THE 2015–2016 KENNESAW STATE UNIVERSITY
HIGH SCHOOL MATHEMATICS COMPETITION

PART I – MULTIPLE CHOICE

For each of the following 25 questions, carefully blacken the appropriate box on the answer sheet with a #2 pencil. Do not fold, bend, or write stray marks on either side of the answer sheet. Each correct answer is worth 6 points. Two points are given if no box is marked. Zero points are given for an incorrect answer or if multiple boxes are marked. Note that wild guessing is likely to lower your score. When the exam is over, give your answer sheet to your proctor. You may keep your copy of the questions.

NO CALCULATORS

90 MINUTES

1. Between 1934 and 2015 there were 13 different presidents of the United States and 16 different vice presidents. If 11 of the vice presidents were never president, how many of the presidents were never vice president?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

2. In the addition "cryptarithm" at the right, each letter represents one of the digits from 0 to 9 (different letters represent different digits). What is the <u>smallest</u> possible value for the four digit number F I V E.

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  F O U R
+   O N E
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  F I V E
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(A) 1345 (B) 1475 (C) 1486 (D) 1627 (E) 1648

3. A group of boys formed a secret club. For dues, each boy paid as many dollars as there were boys in the club. When four more boys joined the club, these four boys each paid as many dollars as there were boys now in the club. After this, no more boys joined, and the club reported \$301 in its treasury. How many boys are now in the club?

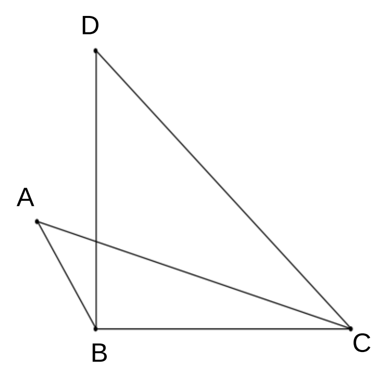
(A) 15 (B) 16 (C) 17 (D) 18 (E) 19

4. Suppose a six-sided die has sides numbered one through six. If a person throws the die two times, what is the probability that the second number will be larger than the first?

(A) $\frac{1}{6}$ (B) $\frac{1}{2}$ (C) $\frac{5}{12}$ (D) $\frac{7}{18}$ (E) None of these

5. In the diagram, $\angle ABD \cong \angle DCA$ and $\overline{BD}$ is perpendicular to $\overline{BC}$. If the measure of $\angle DCB$ is 50°, what is the measure of $\angle A$?

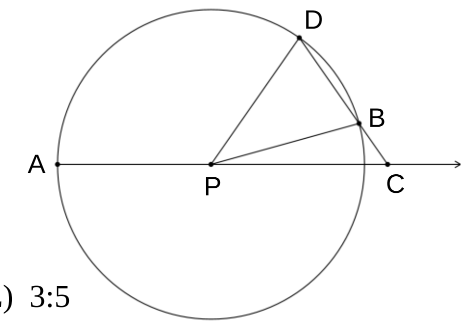
(A) 25° (B) 30° (C) 40° (D) 45° (E) 50°



6. There are two four-digit numbers, each of the form $\underline{A}\,\underline{B}\,\underline{C}\,\underline{A}$, with the property that the two-digit number $\underline{A}\,\underline{B}$ is a prime, the two-digit number $\underline{B}\,\underline{C}$ is a square, and the two-digit number $\underline{C}\,\underline{A}$ is the product of a prime and a square greater than 1. Compute the sum of these two four-digit numbers.

(A) 10,657 (B) 11,531 (C) 12,185 (D) 13,729 (E) 14,363

7. In the circle shown with center P, radius $\overline{AP}$ is extended to point C outside the circle and point D is chosen on the circle in such a way that $\overline{DC}$ intersects the circle at point B and $\overline{DC}$ is equal in length to the radius of the circle. Compute the ratio of the measure of $\angle CPD$ to the measure of $\angle APB$.



(A) 1:2 (B) 1:3 (C) 1:4 (D) 2:5 (E) 3:5

8. The *midrange* of a set of numbers is the average of the greatest value and least value in the set. For a set of six increasing, nonnegative integers, the mean, the median, and the midrange are all 5. How many such sets are there?

(A) 8 (B) 9 (C) 10 (D) 12 (E) None of these

9. On Dr. Garner's last math test, the mean score achieved by 75% of a class was 5 points lower than the mean of the whole class. The mean of the remaining students was how many points above the class mean?

(A) 15 (B) 12 (C) 9 (D) 8 (E) 6

10. The sum of 28 consecutive positive odd integers is a perfect cube. What is the smallest possible first number in such a set of 28 numbers?

(A) 59 (B) 61 (C) 67 (D) 69 (E) 71

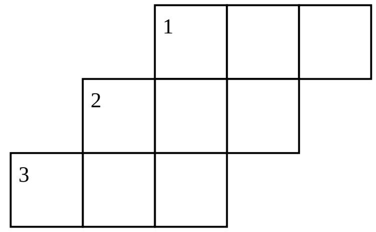
11. Don and Debbie have a total of \$6.07 consisting of pennies, nickels, and quarters. Don has only quarters and nickels, and Debbie has only quarters and pennies. Don has seven times as many nickels as Debbie has pennies. What is the total number of coins that Don and Debbie have?

(A) 93 (B) 97 (C) 103 (D) 105 (E) 109

12. *ABCD* is a square of side length 1. *EFGH* is a square that has one vertex on each side of *ABCD*. If the sides of *EFGH* make an angle θ with the sides of *ABCD*, then the area of *EFGH* is

(A) $\dfrac{1}{4\sin\theta\cos\theta}$ (B) $\dfrac{1}{1+\sin 2\theta}$ (C) $\dfrac{1}{\tan\theta+\cot\theta}$ (D) $\dfrac{1}{2}$ (E) $\dfrac{1}{4\cos^2\theta}$

13. Each integer from 1 to 9 is entered exactly once in the "cross-number" puzzle shown in such a way that the three-digit numbers appearing in 1-across, 2-across, 3-across, and 1-down are perfect squares. Compute the two-digit number appearing in 2-down.



(A) 76 (B) 58 (C) 52 (D) 36 (E) 12

14. The solutions to the equation $x^3 + ax^2 + bx + c = 0$ are three consecutive positive integers, compute the value of $\frac{a^2}{b+1}$.

(A) 2 (B) 3 (C) $\frac{16}{5}$ (D) $\frac{5}{2}$ (E) None of these

15. Two sides of a triangle have lengths of 3 inches and 5 inches, and the area of the triangle is 6 square inches. The triangle, however, is not a right triangle. If the number of inches in the length of the third side of this triangle is $\sqrt{k}$, compute *k*.

(A) 40 (B) 48 (C) 52 (D) 54 (E) 60

16. The first two terms of a geometric sequence are *i* and *i* + 1 (where $i = \sqrt{-1}$), in that order. Compute the value of the fifteenth term.

(A) –64 (B) 64*i* + 64 (C) –64*i* – 64 (D) –128 (E) 128*i* –128

17. If *a*, *b*, *c* are distinct positive integers and *a* + *b* + *c* = 2015 and *ab* – *c* = 2015, compute the value of *c*.

(A) 529 (B) 729 (C) 1,089 (D) 1,369 (E) 1,849

18. Suppose $a = 2^{(\sqrt{n})!}$, $b = 2^{2^n}$, and $c = (2^n)!$ where *n* is a perfect square greater than 1,000,000. From largest to smallest, which of the following is the correct order for *a*, *b*, and *c*?

(A) *a*, *b*, *c* (B) *b*, *c*, *a* (C) *c*, *b*, *a* (D) *c*, *a*, *b* (E) *b*, *a*, *c*

19. A parallelogram has sides of length 4 and 6. The length of one of its diagonals is 8. If the length of the other diagonal is $\sqrt{k}$, what is the value of *k*?

(A) 20 (B) 24 (C) 28 (D) 32 (E) 40

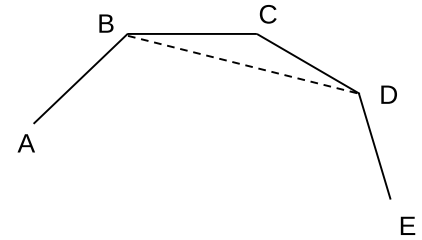
20. If $f(11) = 11$, and for all x, $f(x + 3) = \dfrac{f(x)-1}{f(x)+1}$, compute $f(2015)$.

(A) 11 (B) $-\frac{1}{11}$ (C) $\frac{5}{6}$ (D) $-\frac{6}{5}$ (E) –11

21. Let A be a two-digit integer and let B be the integer obtained by reversing the digits of A. If $A^2 - B^2$ is the square of an integer, compute $A^2 + B^2$.

(A) 4,941 (B) 5,265 (C) 5,913 (D) 6,885 (E) 7,361

22. Shown in the accompanying diagram is part of a regular polygon (ABCDE…) of unspecified number of sides $n > 4$. Another regular polygon having $\overline{BD}$ as one side and angles ABD and EDB as consecutive angles is drawn. Which of the following is a possible value of n?



(A) 55 (B) 56 (C) 57 (D) 58 (E) 59

23. What is the ninth digit from the right in the value of 101^{20}?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

24. If $\log_{\sin x}(\tan x) = \log_{\tan x}(\sin x)$, compute the value of $\cos x$.

(A) $\frac{-1+\sqrt{2}}{2}$ (B) $\frac{1-\sqrt{3}}{2}$ (C) $\frac{-1+\sqrt{3}}{2}$ (D) $\frac{-1+\sqrt{5}}{2}$ (E) $\frac{1-\sqrt{5}}{2}$

25. A triangle has vertices A(0,0), B(3,0), and C(3,4). If ΔABC is rotated counterclockwise around the origin until point C lies on the positive y-axis, compute the area of the region common to the original triangle and the rotated triangle.

(A) $\frac{21}{16}$ (B) $\frac{25}{16}$ (C) $\frac{29}{16}$ (D) $\frac{35}{16}$ (E) None of these