74. $x = y^2$

$1 = 2yy'$

$y' = \dfrac{1}{2y}$, slope of tangent line

Consider the slope of the normal line joining $(x_0, 0)$ and $(x, y) = (y^2, y)$ on the parabola.

$$-2y = \frac{y - 0}{y^2 - x_0}$$

$$y^2 - x_0 = -\frac{1}{2}$$

$$y^2 = x_0 - \frac{1}{2}$$

(a) If $x_0 = \frac{1}{4}$, then $y^2 = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$, which is impossible. So, the only normal line is the x-axis $(y = 0)$.

(b) If $x_0 = \frac{1}{2}$, then $y^2 = 0 \Rightarrow y = 0$. Same as part (a).

(c) If $x_0 = 1$, then $y^2 = \frac{1}{2} = x$ and there are three normal lines.

The x-axis, the line joining $(x_0, 0)$ and $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$, and the line joining $(x_0, 0)$ and $\left(\frac{1}{2}, -\frac{1}{\sqrt{2}}\right)$

If two normals are perpendicular, then their slopes are -1 and 1. So,

$$-2y = -1 = \frac{y - 0}{y^2 - x_0} \Rightarrow y = \frac{1}{2}$$

and

$$\frac{1/2}{(1/4) - x_0} = -1 \Rightarrow \frac{1}{4} - x_0 = -\frac{1}{2} \Rightarrow x_0 = \frac{3}{4}.$$

The perpendicular normal lines are $y = -x + \dfrac{3}{4}$ and $y = x - \dfrac{3}{4}$.

75. (a) $\dfrac{x^2}{32} + \dfrac{y^2}{8} = 1$

$$\frac{2x}{32} + \frac{2yy'}{8} = 0 \Rightarrow y' = \frac{-x}{4y}$$

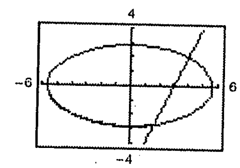
At $(4, 2)$: $y' = \dfrac{-4}{4(2)} = -\dfrac{1}{2}$

Slope of normal line is 2.

$$y - 2 = 2(x - 4)$$

$$y = 2x - 6$$

(b)



(c) $\dfrac{x^2}{32} + \dfrac{(2x - 6)^2}{8} = 1$

$$x^2 + 4(4x^2 - 24x + 36) = 32$$

$$17x^2 - 96x + 112 = 0$$

$$(17x - 28)(x - 4) = 0 \Rightarrow x = 4, \frac{28}{17}$$

Second point: $\left(\dfrac{28}{17}, -\dfrac{46}{17}\right)$

Section 2.6 Related Rates

1. $y = \sqrt{x}$

$$\frac{dy}{dt} = \left(\frac{1}{2\sqrt{x}}\right)\frac{dx}{dt}$$

$$\frac{dx}{dt} = 2\sqrt{x}\,\frac{dy}{dt}$$

(a) When $x = 4$ and $dx/dt = 3$:

$$\frac{dy}{dt} = \frac{1}{2\sqrt{4}}(3) = \frac{3}{4}$$

(b) When $x = 25$ and $dy/dt = 2$:

$$\frac{dx}{dt} = 2\sqrt{25}(2) = 20$$

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2. $y = 3x^2 - 5x$

$$\frac{dy}{dt} = (6x - 5)\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{1}{6x - 5}\frac{dy}{dt}$$

(a) When $x = 3$ and $\frac{dx}{dt} = 2$:

$$\frac{dy}{dt} = [6(3) - 5]2 = 26$$

(b) When $x = 2$ and $\frac{dy}{dt} = 4$:

$$\frac{dx}{dt} = \frac{1}{6(2) - 5}(4) = \frac{4}{7}$$

3. $xy = 4$

$$x\frac{dy}{dt} + y\frac{dx}{dt} = 0$$

$$\frac{dy}{dt} = \left(-\frac{y}{x}\right)\frac{dx}{dt}$$

$$\frac{dx}{dt} = \left(-\frac{x}{y}\right)\frac{dy}{dt}$$

(a) When $x = 8$, $y = 1/2$, and $dx/dt = 10$:

$$\frac{dy}{dt} = -\frac{1/2}{8}(10) = -\frac{5}{8}$$

(b) When $x = 1$, $y = 4$, and $dy/dt = -6$:

$$\frac{dx}{dt} = -\frac{1}{4}(-6) = \frac{3}{2}$$

4. $x^2 + y^2 = 25$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \left(-\frac{x}{y}\right)\frac{dx}{dt}$$

$$\frac{dx}{dt} = \left(-\frac{y}{x}\right)\frac{dy}{dt}$$

(a) When $x = 3$, $y = 4$, and $dx/dt = 8$:

$$\frac{dy}{dt} = -\frac{3}{4}(8) = -6$$

(b) When $x = 4$, $y = 3$, and $dy/dt = -2$:

$$\frac{dx}{dt} = -\frac{3}{4}(-2) = \frac{3}{2}$$

5. $y = 2x^2 + 1$

$$\frac{dx}{dt} = 2$$

$$\frac{dy}{dt} = 4x\frac{dx}{dt}$$

(a) When $x = -1$:

$$\frac{dy}{dt} = 4(-1)(2) = -8 \text{ cm/sec}$$

(b) When $x = 0$:

$$\frac{dy}{dt} = 4(0)(2) = 0 \text{ cm/sec}$$

(c) When $x = 1$:

$$\frac{dy}{dt} = 4(1)(2) = 8 \text{ cm/sec}$$

6. $y = \frac{1}{1 + x^2}$, $\frac{dx}{dt} = 6$

$$\frac{dy}{dt} = \frac{-2x}{(1 + x^2)^2} \cdot \frac{dx}{dt}$$

$$= \frac{-2x}{(1 + x^2)^2}(6) = \frac{-12x}{(1 + x^2)^2}$$

(a) When $x = -2$:

$$\frac{dy}{dt} = \frac{(-12)(-2)}{\left[1 + (-2)^2\right]^2} = \frac{24}{25} \text{ in./sec}$$

(b) When $x = 0$:

$$\frac{dy}{dt} = \frac{-12(0)}{(1 + 0)^2} = 0 \text{ in./sec}$$

(c) When $x = 2$:

$$\frac{dy}{dt} = \frac{(-12)(2)}{(1 + 2^2)^2} = -\frac{24}{25} \text{ in./sec}$$

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7. $y = \tan x, \dfrac{dx}{dt} = 3$

$\dfrac{dy}{dt} = \sec^2 x \cdot \dfrac{dx}{dt} = \sec^2 x(3) = 3\sec^2 x$

(a) When $x = -\dfrac{\pi}{3}$:

$\dfrac{dy}{dt} = 3\sec^2\left(-\dfrac{\pi}{3}\right) = 3(2)^2 = 12$ ft/sec

(b) When $x = -\dfrac{\pi}{4}$:

$\dfrac{dy}{dt} = 3\sec^2\left(-\dfrac{\pi}{4}\right) = 3\left(\sqrt{2}\right)^2 = 6$ ft/sec

(c) When $x = 0$:

$\dfrac{dy}{dt} = 3\sec^2(0) = 3$ ft/sec

8. $y = \cos x, \dfrac{dx}{dt} = 4$

$\dfrac{dy}{dt} = -\sin x \cdot \dfrac{dx}{dt} = -\sin x(4) = -4\sin x$

(a) When $x = \dfrac{\pi}{6}$:

$\dfrac{dy}{dt} = -4\sin\left(\dfrac{\pi}{6}\right) = -4\left(\dfrac{1}{2}\right) = -2$ cm/sec

(b) When $x = \dfrac{\pi}{4}$:

$\dfrac{dy}{dt} = -4\sin\left(\dfrac{\pi}{4}\right) = -4\left(\dfrac{\sqrt{2}}{2}\right) = -2\sqrt{2}$ cm/sec

(c) When $x = \dfrac{\pi}{3}$:

$\dfrac{dy}{dt} = -4\sin\left(\dfrac{\pi}{3}\right) = -4\left(\dfrac{\sqrt{3}}{2}\right) = -2\sqrt{3}$ cm/sec

9. Yes, y changes at a constant rate.

$\dfrac{dy}{dt} = a \cdot \dfrac{dx}{dt}$

No, the rate dy/dt is a multiple of dx/dt.

10. Answers will vary. See page 149.

11. $A = \pi r^2$

$\dfrac{dr}{dt} = 4$

$\dfrac{dA}{dt} = 2\pi r\dfrac{dr}{dt}$

(a) When $r = 8, \dfrac{dA}{dt} = 2\pi(8)(4) = 64\pi$ cm²/min.

(b) When $r = 32, \dfrac{dA}{dt} = 2\pi(32)(4) = 256\pi$ cm²/min.

12. (a) $\sin\dfrac{\theta}{2} = \dfrac{(1/2)b}{s} \Rightarrow b = 2s\sin\dfrac{\theta}{2}$

$\cos\dfrac{\theta}{2} = \dfrac{h}{s} \Rightarrow h = s\cos\dfrac{\theta}{2}$

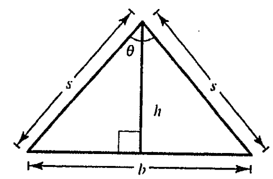
$A = \dfrac{1}{2}bh = \dfrac{1}{2}\left(2s\sin\dfrac{\theta}{2}\right)\left(s\cos\dfrac{\theta}{2}\right)$

$= \dfrac{s^2}{2}\left(2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}\right) = \dfrac{s^2}{2}\sin\theta$

(b) $\dfrac{dA}{dt} = \dfrac{s^2}{2}\cos\theta\dfrac{d\theta}{dt}$ where $\dfrac{d\theta}{dt} = \dfrac{1}{2}$ rad/min.

When $\theta = \dfrac{\pi}{6}, \dfrac{dA}{dt} = \dfrac{s^2}{2}\left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{1}{2}\right) = \dfrac{\sqrt{3}s^2}{8}$.

When $\theta = \dfrac{\pi}{3}, \dfrac{dA}{dt} = \dfrac{s^2}{2}\left(\dfrac{1}{2}\right)\left(\dfrac{1}{2}\right) = \dfrac{s^2}{8}$.



(c) If s and $\dfrac{d\theta}{dt}$ is constant, $\dfrac{dA}{dt}$ is proportional to $\cos\theta$.

13. $V = \dfrac{4}{3}\pi r^3$

$\dfrac{dr}{dt} = 3$

$\dfrac{dV}{dt} = 4\pi r^2\dfrac{dr}{dt}$

(a) When $r = 9$,

$\dfrac{dV}{dt} = 4\pi(9)^2(3) = 972\pi$ in.³/min.

When $r = 36$,

$\dfrac{dV}{dt} = 4\pi(36)^2(3) = 15{,}552\pi$ in.³/min.

(b) If dr/dt is constant, dV/dt is proportional to r^2.

14. $V = \dfrac{4}{3}\pi r^3, \dfrac{dV}{dt} = 800$

$\dfrac{dV}{dt} = 4\pi r^2\dfrac{dr}{dt}$

$\dfrac{dr}{dt} = \dfrac{1}{4\pi r^2}\left(\dfrac{dV}{dt}\right) = \dfrac{1}{4\pi r^2}(800)$

(a) When $r = 30$,

$\dfrac{dr}{dt} = \dfrac{1}{4\pi(30)^2}(800) = \dfrac{2}{9\pi}$ cm/min.

(b) When $r = 60$,

$\dfrac{dr}{dt} = \dfrac{1}{4\pi(60)^2}(800) = \dfrac{1}{18\pi}$ cm/min.

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15. $V = x^3$

$\frac{dx}{dt} = 6$

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

(a) When $x = 2$,

$\frac{dV}{dt} = 3(2)^2(6) = 72 \text{ cm}^3/\text{sec.}$

(b) When $x = 10$,

$\frac{dV}{dt} = 3(10)^2(6) = 1800 \text{ cm}^3/\text{sec.}$

16. $s = 6x^2$

$\frac{dx}{dt} = 6$

$\frac{ds}{dt} = 12x \frac{dx}{dt}$

(a) When $x = 2$,

$\frac{ds}{dt} = 12(2)(6) = 144 \text{ cm}^2/\text{sec.}$

(b) When $x = 10$,

$\frac{ds}{dt} = 12(10)(6) = 720 \text{ cm}^2/\text{sec.}$

17. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{9}{4}h^2\right)h$ [because $2r = 3h$]

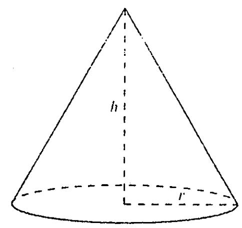
$= \frac{3\pi}{4}h^3$

$\frac{dV}{dt} = 10$

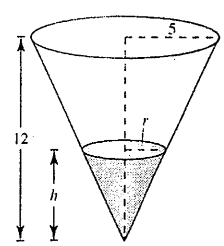
$\frac{dV}{dt} = \frac{9\pi}{4}h^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{4(dV/dt)}{9\pi h^2}$

When $h = 15$,

$\frac{dh}{dt} = \frac{4(10)}{9\pi(15)^2} = \frac{8}{405\pi}$ ft/min.



18. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \frac{25}{144}h^3 = \frac{25\pi}{3(144)}h^3$ $\left(\text{By similar triangles, } \frac{r}{5} = \frac{h}{12} \Rightarrow r = \frac{5}{12}h.\right)$

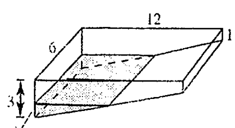


$\frac{dV}{dt} = 10$

$\frac{dV}{dt} = \frac{25\pi}{144}h^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \left(\frac{144}{25\pi h^2}\right)\frac{dV}{dt}$

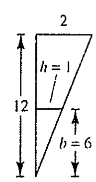
When $h = 8$, $\frac{dh}{dt} = \frac{144}{25\pi(64)}(10) = \frac{9}{10\pi}$ ft/min.

19.



(a) Total volume of pool $= \frac{1}{2}(2)(12)(6) + (1)(6)(12) = 144 \text{ m}^3$

Volume of 1 m of water $= \frac{1}{2}(1)(6)(6) = 18 \text{ m}^3$ (see similar triangle diagram)



% pool filled $= \frac{18}{144}(100\%) = 12.5\%$

(b) Because for $0 \le h \le 2$, $b = 6h$, you have

$V = \frac{1}{2}bh(6) = 3bh = 3(6h)h = 18h^2$

$\frac{dV}{dt} = 36h\frac{dh}{dt} = \frac{1}{4} \Rightarrow \frac{dh}{dt} = \frac{1}{144h} = \frac{1}{144(1)} = \frac{1}{144}$ m/min.

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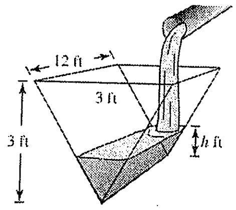
20. $V = \frac{1}{2}bh(12) = 6bh = 6h^2$ (since $b = h$)

(a) $\frac{dV}{dt} = 12h\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{12h}\frac{dV}{dt}$

When $h = 1$ and $\frac{dV}{dt} = 2$, $\frac{dh}{dt} = \frac{1}{12(1)}(2) = \frac{1}{6}$ ft/min.

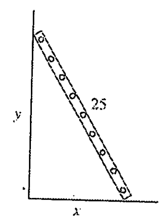


(b) If $\frac{dh}{dt} = \frac{3}{8}$ in./min $= \frac{1}{32}$ ft/min and $h = 2$ ft, then $\frac{dV}{dt} = (12)(2)\left(\frac{1}{32}\right) = \frac{3}{4}$ ft³/min.

21. $x^2 + y^2 = 25^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-x}{y}\cdot\frac{dx}{dt} = \frac{-2x}{y}$ because $\frac{dx}{dt} = 2$.



(a) When $x = 7$, $y = \sqrt{576} = 24$, $\frac{dy}{dt} = \frac{-2(7)}{24} = -\frac{7}{12}$ ft/sec.

When $x = 15$, $y = \sqrt{400} = 20$, $\frac{dy}{dt} = \frac{-2(15)}{20} = -\frac{3}{2}$ ft/sec.

When $x = 24$, $y = 7$, $\frac{dy}{dt} = \frac{-2(24)}{7} = -\frac{48}{7}$ ft/sec.

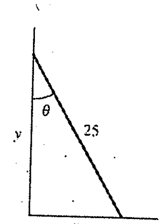
(b) $A = \frac{1}{2}xy$

$\frac{dA}{dt} = \frac{1}{2}\left(x\frac{dy}{dt} + y\frac{dx}{dt}\right)$

From part (a) you have $x = 7$, $y = 24$, $\frac{dx}{dt} = 2$, and $\frac{dy}{dt} = -\frac{7}{12}$. So,

$\frac{dA}{dt} = \frac{1}{2}\left[7\left(-\frac{7}{12}\right) + 24(2)\right] = \frac{527}{24}$ ft²/sec.

(c) $\tan\theta = \frac{x}{y}$



$\sec^2\theta\frac{d\theta}{dt} = \frac{1}{y}\cdot\frac{dx}{dt} - \frac{x}{y^2}\cdot\frac{dy}{dt}$

$\frac{d\theta}{dt} = \cos^2\theta\left[\frac{1}{y}\cdot\frac{dx}{dt} - \frac{x}{y^2}\cdot\frac{dy}{dt}\right]$

Using $x = 7$, $y = 24$, $\frac{dx}{dt} = 2$, $\frac{dy}{dt} = -\frac{7}{12}$ and $\cos\theta = \frac{24}{25}$, you have

$\frac{d\theta}{dt} = \left(\frac{24}{25}\right)^2\left[\frac{1}{24}(2) - \frac{7}{(24)^2}\left(-\frac{7}{12}\right)\right] = \frac{1}{12}$ rad/sec.

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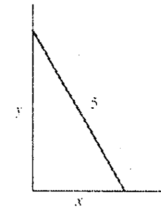
22. $x^2 + y^2 = 25$

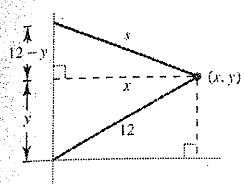
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dx}{dt} = -\frac{y}{x}\cdot\frac{dy}{dt} = -\frac{0.15y}{x} \quad \left(\text{because } \frac{dy}{dt} = 0.15\right)$$



When $x = 2.5$, $y = \sqrt{18.75}$, $\frac{dx}{dt} = -\frac{\sqrt{18.75}}{2.5}0.15 \approx -0.26$ m/sec.

23. When $y = 6$, $x = \sqrt{12^2 - 6^2} = 6\sqrt{3}$, and $s = \sqrt{x^2 + (12 - y)^2} = \sqrt{108 + 36} = 12$.



$$x^2 + (12 - y)^2 = s^2$$

$$2x\frac{dx}{dt} + 2(12 - y)(-1)\frac{dy}{dt} = 2s\frac{ds}{dt}$$

$$x\frac{dx}{dt} + (y - 12)\frac{dy}{dt} = s\frac{ds}{dt}$$

Also, $x^2 + y^2 = 12^2$.

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$$

So, $x\frac{dx}{dt} + (y - 12)\left(\frac{-x}{y}\frac{dx}{dt}\right) = s\frac{ds}{dt}$.

$$\frac{dx}{dt}\left[x - x + \frac{12x}{y}\right] = s\frac{ds}{dt} \Rightarrow \frac{dx}{dt} = \frac{sy}{12x}\cdot\frac{ds}{dt} = \frac{(12)(6)}{(12)(6\sqrt{3})}(-0.2) = \frac{-1}{5\sqrt{3}} = \frac{-\sqrt{3}}{15} \text{ m/sec (horizontal)}$$

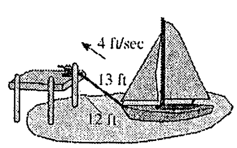
$$\frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt} = \frac{-6\sqrt{3}}{6}\cdot\frac{(-\sqrt{3})}{15} = \frac{1}{5} \text{ m/sec (vertical)}$$

24. Let L be the length of the rope.

(a) $L^2 = 144 + x^2$

$$2L\frac{dL}{dt} = 2x\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{L}{x}\cdot\frac{dL}{dt} = -\frac{4L}{x} \quad \left(\text{since } \frac{dL}{dt} = -4 \text{ ft/sec}\right)$$



When $L = 13$:

$$x = \sqrt{L^2 - 144} = \sqrt{169 - 144} = 5$$

$$\frac{dx}{dt} = -\frac{4(13)}{5} = -\frac{52}{5} = -10.4 \text{ ft/sec}$$

Speed of the boat increases as it approaches the dock.

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(b) If $\frac{dx}{dt} = -4$, and $L = 13$:

$$\frac{dL}{dt} = \frac{x}{L}\frac{dx}{dt} = \frac{5}{13}(-4) = \frac{-20}{13} \text{ ft/sec}$$

$$\frac{dL}{dt} = \frac{x}{L}\frac{dx}{dt} = \frac{\sqrt{L^2 - 144}}{L}(-4)$$

$$\lim_{L\to 12^+}\frac{dL}{dt} = \lim_{L\to 12^+}\frac{-4}{L}\sqrt{L^2 - 144} = 0$$

25. (a)

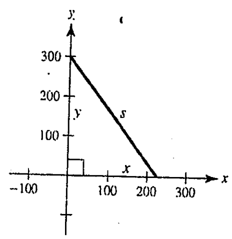
$$s^2 = x^2 + y^2$$

$$\frac{dx}{dt} = -450$$

$$\frac{dy}{dt} = -600$$

$$2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\frac{ds}{dt} = \frac{x(dx/dt) + y(dy/dt)}{s}$$



When $x = 225$ and $y = 300$, $s = 375$ and

$$\frac{ds}{dt} = \frac{225(-450) + 300(-600)}{375} = -750 \text{ mi/h.}$$

(b) $t = \frac{375}{750} = \frac{1}{2}\text{ h} = 30 \text{ min}$

26.

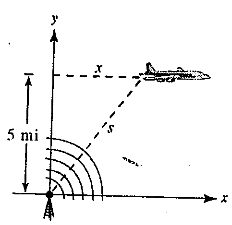
$$x^2 + y^2 = s^2$$

$$2x\frac{dx}{dt} + 0 = 2s\frac{ds}{dt} \qquad \left(\text{because } \frac{dy}{dt} = 0\right)$$

$$\frac{dx}{dt} = \frac{s}{x}\frac{ds}{dt}$$

When $s = 10$, $x = \sqrt{100 - 25} = \sqrt{75} = 5\sqrt{3}$,

$$\frac{dx}{dt} = \frac{10}{5\sqrt{3}}(240) = \frac{480}{\sqrt{3}} = 160\sqrt{3} \approx 277.13 \text{ mi/h.}$$



27.

$$s^2 = 90^2 + x^2$$

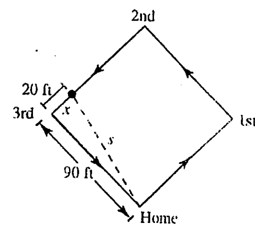
$$x = 20$$

$$\frac{dx}{dt} = -25$$

$$2s\frac{ds}{dt} = 2x\frac{dx}{dt} \Rightarrow \frac{ds}{dt} = \frac{x}{s}\cdot\frac{dx}{dt}$$

When $x = 20$, $s = \sqrt{90^2 + 20^2} = 10\sqrt{85}$,

$$\frac{ds}{dt} = \frac{20}{10\sqrt{85}}(-25) = \frac{-50}{\sqrt{85}} \approx -5.42 \text{ ft/sec.}$$



28.

$$s^2 = 90^2 + x^2$$

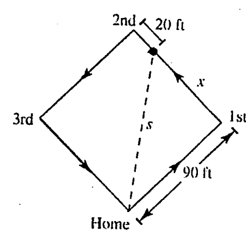
$$x = 90 - 20 = 70$$

$$\frac{dx}{dt} = 25$$

$$\frac{ds}{dt} = \frac{x}{s}\cdot\frac{dx}{dt}$$

When $x = 70$, $s = \sqrt{90^2 + 70^2} = 10\sqrt{130}$,

$$\frac{ds}{dt} = \frac{70}{10\sqrt{130}}(25) = \frac{175}{\sqrt{130}} \approx 15.35 \text{ ft/sec.}$$



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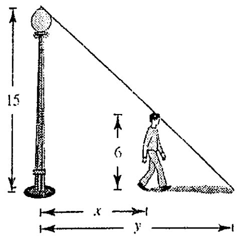
29. (a) $\frac{15}{6} = \frac{y}{y-x} \Rightarrow 15y - 15x = 6y$

$y = \frac{5}{3}x$

$\frac{dx}{dt} = 5$

$\frac{dy}{dt} = \frac{5}{3} \cdot \frac{dx}{dt} = \frac{5}{3}(5) = \frac{25}{3}$ ft/sec



(b) $\frac{d(y-x)}{dt} = \frac{dy}{dt} - \frac{dx}{dt} = \frac{25}{3} - 5 = \frac{10}{3}$ ft/sec

30. (a) $\frac{20}{6} = \frac{y}{y-x}$

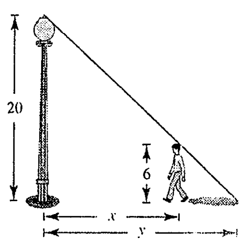
$20y - 20x = 6y$

$14y = 20x$

$y = \frac{10}{7}x$

$\frac{dx}{dt} = -5$

$\frac{dy}{dt} = \frac{10}{7}\frac{dx}{dt} = \frac{10}{7}(-5) = \frac{-50}{7}$ ft/sec



(b) $\frac{d(y-x)}{dt} = \frac{dy}{dt} - \frac{dx}{dt}$

$= \frac{-50}{7} - (-5)$

$= \frac{-50}{7} + \frac{35}{7} = \frac{-15}{7}$ ft/sec

31. $x(t) = \frac{1}{2}\sin\frac{\pi t}{6}, x^2 + y^2 = 1$

(a) Period: $\frac{2\pi}{\pi/6} = 12$ seconds

(b) When $x = \frac{1}{2}, y = \sqrt{1^2 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$ m.

Lowest point: $\left(0, \frac{\sqrt{3}}{2}\right)$

(c) When $x = \frac{1}{4}$,

$y = \sqrt{1 - \left(\frac{1}{4}\right)^2} = \frac{\sqrt{15}}{4}$ and $t = 1$:

$\frac{dx}{dt} = \frac{1}{2}\left(\frac{\pi}{6}\right)\cos\frac{\pi t}{6} = \frac{\pi}{12}\cos\frac{\pi t}{6}$

$x^2 + y^2 = 1$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$

So, $\frac{dy}{dt} = -\frac{1/4}{\sqrt{15}/4} \cdot \frac{\pi}{12}\cos\left(\frac{\pi}{6}\right)$

$= \frac{-\pi}{\sqrt{15}}\left(\frac{1}{12}\right)\frac{\sqrt{3}}{2} = \frac{-\pi}{24}\frac{1}{\sqrt{5}} = \frac{-\sqrt{5}\pi}{120}.$

Speed $= \left|\frac{-\sqrt{5}\pi}{120}\right| = \frac{\sqrt{5}\pi}{120}$ m/sec

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32. $x(t) = \frac{3}{5}\sin \pi t,\ x^2 + y^2 = 1$

(a) Period: $\frac{2\pi}{\pi} = 2$ seconds

(b) When $x = \frac{3}{5}$, $y = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$ m.

Lowest point: $\left(0, \frac{4}{5}\right)$

(c) When $x = \frac{3}{10}$, $y = \sqrt{1 - \left(\frac{1}{4}\right)^2} = \frac{\sqrt{15}}{4}$ and $\frac{3}{10} = \frac{3}{5}\sin \pi t \Rightarrow \sin \pi t = \frac{1}{2} \Rightarrow t = \frac{1}{6}$:

$$\frac{dx}{dt} = \frac{3}{5}\pi \cos \pi t$$

$$x^2 + y^2 = 1$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$$

$$\text{So, } \frac{dy}{dt} = \frac{-3/10}{\sqrt{15}/4} \cdot \frac{3}{5}\pi \cos\left(\frac{\pi}{6}\right)$$

$$= \frac{-9\pi}{25\sqrt{5}} = \frac{-9\sqrt{5}\pi}{125}.$$

$$\text{Speed} = \left|\frac{-9\sqrt{5}\pi}{125}\right| \approx 0.5058 \text{ m/sec}$$

33. Because the evaporation rate is proportional to the surface area, $dV/dt = k(4\pi r^2)$. However, because $V = (4/3)\pi r^3$, you have

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}.$$

Therefore, $k(4\pi r^2) = 4\pi r^2 \frac{dr}{dt} \Rightarrow k = \frac{dr}{dt}.$

34. (i) (a) $\frac{dx}{dt}$ negative $\Rightarrow \frac{dy}{dt}$ positive

(b) $\frac{dy}{dt}$ positive $\Rightarrow \frac{dx}{dt}$ negative

(ii) (a) $\frac{dx}{dt}$ negative $\Rightarrow \frac{dy}{dt}$ negative

(b) $\frac{dy}{dt}$ positive $\Rightarrow \frac{dx}{dt}$ positive

35.
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{dR_1}{dt} = 1$$

$$\frac{dR_2}{dt} = 1.5$$

$$\frac{1}{R^2} \cdot \frac{dR}{dt} = \frac{1}{R_1^2} \cdot \frac{dR_1}{dt} + \frac{1}{R_2^2} \cdot \frac{dR_2}{dt}$$

When $R_1 = 50$ and $R_2 = 75$:

$R = 30$

$$\frac{dR}{dt} = (30)^2\left[\frac{1}{(50)^2}(1) + \frac{1}{(75)^2}(1.5)\right] = 0.6 \text{ ohm/sec}$$

36.
$$pV^{1.3} = k$$

$$1.3pV^{0.3}\frac{dV}{dt} + V^{1.3}\frac{dp}{dt} = 0$$

$$V^{0.3}\left(1.3p\frac{dV}{dt} + V\frac{dp}{dt}\right) = 0$$

$$1.3p\frac{dV}{dt} = -V\frac{dp}{dt}$$

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37. $rg\tan\theta = v^2$

$32r\tan\theta = v^2$, r is a constant.

$$32r\sec^2\theta\frac{d\theta}{dt} = 2v\frac{dv}{dt}$$

$$\frac{dv}{dt} = \frac{16r}{v}\sec^2\theta\frac{d\theta}{dt}$$

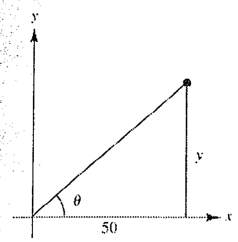
Likewise, $\dfrac{d\theta}{dt} = \dfrac{v}{16r}\cos^2\theta\dfrac{dv}{dt}$.

38. $\tan\theta = \dfrac{y}{50}$

$$\frac{dy}{dt} = 4 \text{ m/sec}$$

$$\sec^2\theta\cdot\frac{d\theta}{dt} = \frac{1}{50}\frac{dy}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{50}\cos^2\theta\cdot\frac{dy}{dt}$$



When $y = 50$, $\theta = \dfrac{\pi}{4}$, and $\cos\theta = \dfrac{\sqrt{2}}{2}$. So,

$$\frac{d\theta}{dt} = \frac{1}{50}\left(\frac{\sqrt{2}}{2}\right)^2(4) = \frac{1}{25}\text{ rad/sec.}$$

39. $\sin\theta = \dfrac{10}{x}$

$$\frac{dx}{dt} = (-1)\text{ ft/sec}$$

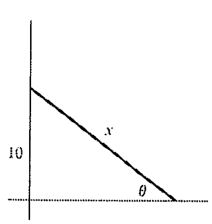
$$\cos\theta\left(\frac{d\theta}{dt}\right) = \frac{-10}{x^2}\cdot\frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \frac{-10}{x^2}\frac{dx}{dt}(\sec\theta)$$

$$= \frac{-10}{25^2}(-1)\frac{25}{\sqrt{25^2 - 10^2}}$$

$$= \frac{10}{25}\frac{1}{5\sqrt{21}} = \frac{2}{25\sqrt{21}}$$

$$= \frac{2\sqrt{21}}{525} \approx 0.017\text{ rad/sec}$$



40. $\tan\theta = \dfrac{y}{x}$, $y = 5$

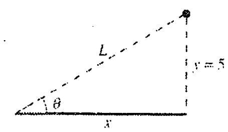
$$\frac{dx}{dt} = -600\text{ mi/h}$$

$$\left(\sec^2\theta\right)\frac{d\theta}{dt} = -\frac{5}{x^2}\cdot\frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \cos^2\theta\left(-\frac{5}{x^2}\right)\frac{dx}{dt} = \frac{x^2}{L^2}\left(-\frac{5}{x^2}\right)\frac{dx}{dt}$$

$$= \left(-\frac{5^2}{L^2}\right)\left(\frac{1}{5}\right)\frac{dx}{dt}$$

$$= \left(-\sin^2\theta\right)\left(\frac{1}{5}\right)(-600) = 120\sin^2\theta$$



(a) When $\theta = 30°$,

$$\frac{d\theta}{dt} = \frac{120}{4} = 30\text{ rad/h} = \frac{1}{2}\text{ rad/min.}$$

(b) When $\theta = 60°$,

$$\frac{d\theta}{dt} = 120\left(\frac{3}{4}\right) = 90\text{ rad/h} = \frac{3}{2}\text{ rad/min.}$$

(c) When $\theta = 75°$,

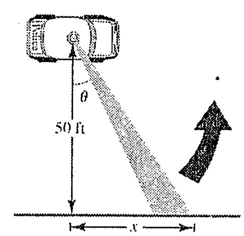
$$\frac{d\theta}{dt} = 120\sin^2 75° \approx 111.96\text{ rad/h} \approx 1.87\text{ rad/min.}$$

41. $\tan\theta = \dfrac{x}{50}$

$$\frac{d\theta}{dt} = 30(2\pi) = 60\pi\text{ rad/min} = \pi\text{ rad/sec}$$

$$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{50}\left(\frac{dx}{dt}\right)$$

$$\frac{dx}{dt} = 50\sec^2\theta\left(\frac{d\theta}{dt}\right)$$



(a) When $\theta = 30°$, $\dfrac{dx}{dt} = \dfrac{200\pi}{3}$ ft/sec.

(b) When $\theta = 60°$, $\dfrac{dx}{dt} = 200\pi$ ft/sec.

(c) When $\theta = 70°$, $\dfrac{dx}{dt} \approx 427.43\pi$ ft/sec.

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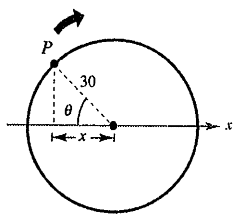
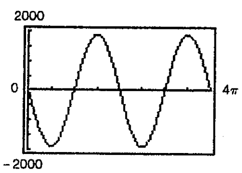
42. $\frac{d\theta}{dt} = (10 \text{ rev/sec})(2\pi \text{ rad/rev}) = 20\pi \text{ rad/sec}$

(a)
$$\cos\theta = \frac{x}{30}$$
$$-\sin\theta\frac{d\theta}{dt} = \frac{1}{30}\frac{dx}{dt}$$
$$\frac{dx}{dt} = -30\sin\theta\frac{d\theta}{dt}$$
$$= -30\sin\theta(20\pi)$$
$$= -600\pi\sin\theta$$



(b)



(c) $|dx/dt| = |-600\pi\sin\theta|$ is greatest when

$|\sin\theta| = 1 \Rightarrow \theta = \frac{\pi}{2} + n\pi$ (or $90° + n \cdot 180°$).

$|dx/dt|$ is least when $\theta = n\pi$ (or $n \cdot 180°$).

(d) For $\theta = 30°$,

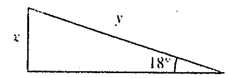
$$\frac{dx}{dt} = -600\pi\sin(30°) = -600\pi\frac{1}{2} = -300\pi \text{ cm/sec.}$$

For $\theta = 60°$,

$$\frac{dx}{dt} = -600\pi\sin(60°)$$
$$= -600\pi\frac{\sqrt{3}}{2} = -300\sqrt{3}\pi \text{ cm/sec.}$$

43. $\sin 18° = \frac{x}{y}$

$$0 = -\frac{x}{y^2}\cdot\frac{dy}{dt} + \frac{1}{y}\cdot\frac{dx}{dt}$$
$$\frac{dx}{dt} = \frac{x}{y}\cdot\frac{dy}{dt} = (\sin 18°)(275) \approx 84.9797 \text{ mi/hr}$$



44. $\tan\theta = \frac{x}{50} \Rightarrow x = 50\tan\theta$

$$\frac{dx}{dt} = 50\sec^2\theta\frac{d\theta}{dt}$$
$$2 = 50\sec^2\theta\frac{d\theta}{dt}$$
$$\frac{d\theta}{dt} = \frac{1}{25}\cos^2\theta, \quad -\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$$

45. (a) $dy/dt = 3(dx/dt)$ means that y changes three times as fast as x changes.

(b) y changes slowly when $x \approx 0$ or $x \approx L$. y changes more rapidly when x is near the middle of the interval.

46. $x^2 + y^2 = 25$; acceleration of the top of the ladder $= \frac{d^2y}{dt^2}$

First derivative: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$

Second derivative: $x\frac{d^2x}{dt^2} + \frac{dx}{dt}\cdot\frac{dx}{dt} + y\frac{d^2y}{dt^2} + \frac{dy}{dt}\cdot\frac{dy}{dt} = 0$

$$\frac{d^2y}{dt^2} = \left(\frac{1}{y}\right)\left[-x\frac{d^2x}{dt^2} - \left(\frac{dx}{dt}\right)^2 - \left(\frac{dy}{dt}\right)^2\right]$$

When $x = 7$, $y = 24$, $\frac{dy}{dt} = -\frac{7}{12}$, and $\frac{dx}{dt} = 2$ (see Exercise 25). Because $\frac{dx}{dt}$ is constant, $\frac{d^2x}{dt^2} = 0$.

$$\frac{d^2y}{dt^2} = \frac{1}{24}\left[-7(0) - (2)^2 - \left(-\frac{7}{12}\right)^2\right] = \frac{1}{24}\left[-4 - \frac{49}{144}\right] = \frac{1}{24}\left[-\frac{625}{144}\right] \approx -0.1808 \text{ ft/sec}^2$$

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47. $L^2 = 144 + x^2$; acceleration of the boat $= \dfrac{d^2x}{dt^2}$

First derivative: $2L\dfrac{dL}{dt} = 2x\dfrac{dx}{dt}$

$$L\frac{dL}{dt} = x\frac{dx}{dt}$$

Second derivative: $L\dfrac{d^2L}{dt^2} + \dfrac{dL}{dt}\cdot\dfrac{dL}{dt} = x\dfrac{d^2x}{dt^2} + \dfrac{dx}{dt}\cdot\dfrac{dx}{dt}$

$$\frac{d^2x}{dt^2} = \left(\frac{1}{x}\right)\left[L\frac{d^2L}{dt^2} + \left(\frac{dL}{dt}\right)^2 - \left(\frac{dx}{dt}\right)^2\right]$$

When $L = 13, x = 5, \dfrac{dx}{dt} = -10.4$, and $\dfrac{dL}{dt} = -4$ (see Exercise 28). Because $\dfrac{dL}{dt}$ is constant, $\dfrac{d^2L}{dt^2} = 0$.

$$\frac{d^2x}{dt^2} = \frac{1}{5}\left[13(0) + (-4)^2 - (-10.4)^2\right]$$

$$= \frac{1}{5}[16 - 108.16] = \frac{1}{5}[-92.16] = -18.432 \text{ ft/sec}^2$$

48. (a) Using a graphing utility,

$m(s) = -1.24449\,s^3 + 72.7661\,s^2 - 1416.428\,s + 9215.21.$

(b) $\dfrac{dm}{dt} = \left(-3.73347\,s^2 + 145.5322\,s - 1416.428\right)\dfrac{ds}{dt}$

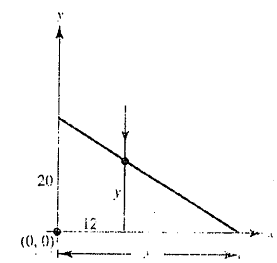
If $\dfrac{ds}{dt} = 0.75$ and $t = 7$, then $s = 19.7$ and $\dfrac{dm}{dt} \approx 1.23$ million/year.

49. $y(t) = -4.9t^2 + 20$

$\dfrac{dy}{dt} = -9.8t$

$y(1) = -4.9 + 20 = 15.1$

$y'(1) = -9.8$



By similar triangles: $\dfrac{20}{x} = \dfrac{y}{x - 12}$

$$20x - 240 = xy$$

When $y = 15.1$: $20x - 240 = x(15.1)$

$$(20 - 15.1)x = 240$$

$$x = \frac{240}{4.9}$$

$$20x - 240 = xy$$

$$20\frac{dx}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{x}{20 - y}\frac{dy}{dt}$$

At $t = 1$, $\dfrac{dx}{dt} = \dfrac{240/4.9}{20 - 15.1}(-9.8) \approx -97.96$ m/sec.

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