$$(x^2+2)^2 \int \left[\frac{8x^3+13x}{(x^2+2)^2} = \frac{Ax+B}{x^2+2} + \frac{Cx+D}{(x^2+2)^2} \right]$$

$$8x^3+13x = (Ax+B)(x^2+2) + Cx + D$$

$$= Ax^3 + Bx^2 + 2Ax + 2B + Cx + D$$

$$8x^3 + 0x^2 + 13x + 0 = Ax^3 + Bx^2 + (2A+C)x + \boxed{2B+D}$$

$A=8$ $\quad 2A+C=13 \quad 2(8)+C=13 \quad C=-3$

$B=0 \quad 2B+D=0 \quad 2(0)+D=0 \quad D=0$

$$\int \frac{8x+0}{x^2+2}dx + \int \frac{-3x+0}{(x^2+2)^2}dx$$

$u=x^2+2$

$\frac{du}{dx}=2x$

$dx = \frac{du}{2x}$

$$\int \frac{8\cancel{x}}{u} \cdot \frac{du}{2\cancel{x}}$$

$$4\int \frac{du}{u}$$

$u=x^2+2 \quad \frac{du}{dx}=2x \quad dx=\frac{du}{2x}$

$$\int \frac{-3\cancel{x}}{u^2} \cdot \frac{du}{2\cancel{x}} = -\frac{3}{2}\int u^{-2}du = -\frac{3}{2}\frac{u^{-1}}{-1}$$

$$\boxed{4\ln|x^2+2| + \frac{3}{2(x^2+2)} + C}$$

$$x(x+1)^2\left[\frac{5x^2+20x+6}{x(x+1)(x+1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}\right]$$

$$5x^2+20x+6 = A(x+1)^2 + B(x)(x+1) + Cx$$

$\underline{\underline{x=0}}$ $\quad 0+0+6 = A(0+1)^2 + \cancel{B(0)} + \cancel{C(0)}$

$$\boxed{6 = A}$$

$$5x^2+20x+6 = 6(x+1)^2 + Bx(x+1) + Cx$$

$\underline{\underline{x=-1}}$ $\quad +5-20+6 = \cancel{6(0)^2} + B\cancel{(-1)(0)} + C(-1)$

$$-9 = -C \quad \boxed{C=9}$$

$$5x^2+20x+6 = 6(x+1)^2 + Bx(x+1) + 9x$$

$\underline{\underline{x=1}}$ $\quad 5+20+6 = 6(2)^2 + B(2) + 9$

$$31 = 33+2B \quad -2=2B \quad \boxed{B=-1}$$

$$\int \frac{6}{x} + \frac{-1}{x+1} + \frac{9}{(x+1)^2}\,dx$$

$$9\int (x+1)^{-2}\,dx$$

$u=x+1$

$\frac{du}{dx}=1$

$$9\int u^{-2}\,du$$

$$9\frac{u^{-1}}{-1}$$

$$\boxed{6\ln|x| - \ln|x+1| - \frac{9}{(x+1)} + C}$$

$$x(x-1)(x^2+4)\left[\frac{2x^3-4x-8}{x(x-1)(x^2+4)} = \frac{A}{x} + \frac{B}{x-1} + \frac{Cx+D}{x^2+4}\right]$$

$$2x^3-4x-8 = A(x-1)(x^2+4) + Bx(x^2+4) + (Cx+D)(x)(x-1)$$

$\underline{\underline{x=0}}$ $0-0-8 = A(-1)(4) + B\cancel{(0)} + \cancel{(Cx+D)}(0)$

$-8 = -4A$ $\boxed{A=2}$

$2x^3-4x-8 = 2(x-1)(x^2+4) + Bx(x^2+4) + (Cx+D)(x)(x-1)$

$\underline{\underline{x=1}}$ $2-4-8 = 2\cancel{(0)}(x^2+4) + B(5) + \cancel{(Cx+D)}(0)$

$-10 = 5B$ $\boxed{B=-2}$

$\underline{\underline{x=-1}}$ $2x^3-4x-8 = 2(x-1)(x^2+4) - 2x(x^2+4) + (Cx+D)(x)(x-1)$

$-2+4-8 = 2(-2)(5) + 2(5) + [-C+D](-1)(-2)$

$-6 = -20 + 10 - 2C + 2D$

$4 = -2C + 2D \rightarrow \underline{2 = -C + D}$

$\underline{\underline{x=2}}$ $2(2)^3 - 4(2) - 8 = 2(1)(8) - 2(2)(8) + [2C+D]2$

$16 - 8 - 8 = 16 - 32 + 4C + 2D$

$0 = -16 + 4C + 2D$ $\quad 16 = 4C + 2D$

$\underline{8 = 2C + D}$

$2(2 = -C + D)$ $\quad 4 = -2C + 2D$

$8 = 2C + D$ $\quad 8 = 2C + D$

$12 = 3D$ $\boxed{D=4}$ $\boxed{C=2}$

$$\int \frac{2}{x} + \frac{-2}{x-1} + \frac{2x+4}{x^2+4}\,dx = \int \frac{2}{x} - \frac{2}{x-1} + \frac{2x}{x^2+4} + \frac{4}{x^2+4}\,dx$$

$$\boxed{2\ln|x| - 2\ln|x-1| + \ln|x^2+4| + \frac{4}{2}\arctan\left(\frac{x}{2}\right) + C}$$

$u = x^2+4$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$2\int \frac{1x}{u}\cdot\frac{du}{2x}$

$$\int \frac{4}{(x)^2+(2)^2}\,dx$$