frac{\pi t}{6}\right) = 1$, $x(t) = \frac{1}{2}(1) = \frac{1}{2}$

when $x = \frac{1}{2}$, $\left(\frac{1}{2}\right)^2 + y^2 = 1$ $\quad y^2 = 1 - \frac{1}{4}$, $\quad y = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$

$y = \frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$

Lowest point: $\left(0, \frac{\sqrt{3}}{2}\right)$

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c) Find $\frac{dy}{dt}$ when $x = 1/4$

$y = \sqrt{1 - (1/4)^2} = \frac{\sqrt{15}}{4}$ and $t = 1$

$x = \frac{1}{2}\sin\left(\frac{\pi t}{6}\right)$

$\frac{dx}{dt} = \frac{1}{2}\cos\left(\frac{\pi t}{6}\right) \cdot \frac{\pi}{6}\left(\frac{dt}{dt}\right)$

$\frac{dx}{dt} = \frac{\pi}{12}\cos\left(\frac{\pi t}{6}\right)$

$x^2 + y^2 = 1$

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 0$ $\quad \frac{dy}{dt} = \frac{-x}{y}\left(\frac{dx}{dt}\right)$

$\frac{dy}{dt} = \frac{-1/4}{\sqrt{15}/4} \cdot \frac{\pi}{12}\cos\left(\frac{\pi}{6}(1)\right)$

$\frac{dy}{dt} = \frac{-1}{4} \cdot \frac{4}{\sqrt{15}} \cdot \frac{\pi}{12} \cdot \frac{\sqrt{3}}{2} = \frac{-\pi}{24\sqrt{5}}$ or $-\frac{\sqrt{5}\pi}{120}$

speed $= \left|-\frac{\sqrt{5}\pi}{120}\right| = \boxed{\frac{\sqrt{5}\pi}{120}\ m/sec}$

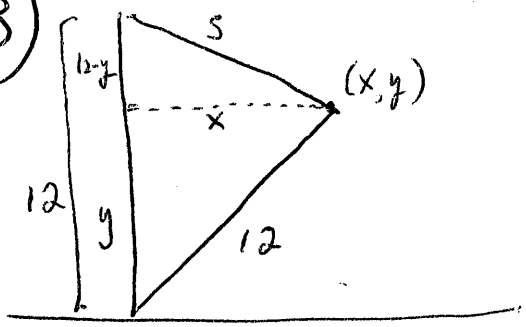
# Related Rates  Selected HW problems

26

$\frac{ds}{dt} = -0.2$ m/s  Find $\frac{dx}{dt}$, $\frac{dy}{dt}$

$y = 6$, $x = 6\sqrt{3}$, $s = 12$

23)




$x^2 + (12-y)^2 = s^2$ and $x^2 + y^2 = 12^2$

$2x\left(\frac{dx}{dt}\right) + 2(12-y)(-1)\left(\frac{dy}{dt}\right) = 2s\left(\frac{ds}{dt}\right)$

$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 0$

$\frac{dy}{dt} = \boxed{\frac{-x}{y}\left(\frac{dx}{dt}\right)}$

$x\left(\frac{dx}{dt}\right) + (y-12)\left[\frac{-x}{y}\left(\frac{dx}{dt}\right)\right] = s\left(\frac{ds}{dt}\right)$

$x\left(\frac{dx}{dt}\right) - x\left(\frac{dx}{dt}\right) + \frac{12x}{y}\left(\frac{dx}{dt}\right) = s\left(\frac{ds}{dt}\right)$

$\frac{dx}{dt} = \frac{sy}{12x}\left(\frac{ds}{dt}\right)$

$\frac{dx}{dt} = \frac{(12)(6)}{12(6\sqrt{3})}(-0.2) = \frac{-1}{5\sqrt{3}} = \boxed{\frac{-\sqrt{3}}{15}\text{ m/sec}}$ (horizontal)

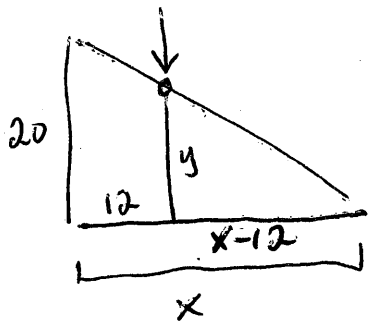
$\frac{dy}{dt} = \frac{-6\sqrt{3}}{6}\left(\frac{-\sqrt{3}}{15}\right) = \frac{1}{5}$ m/sec (vertical)

49) Moving Shadow $y(t) = -4.9t^2 + 20$

$\frac{dy}{dt} = -9.8t$

$y(1) = -4.9 + 20 = 15.1$

$y'(1) = -9.8$




$\frac{y}{20} = \frac{x-12}{x}$ $\quad \frac{15.1}{20} = \frac{x-12}{x}$

$15.1x = 20x - 240$

$-4.9x = -240$

$x = 48.979$

$\frac{y}{20} = \frac{x-12}{x}$

$yx = 20(x-12)$

$xy = 20x - 240$

$\frac{dx}{dt}y + x\left(\frac{dy}{dt}\right) = 20\left(\frac{dx}{dt}\right) - 0$

$\frac{dx}{dt}y - 20\left(\frac{dx}{dt}\right) = -x\left(\frac{dy}{dt}\right)$

$\frac{dx}{dt}(y-20) = -x\left(\frac{dy}{dt}\right)$

$\frac{dx}{dt} = \frac{-x\left(\frac{dy}{dt}\right)}{y-20} = \frac{-48.979(-9.8)}{15.1-20} = \boxed{-97.958 \text{ m/sec}}$